EXECUTIVE SUMMARIES

2024

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An Executive Summary consists of a brief description of the problem followed by a largely equation-free summary of the progress made and the results obtained by the study group.

THE BATCH PAN SCHEDULING PROBLEM IN A WHITE SUGAR REFINERY

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Executive Summary

In a typical white sugar refinery, the sugar is crystallised in batch "Vacuum Pans". These are evaporative crystallisers where evaporation and crystallisation takes place simultaneously under vacuum. At the end of cach batch pan cycle the pan will contain a mixture of crystals and mother liquor with 50% of the sugar being in crystal form. The crystallisation is followed by centrifugation to separate the crystal sugar from the mother liquor. There is some water addition at this stage to wash the sugar and dissolve small crystals that can pass into the separated mother liquor. The crystallisation is done in multiple stages with the impurities, predominantly colour", remaining in the mother liquor. The crystallisation takes place in a batch pan where water is removed from the liquor fed to the pan, using heating steam to drive the evaporation. When each batch is complete, the contents of the batch pan are discharged into a "strike receiver" which will have a capacity of 1.5 times the size of the batch. This means that the "strike receiver" does not have to be completely empty before a batch pan is discharged into it.

The mixture of crystals and mother liquor (called massecuite) will be processed in centrifugals to separate the crystals. Although this is a batch process, it can be

considered as continuous for this exercise as the centrifugal batch sizes are small relative to the pan batch sizes. The colour in the sugar is dependent on the colour remaining in the mother liquor. With 50% of the sucrose being removed from the liquor in each stage of crystallisation the mother liquor colour and the sugar colour doubles for cach stage. At the same time the quantity of sugar produced in each stage halves. The length of the batch cycles becomes longer for the later crystallisation stages as a result of the concentrating impurities slowing down the crystallisation rate.

The task for this study group was to develop a scheduling program that allocates pans appropriately to be able to process a specified quantity of fine liquor with a certain number of operational constraints. The overall objective to achieve was to smooth out the steam demand requirement for each cycle of batch processing. Indeed, given that the duration of batch cycle processing as well as the required operation are fixed and continuous, our proposed optimization model considers binary decision variables to mark and track the time period at which any cycle of any stage can commence. As such, the steam demand for each cycle of any stage can easily be calculated, since it is a constant demand for each cycle that only varies with the stages. Since the overall objective was to smooth out the steam demand requirement across the planing horizon, we have considered an objective function defined as the standard deviation (also modeled as the variance, without loss of generality) between the steam demand of each cycle and the overall average steam demand of the schedule.

Besides the overall scheduling objective,there were operational constraints that needed to be achieved by our mathematical model. Namely:

- Achieve the smoothest possible total steam demand from all pans: This is achieved through the above mentioned objective function.
- Add waiting times between batches to achieve the required scheduling: This has been explicitly accounted for in the definition of the decision variables of the model and the pre-processing carried out when discretising the planning horizon.
- Do not start a batch pan unless there is sufficient liquor/jet in the feed tank to complete the batch: We have defined precedence relationship constraints in order to guarantee this requirement.
- There must be sufficient space in the strike receiver to accommodate the contents of a batch pan at the end of its cycle: The above mentioned precedence relationship constraints also aims to achieve this requirement.
- Processing of sugar through the centrifugals must match the requirements for correct proportional mixing of sugars of different grades (so as to maintain the required average sugar colour). By defining a decision variable that is able to distinguish the stages of processing each batch, we are able to capture this requirement

- Determine the minimum number of pans necessary to meet the requirements: This will only be accounted for when we will develop a network version of the model, wherein, we consider more than one pan at each stage.

The overall model proposed is therefore an mixed-integer nonlinear model, wherein the continuous decision variable are control variables that check the volume of the pans at cach stage, which will help to decide whether a cycle can be started or not. The discrete decision variables are binary decision variables which mark and track the time period at which any cycle of any stage can commence. The non-linearity of the model is found in the objective function, where we minimize a variance function. The model will first be attempted to solved directly using commercial optimization solvers such as CPLEX or GUROBI. later on, some linearization techniques will also be attempted as well as some heuristic techniques can be developed in order to quickly find a feasible solution within a reasonable amount of time.

