Vehicle Routing Problem with Inventory Management and Selective Delivery vs Fixed Route CVRP

Keaoleboga Thubisi, Boikanyo Molele, Ibrahim Dan-Dije, Pia Rielly, Ayanda Ndimande, Mazwi Masango, Nompumelelo Ndlovu and Tshepang Patose

Supervisors:
Professor Montaz Ali, Dr Keegan Anderson

Industry Representative:
Dr Ian Campbell

University of the Witwatersrand

Outline

1 Introduction

2 Models & Results

3 Conclusion

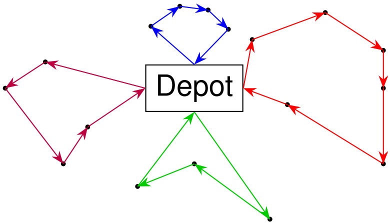
Vehicle Routing Problem (VRP)

Goal: Plan routes for a fleet to serve customers while minimizing cost or time.

Constraints: Vehicle capacity, delivery time windows, service levels.

Variants:

- Capacitated VRP (CVRP)
- VRP with Time Windows (VRPTW)
- Inventory Routing Problem (IRP)
- Stochastic VRP
- Capacitated Vehicle Routing Problem with Inventory Management and Selective Delivery (CVRPIMSD)



Our Model

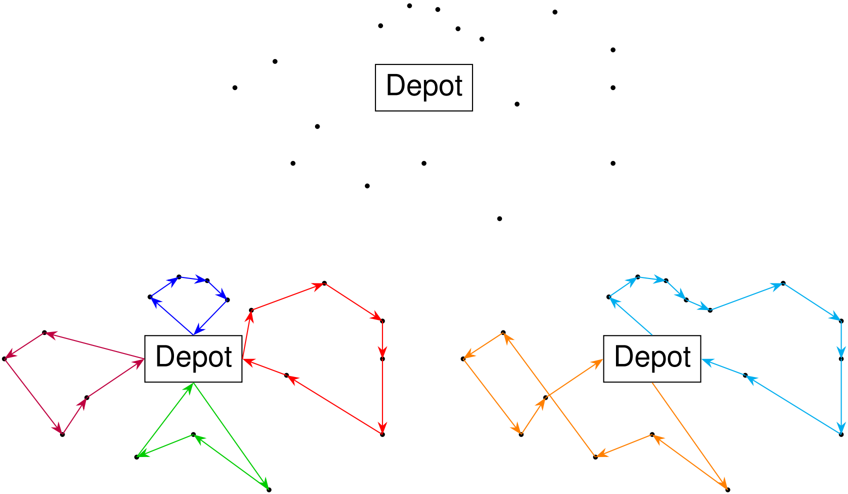
CVRPIMSD

- **Customers:** stores with deterministic or stochastic demand, arriving in the warehouse daily.
- **Demand:**
  - Deterministic or stochastic weekly demand, split across SLA delivery days.
  - Demand is heterogenous i.e. distinct customer demands considered distiguishable.
- **Vehicles:** Homogenous fleet with capacity limits; extra vehicles can be added at cost.
- **Routing Options:**
  - Fixed master-routes, triggered by SLA days.
  - Variable routes determined daily by SLA days and stock-accumulation.
  - Each routing option associated with a turn-around time parameter.

Key Research Objective

How much should turnaround time decrease to make fixed routes more profitable than variable routes?

Optimal versus Fixed Routes



Deterministic Model

Modelling Assumptions

- 60 customers classified by weekly demand:
  - 1–2 cubes: 1 visit/week (e.g. Day 3)
  - 3–4 cubes: 2 visits/week (e.g. Day 2, 4)
  - 5 cubes: 3 visits/week (e.g. Day 1, 3, 5)
- Demand per visit:

$$\text{Delivery per visit} = \text{Weekly demand}/\text{Number of visits}$$

- Execution Policy:
  - Inbound Supply triggered before each delivery day
  - Incoming stock dispatched immediately
  - Route triggered by schedule
- Fleet & Routing:
  - Feasibility: Travel Time $+$ Stoppage Time $\leq$ 8 h
  - Capacity: $\sum_{\text{stores}}$ demand at each store $\leq$ vehicle capacity
  - Extra vehicles added if daily demand exceeds available capacity

