Inventory Optimization Problem with Two Conflicting Objectives

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Outline

Minimize $f(x, \xi)$ such that (A)

$$x \in X \subseteq \mathbb{R}^n$$
$$x \in \mathbb{R}^n,\ \xi \in \mathbb{R}^m.$$

Maximize $g(x, \xi)$ such that (B)

$$x \in X \subseteq \mathbb{R}^n$$
$$x \in \mathbb{R}^n,\ \xi \in \mathbb{R}^m.$$

Problem (A) and (B) Are Conflicting in Nature

Problem: (Minimize $f(x, \xi)$, Maximize $g(x, \xi)$)

- Customers Arrive According to a Poisson Process at a Single Queue, Single Service Channel (Retailer)
- Customer Demand and Replenishment Lead Time are Random

- The Deterministic Decision Variables are Reorder Point r and Reorder Quantity Q, both Discrete, $x^T = (r, q) \in \mathbb{R}^2$
- Random Vector Variable $\xi^T = (\xi_1, \xi_2, \xi_3)$, $\xi_1 \sim$ Poisson (Customer), $\xi_2 \sim$ Weibull (Demand), $\xi_3 \sim$ Replenishment Lead Time Distribution

- Service level (maximized $g(x, \xi)$) and total Inventory Cost (minimized $f(x, \xi)$).

The Cost Functions

- Inventory Holding Cost: $f(x, \xi)$
- Service Level: $g(x, \xi)$

$$\text{Service level} = \frac{\text{Number of customers serviced}}{\text{Number of customers requiring service}}$$

Table: Ranges of r and Q

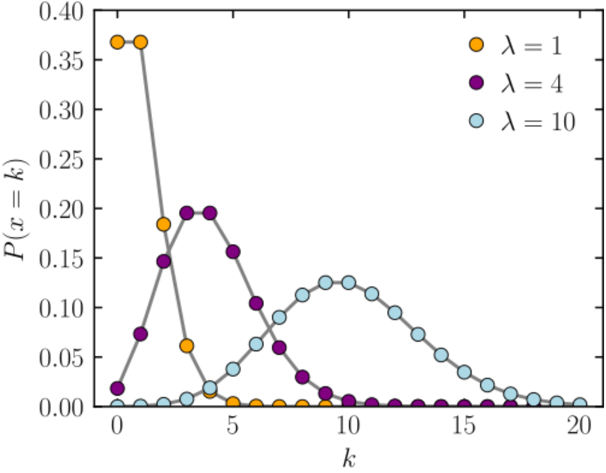
Variables	Min	Max
r	100	400
Q	100	400

Time Delayed Differential Equation

$$\frac{dI}{dt} = P(t - \tau) - D(t) \tag{1}$$

- Probability(Lead time=1)=0.25; Probability(Lead time=2)=0.5; Probability(Lead time=3)=0.25.
- Discrete Uniform Lead Time in $\{1, 2, 3\}$.

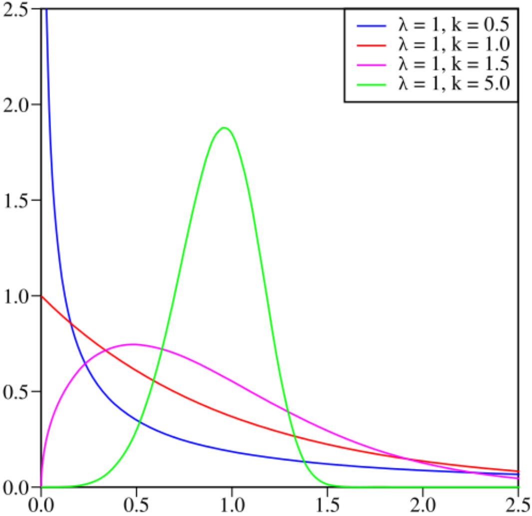
- Poission Distribution $f(k, \lambda) = P\{X = k\} = \frac{\lambda^k e^{-\lambda}}{k!}$



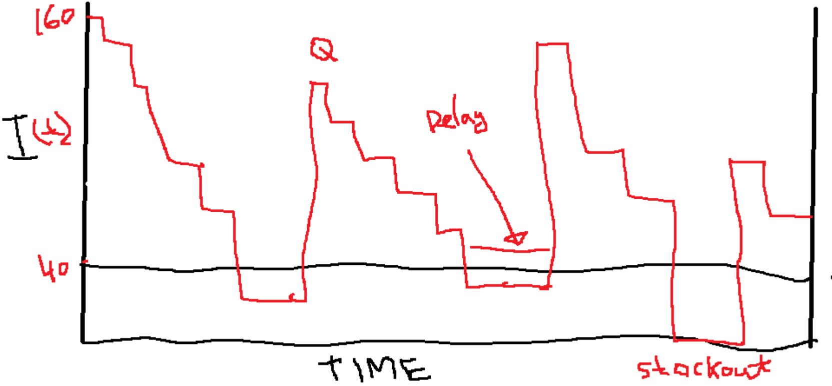
- Exponential Distribution $f(t) = \lambda e^{-\lambda t}$

Weibull Distribution

$$F(x) = 1 - e^{-(x/\lambda)^k}$$



The Simulation Process



Thank You!