ROGUE WAVES IN THE AGULHAS REGION

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Executive Summary

A high frequency of rogue or freak waves have recently been measured around the South African coast, especially in the Agulhas region (south-east). Some have resulted in loss of life or livelihood, flooding, and damage to infrastructure. The group was asked to look at the generation and cause of rogue waves in particular related to this region. Rogue waves have a formal definition of having a peak height of double the significant wave height, defined as the average height of the highest one third of waves over the relevant period. The main danger is that they seem to appear "out of nowhere" with no warning.

A number of different approaches were considered. including some simple simulations, review of relevant literature and analysis of existing data with a view to noting conditions under which such waves have been observed.

The group decided to attempt to simulate the large model study done at the University of Edinburgh [4], in which a circular wave tank was used to generate a wave with the same properties as a rogue wave. A linearized model of the free surface hydrodynamic equations was solved in circular coordinates and a spike in the center of the basin suggested that it would be possible to generate such a wave by the focusing of incoming waves. This model was then extended to include nonlinear effects and produced a slightly higher central wave. These results indicate that a focusing of the waves appears to be very likely to be able to generate a rogue wave.

A simple model was set up to demonstrate the currents around the southern part of South Africa, including the gulf stream, the Agulhas current and the main Southern Ocean easterly current. This model currently includes no topography but it is hoped that future work might incorporate it so that a simplified wave-height model might be developed.

A final effort concentrated on the so-called Korteweg de Vries equation [3] which assumes waves have a long wavelength compared to the depth of the water and have been shown to generate soliton solutions. These are speculated to be an example of a rogue wave and so their formation is of particular interest. The forced Korteweg de Vries equation of Grimshaw et al [2] was used and a series of solitions were simulated as the ocean current flowed over a step in the bottom topography.

There are a large number of papers speculating on the nature and propagation of rogue waves, but most do not examine the origins of such waves. A paper of particular importance to this problem is that of Christou and Ewans (2014) [1]. A comprehensive analysis of 122 million waves collected from reliable and verifiable sources found around 3649 rogue waves, and these were analysed alongside the regular ocean waves. They drew the following conclusions from their detailed analyses.

- A rogue wave is generally steeper than normal waves but not all steep waves are "rogue". The average rogue wave shape had higher crests as well as deeper troughs than the highest 1% of normal waves.
- Rogue waves were slightly more narrow-banded than the highest 1
- The rogue wave samples exhibit dispersive focusing, resulting in the majority of frequency components coming into phase with each other at the time of the rogue wave events.
  □ The study presented evidence to suggest that rogue waves are extraordinary and rare occurrences of the normal population that are caused by dispersive focusing.

The group was unable to obtain full data sets for an extended period, but was able to collect data from 4 locations over a 12 day period. For example with the sample data we estimated the significant wave height at the four locations and found the values of Cape Agulhas (3.04), Cape Point (3.14), East London (Ngqura) (2.31) and Durban (1.91). It is clear that higher significant wave height might lead to more likelihood of a damaging rogue wave. These data were used to create some plots that might be used to consider the major factors such as wind speed and direction and ocean current that may lead to the formation of rogue waves. When coupled with sea bottom topography it is likely that the conditions in which such waves may form could be predicted in advance, thus allowing warnings to be delivered.

This is a very difficult problem. Trying to predict an event that is 1 in 20,000 is fraught with uncertainty. Even if we did have a simulation model, it would not be able to predict exact times and locations, but rather locations in which they may be likely.

We have demonstrated that we can reproduce the conditions for the formation of rogue waves by almost reproducing the Draupner Wave Tank Experiment and using the KdV equation to generate solitons for flow over bottom obstructions. However, these can not resolve all of the topography of the Southern African Coast and so further work is necessary.

References

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MODELLING A GAS CHROMATOGRAPH

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Executive Summary

Gas chromatography (GC) is a widely used technique for separating and analysing volatile compounds in gas mixtures. GC has a broad range of applications, for example in detecting and quantifying pollutants, pesticides and environmental contaminants in air, water and soil samples. In food and beverage analysis, it can be used to determine the presence and con- centrations of flavour compounds or additives. In the pharmaceutical industry, it permits the analysis of drugs, including purity and quantifications of active ingredients. In forensic analysis it plays a crucial role in toxicology and arson investigations. Additionally, in clinical and medical laboratories, GC is used for analysing blood, urine and other biological samples.

