Little penguins vs Fishermen

Graduate Modeling Camp Programme

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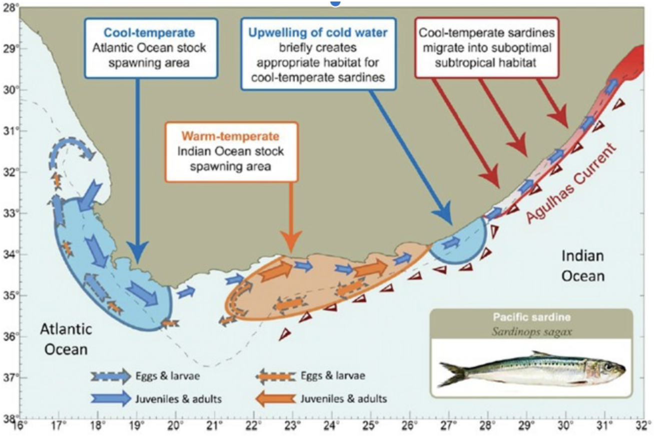
Overview

Sardine Run



The spawning area in the Atlantic Ocean (blue) is dominated by cool-temperate sardines and the spawning area in the Indian Ocean (orange) is dominated by warm-temperate sardines. Upwelling of cold waters on the southeast coast attracts cool-temperate sardines present on the south coast, which then move northward as part of the Sardine Run. When the upwelling ceases, these sardines eventually find themselves in an ecological trap of

Penguin's life cycle

Penguins have a fascinating life cycle that varies slightly among species, but generally includes the following stages:

1. **Nesting and Egg Laying**
   Penguins typically lay one or two eggs, depending on the species. The eggs are incubated by one or both parents, with incubation periods averaging 40 days [Robinson et al., 2015].
2. **Fledgling**
   Once their juvenile feathers grow in, chicks are ready to leave the nest or colony and fend for themselves. Penguins typically take several months to reach this stage, with an average duration of 2 months [Sherley et al., 2020].

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3. **Juvenile Stage**
   After fledging, young penguins often spend 1–2 years at sea, feeding and growing before returning to land to molt and eventually breed. Penguins typically take around 2 years to reach this stage.
4. **Moulting**
   Penguins undergo an annual moult, during which they shed old feathers and grow new ones. While moulting, they remain on land and cannot swim or hunt, relying on stored body fat for survival. The moulting period typically lasts around 3 weeks.
5. **Adult**
   The adult stage is reached when a penguin attains sexual maturity and develops its full adult plumage. Penguins generally take several years to reach this stage, with an average lifespan of up to 15 years in the wild.

Aim of the study

- Quantify the effect of the fishing industry on population numbers.
- Quantify the effect of sardine supply (the sardine run, ect) on the penguin population.

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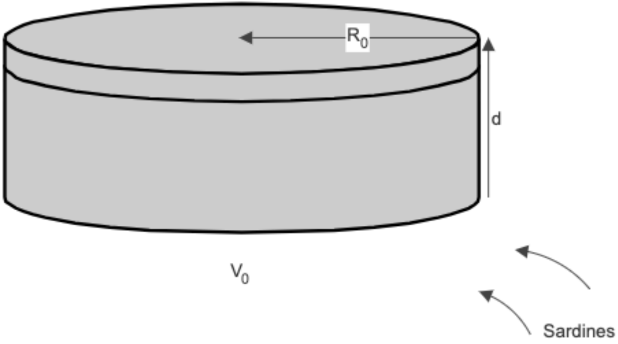

R_0 =45 KM
d = 20m

We formulate a mathematical model to describe the dynamic interactions between penguins and sardines. The population variables in the model are defined as follows:

- N_p: penguin population.
- S: sardine biomass.

Model assumptions

The following assumptions are made for our model:

1. **Constant source of sardines**
   There is a consistent replenishment of sardines from an external source or natural reproduction, modeled as $R(t) + \alpha_s S$, a constant term.
2. **Penguin population on sardines:**
   Penguins hunt sardines at a rate proportional to the product of the sardine biomass $S(t)$ and the penguin biomass $N_p(t)$, represented by $\beta_s SN$, where β_s is the predation rate coefficient.

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3. **Sardine losses**
   Sardines experience losses due to fishing, natural death, and predation by other species (e.g., sharks), represented by $(F(t) + L(t))S$, where $F(t) + L(t)$ is the rate of sardine depletion from these combined effects.
4. **Food supply for penguins**
   Penguin population growth is directly dependent on the availability of sardines as a food source, modeled by $\beta_p SN$, where β_p also influences penguin reproduction or survival rates through adequate food consumption.
5. **Natural death of penguins**
   Penguins experience a natural death rate that is proportional to their population size, represented by γN_p, where γ is the penguin mortality rate.

