Mathematical Modelling of Tornadoes MISG 2020 University of Zululand

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Outline

What is a Tornado?

- A tornado is a violent rotating column of air extending from a thunderstorm to the ground.
- The most violent tornadoes are capable of tremendous destruction with wind speeds of up to $300mph$ $(482.8km/h)$.
- They can destroy large buildings, uproot trees and hurl vehicles hundreds of yards.
- They can also drive straw into trees.
- Damage paths can be in excess of one mile $(1,6km)$ wide to $50miles$ $(80.47km)$ long.
- In an average year, 1000 tornadoes are reported worldwide.

Types of Tornadoes

FIGURE 1: **Tornado**

Types of Tornadoes conti...

Figure 2: **Waterspout tornado:** A weak tornado that forms over water.

Types of Tornadoes Conti...

Figure 3: **Landspout tornado:** A very weak tornado that is associated with the land which is equivalent of a waterspout. For example, a Landspout tornado was spotted in New Hanover, Kwazulu-Natal, on Tuesday afternoon. The South African Weather Service confirmed that the area had experienced a tornado, but said it was not a result of severe thunderstorms.

Types of Tornadoes Conti...

Figure 4: **Multiple Vortex tornado:** Two or more tornadoes share the same thunderstorm but different axes.

How tornado forms

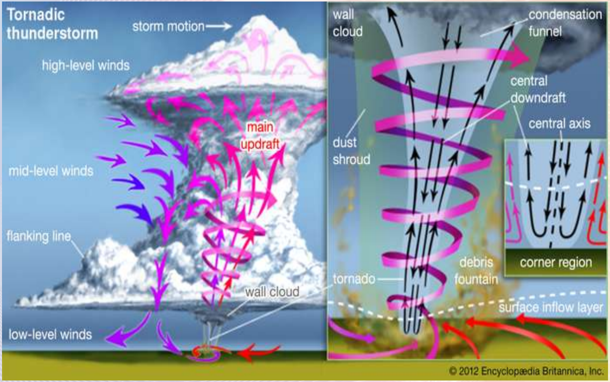

Figure 5: Formation of Tornado [1]

- A tornado is a rappid violent rotation of air that extends from a thunderstorm.
- They mostly happened in the afternoon, when the ground and the atmpsphere are heated enough.
- Tornados will form when thwre are an interaction between the worm humid air and cold air, the worm air rises through the cold air causing an updraft.
- When the wind speed differs significantly in speed and direction the updraft begin to rotate creating a funnel.
- The funnel continues to grow and eventually it descends from the cloud. When it touches the ground, it becomes a tornado.

Measurements of Tornado intensities

Enhanced Fujita Scale
(Implemented February 2007)

Rating	Winds	Expected Damage
EF0	65-85 mph	**Minor damage.** Shingles or parts of roof peeled off; damage to gutters/siding; branches broken off; shallow-rooted trees toppled.
EF1	86-110 mph	**Moderate damage.** More significant roof damage; windows broken; exterior doors damaged or lost; mobile homes badly damaged or overturned.
EF2	111-135 mph	**Considerable damage.** Roofs torn off well-constructed homes; homes shifted off their foundation; mobile homes completely destroyed; large trees snapped or uprooted; cars may be tossed.
EF3	136-165 mph	**Severe damage.** Entire stories of well-constructed homes destroyed; significant damage to large buildings; homes with weak foundations may be blown away; trees begin to lose bark.
EF4	166-200 mph	**Extreme damage.** Well-constructed homes leveled; cars thrown significant distances; top story exterior walls of masonry buildings likely collapse.
EF5	> 200 mph	**Incredible damage.** Well-constructed homes swept away; steel-reinforced concrete structures critically damaged; high-rise buildings sustain severe structural damage; trees usually completely debarked, stripped of branches, and snapped.

Figure 6: Measurements of Tornado intensities in Fujita Scale

Global distribution of Tornado

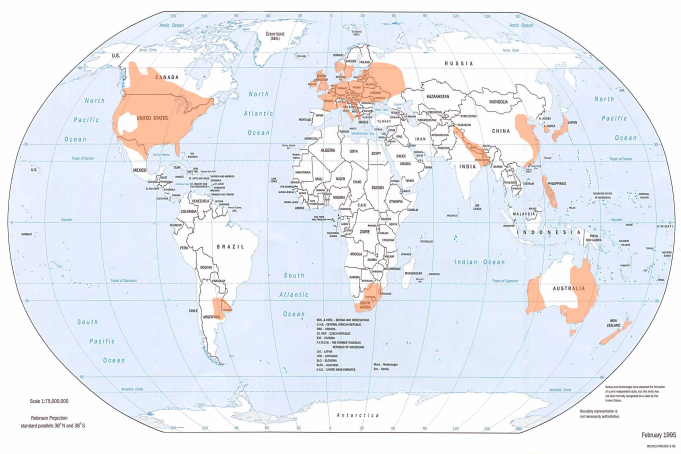

FIGURE 7: Areas worldwide with the highest frequency of tornadoes are indicated by orange shading [2].