The GC process works by adding a small amount of the sample containing different compounds into a carrier gas stream which is then forced through the GC column. The sample adsorbs and desorbs to a stationary material inside the column - the different attachment and detachment rates cause the sample components to separate. At the end of the column a detector responds to the presence of compounds and generates signals proportional to the concentration. These signals are then processed to produce the chromatograph.

During the meeting we developed a mathematical model for the chromatography process and analysed it in two different ways. First a Laplace Transform approach was used to solve the system for a single component sample. In the case of multiple components it was noted that there was negligible interaction or competition

between the components, consequently they could be dealt with separately. In which case the single component transform solution could be applied to all components. A finite difference solution was also developed and used to confirm the accuracy of the Laplace Transform approach. Comparison with experimental data confirmed the accuracy of both methods.

The Laplace Transform approach provided a solution for the combined flow, capture and release of compounds along a GC column. The solution was in the form of an integral which must be evaluated numerically. This is a much simpler prospect than solving the coupled PDE/ODE system which constitutes the general model. Further, the fact that a single solution holds for all contaminants (differences arise due to scaling and wave speed, not the form) means that the integral solution can be a powerful tool in analysing GC.

With chromatography, or any adsorption/desorption process, the key unknowns are the adsorption and desorption rate coefficients. During the week we used a rather ad hoc fitting method to calculate appropriate values. A main focus of future work will then be to improve the fitting to give greater confidence in the ability of our technique to describe flow in a GC.

AXIAL STRAIN EVALUATION WITHOUT THE USE OF STRAIN GAUGES

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Executive Summary

A uni-axial stress test is used to determine the Young's modulus and Poisson's ratio of rock samples. The testing machine applies a uni-axial stress to the sample which it measures and the resultant strain is measured using a strain gauge which consists of a strip that needs is attached to the sample. Attaching the strip to the sample is labor intensive and the strips are expensive and are discarded after one use. The uni-axial stress is typically applied using a descending beam whose displacement can be measured. Could this measurement be used to determine the strain, thus eliminating the need to usc a strain gauge? This is the question brought to the MISG by The Rock Mechanics Laboratory.

Stress verses strain results obtained using a strain gauge, and loading verses beam displacement results as recorded by crosshair movement, were provided by The Rock Mechanics Laboratory. The results indicated that the sample behaved as a linear elastic material and that the loading verses crosshair movement response was also linear except during the very first stages of loading; such early 'adjustment' stages results should be discarded. The proportionality factor was different for the strain gauge measurement to that of the descending beam gauge because components of the testing machine also move in response to loading. The fact that the crosshair response was linear strongly suggested that the machine response was both 'robust', uniform and 'predictable', and under such circumstances the machine response proportionality factor could be determined 'once and for all'. A variety of

dimensionality arguments/models were used to theoretically support this suggestion and under such circumstances the machine response could be represented as an equivalent linear elastic element in the system whose response could be determined using a known sample and known results as recorded by the strain gauge. Using the results provided by the laboratory the effective machine proportionality factor was determined.

The above work assumes that the machine response as recorded by crosshair movement is ‘repeatable’, and this can only be tested in the laboratory. Additional laboratory results have been obtained and being examined.

TOURIST ATTRACTIONS CAPPING VISITOR NUMBERS

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Executive Summary

Mpumalanga is one of the nine provinces in South Africa and is known as "the place where the sun rises" in the Nguni languages. Mpumalanga lies in eastern South Africa, bordering Eswatini and Mozambique. The province stands as a premier destination for tourists in South Africa and is celebrated for its rich heritage as well as diverse natural landscapes. Key tourist attractions include the Kruger National Park, Blyde River Canyon Nature Reserve, Manyeleti Nature Reserve, Mariepskop Nature Reserve, Sabi Sand Game Reserve, and Bushbuckridge Nature Reserve. Several of these attractions are currently undergoing to enhance their appeal and support sustainable tourism growth.

Successful revitalization could lead to significant growth in tourism, bringing more visitors to these attractions. Post-revitalization, it is anticipated that enhanced marketing efforts will attract more tourists, drawn by the attraction's unique and

improved features. A crucial aspect of revitalizing tourist attractions is the management of daily visitor numbers to balance demand and experience quality. A key challenge for managers of these tourist attractions is determining the maximum number of visitors that can be accommodated without compromising the quality of the visitor's experience. Addressing this challenge involves calculating each attraction's Social Carrying Capacity (SCC), which takes into account the Social Comfort Level (SCL) of visitors.