Un-scaled model

Based on the assumptions outlined above, we can formulate our model as follows:

$$\underbrace{\frac{dN_p}{dt}}_{\text{Penguin population}} = \underbrace{\beta_p S N_p}_{\text{Food supply}} - \underbrace{\gamma N_p}_{\text{Natural death}} \tag{1}$$

$$\underbrace{\frac{dS}{dt}}_{\text{Sardine biomass}} = \underbrace{R(t) + \alpha S}_{\text{sardine suppl}} - \underbrace{\beta_s S N_p}_{\text{Eaten by the Penguins}} - \underbrace{(F(t) + L(t))S}_{\text{sardines death rate}} \tag{2}$$

The re-scaled model

To re-scale the model we set

$$N_p = N_p^0 N_p', S = S^0 S', t = t^0 t', R = R^0 R', F = F^0 F', \text{ and } L = L^0 L'.$$

By dropping the primes, we can rewrite the re-scaled model as following:

$$\underbrace{\frac{dN_p}{dt}}_{\text{Penguin population}} = \underbrace{SN_p}_{\text{Food supply}} - \underbrace{\xi_p N_p}_{\text{Natural death}}, \tag{3}$$

$$\underbrace{\frac{dS}{dt}}_{\text{Sardine biomass}} = \underbrace{R^* R(t) + \alpha_s^* S}_{\text{sardine supply}} - \underbrace{\beta_s^* S N_p}_{\text{Eaten by the Penguins}} - \underbrace{(F^* F(t) + L^* L(t)) S}_{\text{sardines death rate}}. \tag{4}$$

Where

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$$t^0 = \frac{1}{\beta_p S^0}, \quad \xi_p = \frac{\gamma}{\beta_p S^0}, \quad R^* = \frac{R^0}{\beta_p {S^0}^2}, \quad \alpha_s^* = \frac{\alpha_s}{\beta_p S^0},$$
$$\beta_s^* = \frac{\beta_s N_p^*}{\beta_p S^0}, \quad F^* = \frac{F^0}{\beta_p S^0}, \quad L^* = \frac{L^0}{\beta_p S^0}.$$

The first model

We will first explore the equations under the simple assumption that the sardine input rate remains constant throughout the year. Specifically, we assume that all input parameters are time-independent. We consider only a constant source of sardines without accounting for the exponential growth of the population. Additionally, we assume that only fishermen impact the sardine biomass, ignoring other factors. The reduced model can be written as:

$$\underbrace{\frac{dN_p}{dt}}_{\text{Penguin population}} = \underbrace{SN_p}_{\text{Food supply}} - \underbrace{\xi_p N_p}_{\text{Natural death}}, \tag{5}$$

$$\underbrace{\frac{dS}{dt}}_{\text{Sardine biomass}} = \underbrace{R^* R}_{\text{sardine supply}} - \underbrace{\beta_s^* SN_p}_{\text{Eaten by the Penguins}} - \underbrace{F^* FS}_{\text{Fished}}. \tag{6}$$

The equilibrium point

To find the equilibria of the system (5)-(6) we set the right-hand side of the system to zero, and we have the unique equilibrium point,

$$(N_p^*, S^*) = (\xi_p, \frac{R + \alpha_s^* \xi_p - F^* \xi_p}{\beta_s^* \xi_p}). \tag{7}$$

This equilibrium point is valid only if $R + \alpha_s^* \xi_p > F^* \xi_p$

Stability of the equilibrium point

To discuss the stability of the equilibrium point we used the Jacobian matrix evaluated at the equilibrium point, we get

$$J = \begin{pmatrix} (S^* - \xi_p) & N_p^* \\ -\beta_s^* S^* & (\alpha_s^* - \beta_s^* N_p^* - F^* F) \end{pmatrix}. \tag{8}$$

The eigenvalues of the above Jacobian matrix are the solution of the following quadratic equation

$$\lambda^2 + b\lambda + c = 0, \tag{9}$$

where

$$b = -\left((S - \xi_p) + (\alpha_s^* - \beta_s^* N_p - F^* F)\right), \tag{10}$$

$$c = (S - \xi_p)\left(\alpha_s^* - \beta_s^* N_p - F^* F\right) + \beta_s^* S N_p. \tag{11}$$

The eigenvalues are given by:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4c}}{2}. \tag{12}$$

Numerical simulations

In this part, we will present our numerical simulations. We assumed that $R^* = \beta_s^* = F^* = 1$.