Stochastic Model

Modelling Assumptions

- Weekly demand: $D_{\text{week}} \sim \mathcal{N}(\mu_{\text{week}}, \sigma^2_{\text{week}})$
- Daily demand via multinomial split:

$$(d_1, \ldots, d_n) \sim \text{Multinomial}(D_{\text{week}}, p), \quad \sum_{i=1}^{n} p_i = 1$$

- Properties: $\sum_{i=1}^{n} d_i = D_{\text{week}}, \quad \mathbb{E}[d_i] = D_{\text{week}}\, p_i$
- Execution Policy:
  - Stock ages in warehouse; Daily demand arrives each morning
  - Stock dispatched on active SLA days; Expiry possible
- Routing & Inventory:
  - Daily demand packed FIFO
  - Feasibility: Travel Time $+$ Stoppage Time $\leq$ 8 h
  - Capacity: $\sum_{\text{stores}}$ demand at each store $\leq$ vehicle capacity

Optimal Route Model

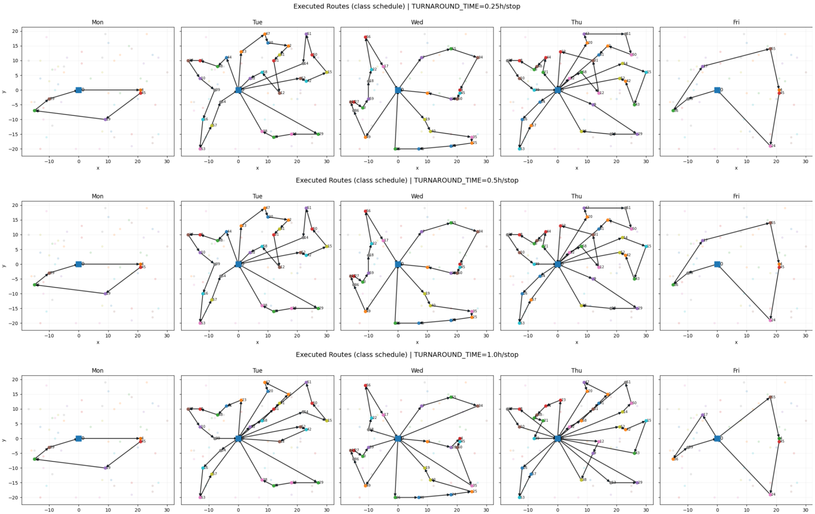
Stochastic Demand

Optimal Route Distribution Process

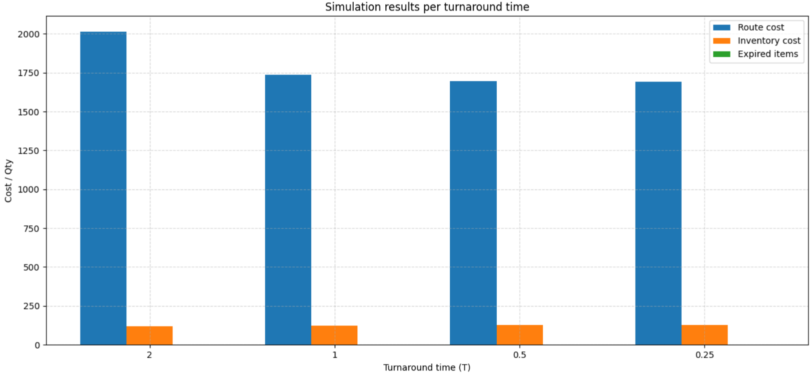
- Demand $\rightarrow$ Multinomial split per shop
- Shop *i* is active if
  - Inventory_shop $i > 0$
  - Scheduled for that day
- Active shops are clustered, and the shortest route per cluster is found
- Vehicles are then loaded and dispatched with extra vehicles at an extra cost.
- Routes optimized every day