When the number of visitors exceeds the carrying capacity of a destination, it can lead to overcrowding, strain on local infrastructure, cultural degradation, and a decline in the quality of life for both residents and visitors. In the long run, the effects of overcrowding heighten the risk of over-tourism in these popular attractions. Overtourism occurs when the number of visitors becomes excessive and unsustainable, leading to further infrastructure strain and environmental degradation. In the short-run, a key strategy to mitigate over-tourism in the long-run involves capping daily visitor numbers, thereby managing overcrowding and its negative impacts.

The study group proposed a Capacity Management Model (CMM) that aims to visitor across various times and activities, striking a balance between maximizing revenue and ensuring visitor satisfaction. This model incorporates parameters such as activity transition probabilities, tourist arrival rates, and queue lengths. Inspired by principles from traffic flow and inventory management, it formulates a mixed integer optimization problem, with visitor satisfaction inversely related to queue length. The model's objective function seeks to maximize profits while minimizing queues, subject to constraints on flow, visitor movement, resource utilization, activity capacity, and queue length tolerances.

Without actual data, we simulated scenarios to validate the model: one with minimal crowding and another with excessive overcrowding. The simplex method was employed to solve these scenarios, demonstrating the model's potential capability to efficiently manage visitor capacity and flow.

However, further work is required to tackle overcrowding and over-tourism effectively. Gathering real data from tourist sites is essential for validating and refining the model. Our study primarily addresses overcrowding; future iterations should incorporate sustainable tourism principles, including environmental conservation, community engagement, and cultural preservation. Our ultimate aim is to safeguard tourist attractions for future generations, promoting sustainable and responsible tourism growth.

WIND TURBINE BLADE FAULTS DETECTION

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Executive Summary

Wind turbine blades are expected to last about twenty years but may last only four or five years because of the formation of faults. The faults are not easy to see or detect. One method of fault detection is using fatigue loading with acoustic wave emission detection. The faults are actuated by twirling the blade. The faults cause vibrations in the blade which can be measured using displacement detectors embedded in the blade. The Study Group was asked to interpret the results for fault detection.

A blade is typically made using fibreglass infused polyester. The main body is hollow. The structures are made by a lamination process and most faults are caused by delamination. A rinkle may or may not be seen on the surface.

The vertical displacement of a uniform blade was modelled by the fourth order Euler-Bernoulli beam equation. The Study Group considered the flapping mode caused by vertically oscillating the base end of the beam with the tip end free to move. Both vertical (flapping) and span-wise modes are excited. The forced solution of this beam problem was derived in terms of circular and hyperbolic functions. A resonant response was found for an infinite range of eigenvalues. A dimensionless combination

$$J = \frac{EI}{\rho\,\omega^2 L^4}$$

determines the dynamic behaviour of the beam where EI is the flexural rigidity of the beam, ρ is its density, L is its length and ω is the frequency of the vertical movement. This means that a small laboratory model can be used to exactly determine the dynamic behavior of a much larger real scale beam. The value of J for the laboratory model needs to equal that of the beam. It is possible to tune a small scale laboratory model to duplicate the behavior of the larger beam by adjusting ω. The dynamic behaviour depends very strongly on the beam length L. The frequency ω can be chosen so that J is close to a resonant value in the laboratory

The presence of a fault on a moving turbine blade causes elastic waves to be generated; body waves, surface waves and bending waves. Body waves are of two types, longitudinal pressure waves and transverse shear waves. A transverse wave on a free boundary such as a crack or fault will generate both reflected longitudinal and transverse waves provided the incidence angle is greater than a critical value. Elastic surface waves (Rayleigh or Love waves) may be generated when a body wave hits a crack or fault when the incidence angle is less than the critical angle. Bending waves cause motion at right angles to the surface of the beam with a velocity proportional to the thickness of the beam.

Body wave composition of an incident wave will change when it hits a fault and the resultant waves will travel at different speeds making detection possible. However plastics and composites do not behave like pure elastic materials and waves generally lose energy. It seems unlikely that fault detection would work except over small distances. It would be useful to set up a simpler experimental configuration which is smaller, less expensive with simpler forcing, to explore the effects of faults in isolation. Much more would need to be known concerning the material properties and structure of the blades to proceed further.