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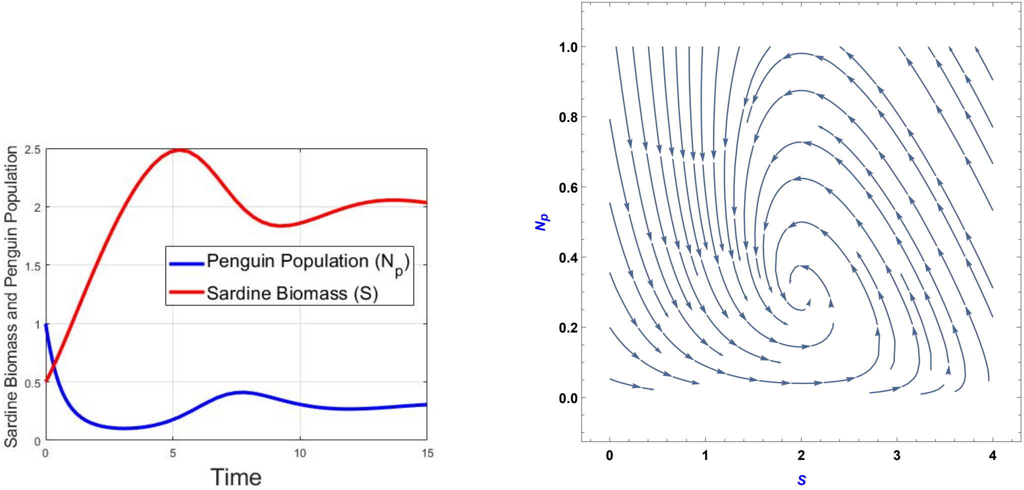

Figure: The plot shows the dynamics of sardine and penguin interactions.

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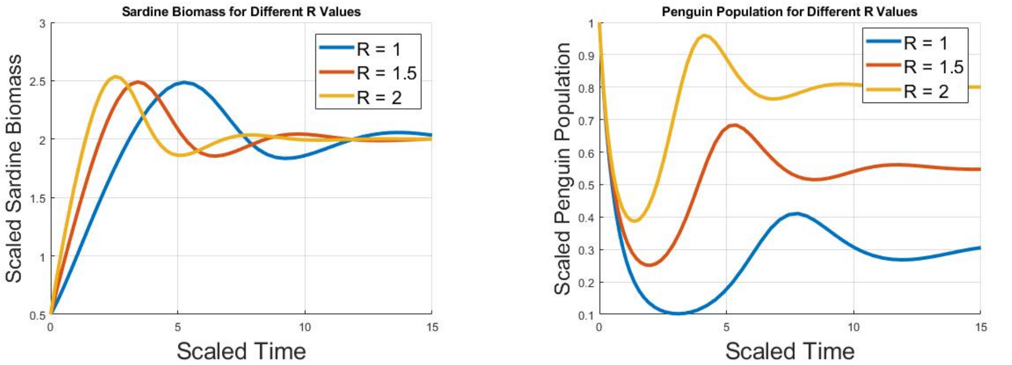

Figure: The plot shows the dynamics of sardine and penguin interactions. under different R values.

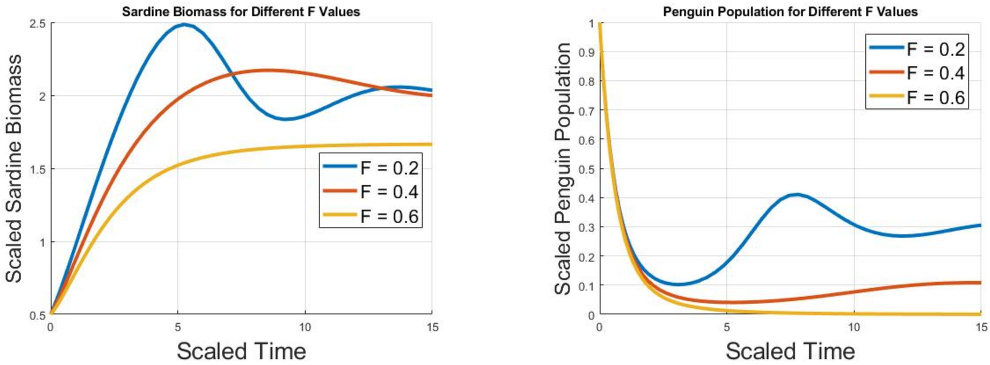

Figure: The plot shows the dynamics of sardine and penguin interactions under different F values.

Conclusion

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Future work

- The predator-prey dynamic, as described in [Sherley et al., 2013], will be considered to model the relationship between penguins and sardines.
- the group will consider the second model under the assumption that the sardine input rate R is a function of time throughout the year.
- This problem relates to the next week problem which is concerned with the effect of climate change on the survivability of the penguin.

References

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Thank you for listening