Navier-Stokes equation

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} &= \nu \Delta \mathbf{v} - \nabla p + F, \\ \nabla.\mathbf{v} &= 0. \end{aligned} \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} = \text{Steady state term},$$
$$(\mathbf{v} \cdot \nabla)\mathbf{v} = \text{Convection term},$$
$$\nu \Delta \mathbf{v} = \text{Diffusion term},$$
$$\nabla p = \text{Pressure term},$$
$$F = \text{External forces term}.$$

The vorticity

$$\vec{w} = \nabla \times \vec{v}$$

and

$$\frac{\partial \vec{w}}{\partial t} + (\vec{v} \cdot \nabla)\vec{w} = (\vec{w} \cdot \nabla)\vec{v}$$

We seek the solution of the form

$$\vec{v} = v_r(r)\vec{e_r} + v_\theta(r)\vec{e_\theta} + v_z(z)\vec{e_z}$$

Assumptions

- Steady state.
- External force negligible.
- No-slip boundary conditions

Burgers-Rott model

We assumed axisymmetry

$$\mathbf{v}(r,\theta,z) = v_r(r)\mathbf{e}_r + v_\theta(r)\mathbf{e}_\theta + v_z(z)\mathbf{e}_z, \tag{2}$$

whose solutions are:

$$\begin{cases} v_r(r) = -ar + \frac{b}{r}, \\ v_\theta(r) = \frac{\Gamma_\infty}{2\pi r^2}\left[1 - e^{-\frac{ar^2}{2\nu}}\right] + \frac{c}{r}, \\ v_z(z) = 2a(z - z_0), \end{cases} \tag{3}$$

where $a > 0$, $\Gamma_\infty > 0$, b and c are constants.

Burgers-Rott model cont...

possibilities of v_r terms depending on b,

- $b < 0 \implies$ a sink of fluid on the $z-$axis.
- $b = 0 \implies$ a tornado.
- $b > 0 \implies$ a source of fluid on the $z-$axis [3].

Discussion of Burgers-Rott Model

- When $r \rightarrow \infty$, v_r is large, implying the tornado affects anyone no matter how far they are from the vortex, which is unrealistic
- Vortex starting from the bottom contradicts the mechanism in which a tornado is formed

Donaldson-Sullivan model

We assumed $v_z = v_z(r, z)$, then

$$\begin{cases} v_r(r) = -ar + \frac{6\nu}{r}\left(1 - e^{-\frac{ar^2}{2\nu}}\right) \\ v_\theta(r) = \frac{\Gamma_\infty}{2\pi r H(\infty)} H(\frac{ar^2}{2\nu}) \\ v_z(r, z) = 2a(-z + z_0)\left(1 - 3e^{-\frac{ar^2}{2\nu}}\right) \end{cases} \tag{4}$$

- Allows for two cell vortex with upstream and down drift.
- $V_r(r) \to -\infty$ as $r \to \infty$.
- Can be generalized, $v_z \neq 0$ as $r \to \infty$.

Bakers Model

$$\begin{cases} v_r = \frac{-4rz}{(1+r^2)(1+z^2)}, \\ v_\theta = \frac{kr^{r-1}\left(\ln(1+z^2)\right)^{\frac{\gamma}{2}}}{1+r^2}, \\ v_z = \frac{4\delta\ln(1+z^2)}{(1+r^2)}. \end{cases} \tag{5}$$

- More realistic model of Tornado far away the vortex has little effect.
- It does not allow for two cell vortex.

