# Conductivity, thickness and location determination for an embedded strip of material

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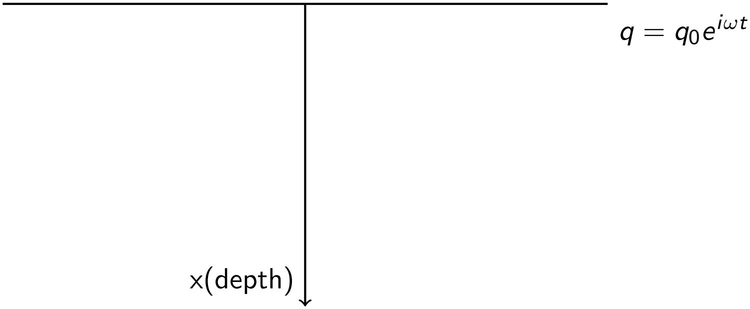
# Main objectives

- Determine the conductivity of a thin strip embedded in a material of different conductivity given its thickness.
- Determine the thickness of a strip embedded in a material of different conductivity given its conductivity.

# Problem 1: Determining conductivity of a given material



# Problem 1: Determining conductivity of a given material

The heat supplied at the surface:

- $q = q_0 \cos \omega t = q_0 e^{i\omega t}$........(1)

Heat is transported mainly by conduction and the heat flux is adequately described by Fourier's law:

- $q = -k\frac{\partial T}{\partial x}$.....................(2)

where $k$ is the conductivity of the material. So by equation (1), (2) can be written as;

$-k\frac{\partial T}{\partial x} = q_0 e^{i\omega t}$................(3)

The heat flow is described by the heat equation:

- $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$......................(4)

At infinity we have

- $T \to 0$ as $x \to \infty$............(5)

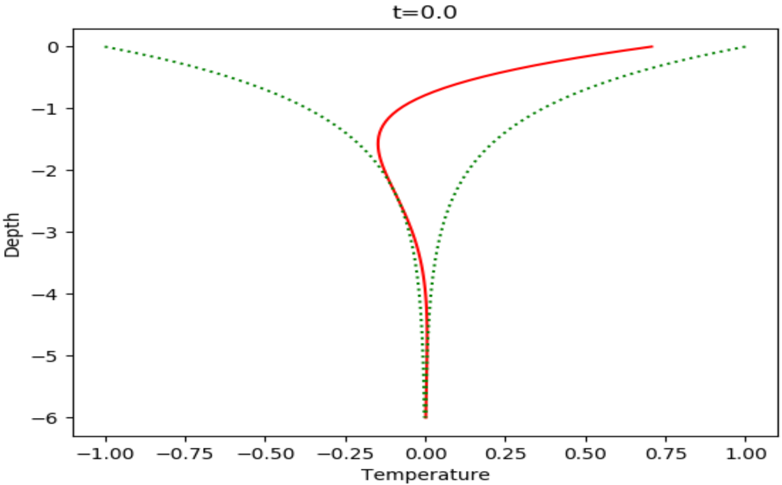
Solving the Heat equation (4) above subject to the boundary condition (3) we get the solution to be;