Optimal Route Results

Daily Routes for Different Turnaround Times



Optimal Route Results: Cost vs Turnaround Time



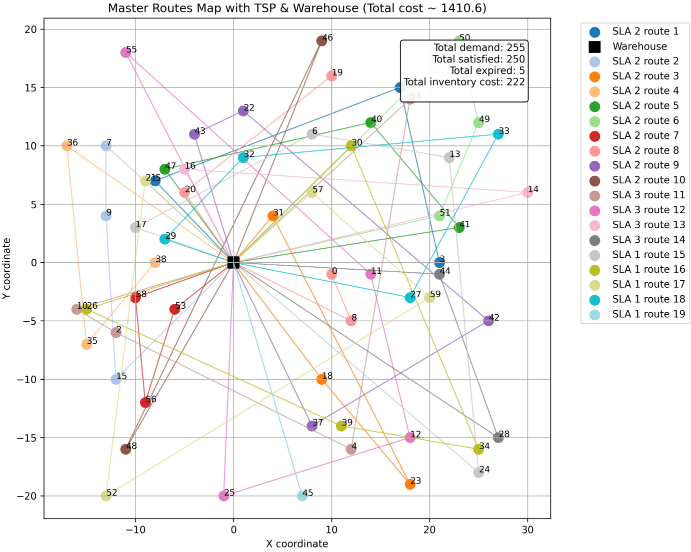
Fixed Route Model

Stochastic Demand

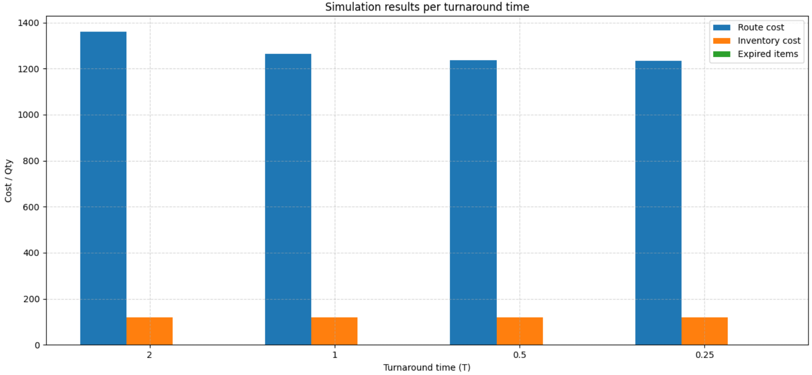
Fixed Route Distribution Process

- Demand $\rightarrow$ Multinomial split per shop
- Shop *i* is active if
    - Scheduled for that day
- All shops grouped by SLA category and each group clustered by TSP.
- Vehicles are loaded and dispatched according to weekly schedules.
- Routes remain fixed.

Fixed Route Results: Weekly Master Routes



Fixed Route Results: Cost vs Turnaround Time



Observations

- **Turnaround time is a dominant driver of feasibility and cost.**
- **Fixed routes trade flexibility for operational simplicity.**
- **Cost differences between fixed and variable routing expected to diverge as turnaround time increases.**
- **Increasing turn-around time further restricts route optimisation, and can result in needing extra vehicles at higher cost.**

Conclusion

- **What have we done?**
  Developed and simulated a CVRPIMSD, comparing fixed master routes with daily re-optimised routing under stochastic demand and explicit turnaround-time constraints.
- **What have we learned?**
  Fixed routing performs well under fixed SLA schedules, while variable routing becomes competitive at higher demand levels or under greater demand variability. Results suggest a possible turnaround-time threshold, below which further reductions have limited impact on routing costs.
- **Have we answered the research question?**
  The study provides initial indicative evidence on the role of turnaround time in fixed versus variable routing, offering a useful contribution while highlighting directions for further investigation.

Future Work

- Model turn-around times as stochastic or store-dependent.
- Optimise delivery frequencies jointly with routing and inventory decisions (flexible SLA days).
- Incorporate inventory holding, shortage, and expiry costs explicitly.
- Allow multiple trips per vehicle and driver-hour constraints.
- Introduce stochastic or time-dependent travel times.

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References I

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Thank You!

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We appreciate your attention