Plots

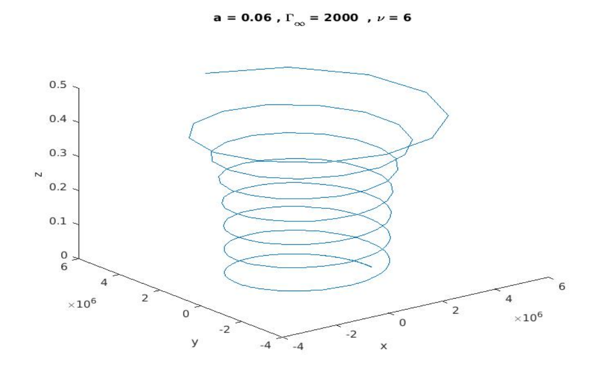

Figure 8: Burgers-Rott model

Plots

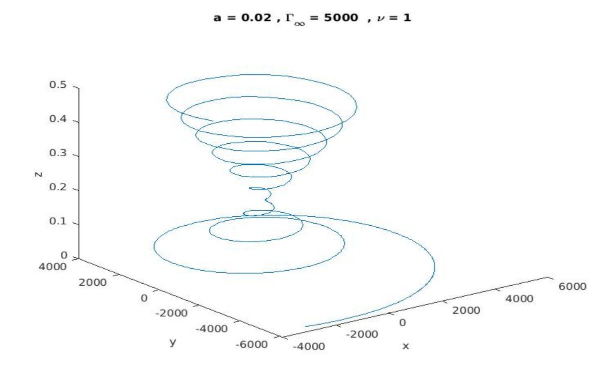

Figure 9: Burgers-Rott model

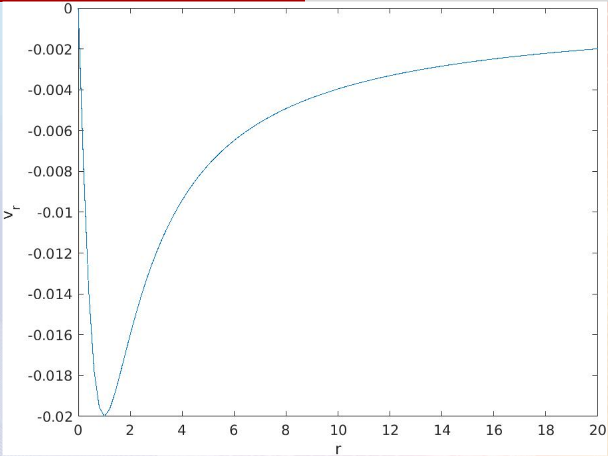

FIGURE 10: Velocity profile of tangential part of Baker's model, v_r

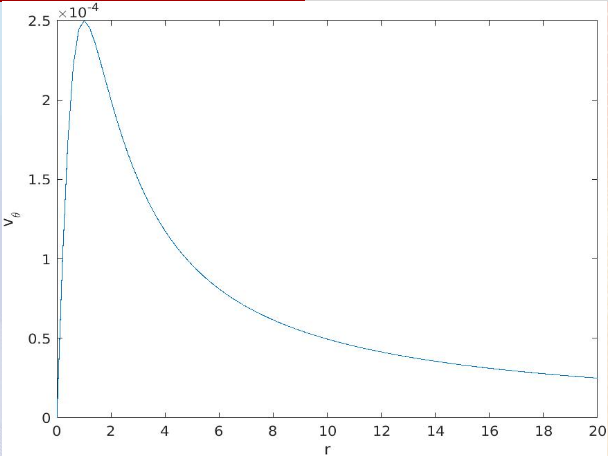

FIGURE 11: Velocity profile of radial part of Baker's model, v_θ

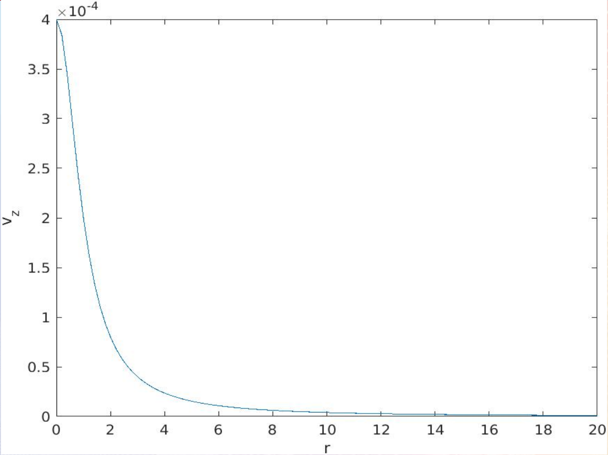

FIGURE 12: Velocity profile of vertical part of Baker's model, v_z

Discussion of velocity profiles

- From figure(9) and figure(11) as $r \longrightarrow \infty$ we see that the velocity $v_r, v_\theta \longrightarrow 0$ which is more consistent with what we observe, which is that the further away from the tornado the less you are affected.

Conclusion and Possible Future Work

- We may consider heat to add energy source to the model
- We may consider a model with translational rotation

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THANK YOU-:)