# Problem 1: Determining conductivity of a given material

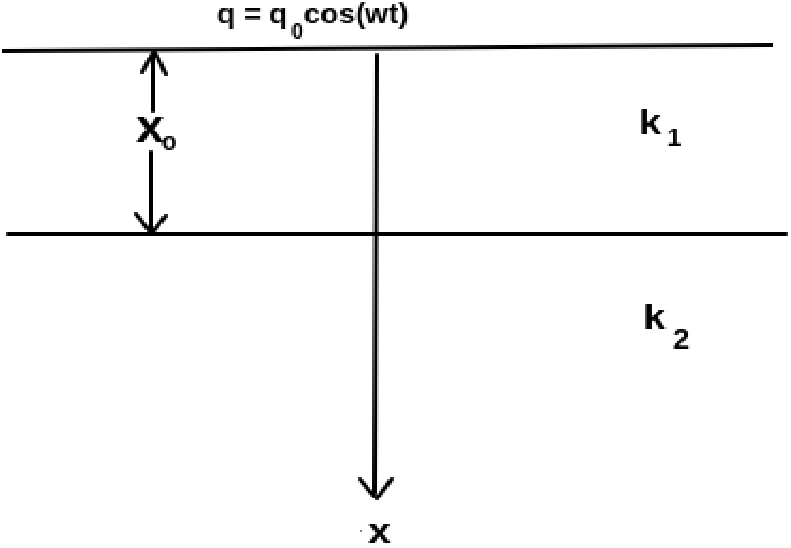
- $T(x,t) = \frac{q_0}{k}\sqrt{\frac{\kappa}{\omega}}e^{-\sqrt{\frac{\omega}{2\kappa}}x}e^{(-\sqrt{\frac{\omega}{2\kappa}}x+\omega t+\frac{7}{4}\pi)i}$
  and it follows that at the surface;
- $T(o,t) = \frac{q_0}{k}\sqrt{\frac{\kappa}{\omega}}e^{(\omega t+\frac{7}{4}\pi)i}$
  The maximum temperature at the surface is given by
- $T_{max} = \frac{q_0}{\sqrt{\rho c\omega k}}$ where $\kappa = \frac{k}{\rho c}$ and $\rho$ and $c$ are density and specific heat capacity respectively.
- Conductivity $k$ of the material is then determined by making $k$ subject from the above to obtain it as;

$$k = \frac{{q^2}_0}{\rho c\omega {T^2}_{max}}$$

# Problem 1 plot: Temperature decay in the material with heat input



# Problem 2 and 3



# Problem 2 and 3

We need to determine $x_0$ given $q_0, \omega, k_1$ and $k_2$ subject to the following;

- $q = q_0 \cos \omega t = q_0 e^{i\omega t}$........(1)
- $q_0 e^{(i\omega t)} = -k_1 \frac{\partial T_1}{\partial x}|_{(0,t)}$........(2)
- $\frac{\partial T_1}{\partial t} = \kappa_1 \frac{\partial^2 T_1}{\partial x^2}$ and $\frac{\partial T_2}{\partial t} = \kappa_2 \frac{\partial^2 T_2}{\partial x^2}$...............(3)
- $T_1(x_0, t) = T_2(x_0, t)$..................(4)
- $k_1 \frac{\partial T_1}{\partial x}|_{(x_0,t)} = k_2 \frac{\partial T_2}{\partial x}|_{(x_0,t)}$..................(5)
- $T_2 \to 0$ as $x \to \infty$..........(6)

# Problem 2 and 3

Solving the PDEs mentioned previously, we obtain:

$$T_1(x,t) = e^{i\omega t}(Ae^{\lambda x} + Be^{-\lambda x})$$

and

$$T_2(x,t) = e^{i\omega t}(Ce^{-\gamma x})$$

where

- $\lambda = \sqrt{\frac{\omega}{2\kappa_1}}(1+i)$
- $\gamma = \sqrt{\frac{\omega}{2\kappa_2}}(1+i)$
- $A = \frac{q_0}{\lambda k_1} * \frac{(\frac{\alpha\lambda}{\gamma}-1)}{(1+\frac{\lambda\alpha}{\gamma})e^{(2\lambda x_0)}+(1-\frac{\lambda\alpha}{\gamma})}$
- $B = A + \frac{q_0}{\lambda k_1}$
- $C = [Ae^{\lambda x_0} + Be^{-\lambda x_0}]e^{\gamma x_0}$

# Problem 2 and 3

The equation of interest is:

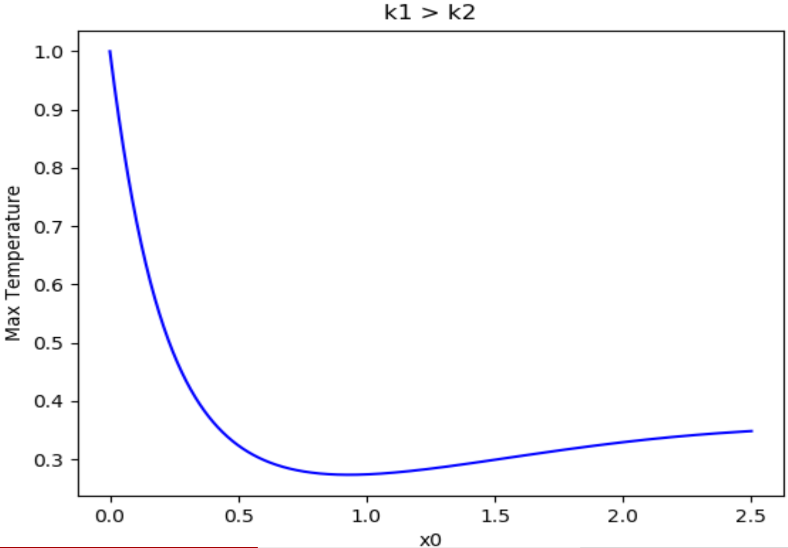
$$T_{max} = A + B = 2A + \frac{q_0}{\lambda k_1}$$

More particularly the solution to

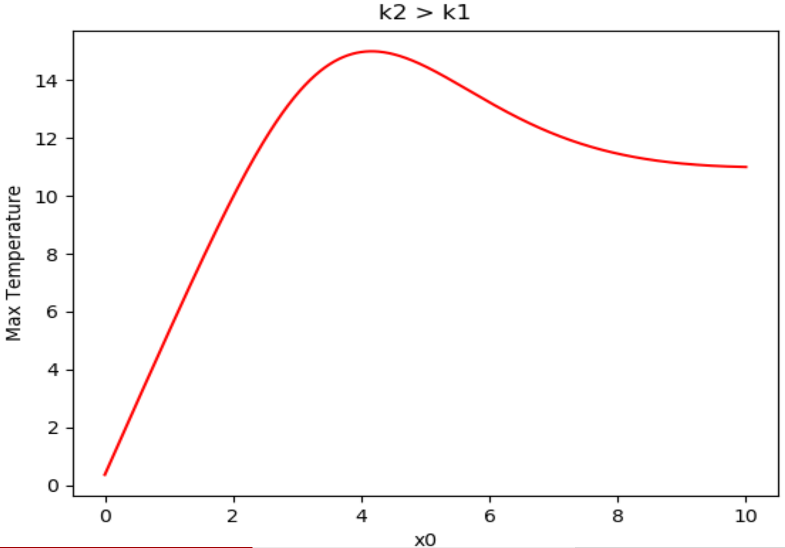
$$Re(T_{max})$$

which is the observable temperature at the surface.

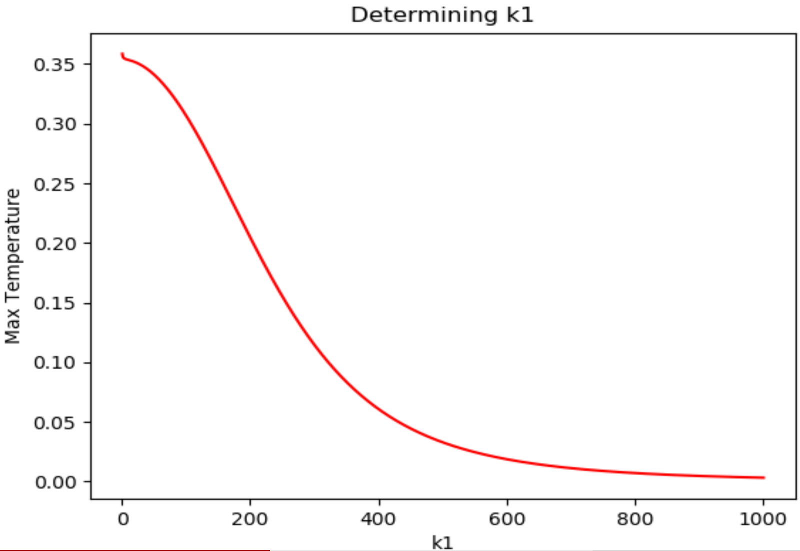
# Problem 2: Determining the depth of the material



# Problem 2: Determining the depth of the material



# Problem 3: Determining conductivity of a thin strip



# Conclusions

- The depth of penetration is of the order $\sqrt{\frac{\kappa}{\omega}}$ which is small when $\omega$ is large.
- The thickness and conductivity can only be determined when the thickness of the top layer is of the same order as $\sqrt{\frac{\kappa}{\omega}}$.
- To achieve relatively accurate results we need to ensure that $\omega$ is small enough so that the readings can be measured from the plots.