THE EFFECTS OF LANE POSITION IN A SWIMMING RACE

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Abstract

The forces and disturbances on swimmers in a multi-lane swimming pool during a race are considered to determine whether swimmers in the lanes closest to the edge of the pool are disadvantaged. Wave and pressure drag, wall effect and wave patterns are computed and considered as possible factors. Only wall effect and waves emerge as possible factors, and it seems likely that the conditions in the outside lane are no more or less favourable than elsewhere, but that conditions everywhere are also dependent on the competitor's location in the race and the waves generated by the swimmers ahead.

1 Introduction

Swimmers in a race are subjected to a number of forces as they progress down the pool. Apart from the forces within the water acting on the swimmer there are forces generated due to the presence of the waves. The problem was to consider the interaction of the various forces on the swimmers and study how that affects the "fairness" of a race given that the conditions in each lane are not identical.

A number of factors were considered including form drag and the turbulent wake, waves and reflections and the influence of the walls of the pool. After doing some

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preliminary scaling calculations and viewing video of some races, it was determined that the wake generated seemed to be restricted to the region directly behind the swimmer and remained in the same lane. Form drag due to the presence of the side wall was also shown to be of no relevance once several centimeters away from the wall. Preliminary calculations were conducted on the side force generated by the presence of the wall (lift) and these also seemed to indicate that the effect was very small.

The remaining factor that may play a role was therefore determined to be wave activity. The computation of solutions to the problem of waves on a water surface generated by a moving body is surprisingly complicated. However, since the waves generated by a swimmer are relatively small, sensible approximations to the wave field can be obtained using linear theory.

As a starting model, the waves generated by a single object in two dimensions were computed. They suggest that the wave train behind the first swimmer may either hinder (if they are just behind a wave crest) or assist (if they are just ahead and can surf down the wave front), a following swimmer depending on their distance behind.

A review of the literature showed that in relatively deep water, the wave pattern behind a moving object has a characteristic angle (called the Kelvin angle) of around 19.5 degrees. It was also shown that the largest disturbance to the water is along this line. Simple simulations were performed to show the interaction between a group of swimmers in a race with their wake-lines indicated. A swimmer just ahead of this line is essentially surfing, while one behind is swimming "uphill" and hence would be slightly retarded. This effect is due to the pressure difference between the water under a wave and that in the air. It was demonstrated that although the lead swimmer is unlikely to be affected by the disturbance (except in a multiple lap race) those behind may be positively or negatively affected by the wave field generated by others. It also seems likely that the swimmer closest to the wall may suffer the greatest buffeting due to the wave reflections of all of the other swimmers.

While these affects are generally very small, in a close race they may have some impact. The conclusion is therefore that there is definitely some affect of lane position, but it is really dependent on where the swimmer will place in the race and which swimmer they are next to, rather than the lane position itself.

2 Drag and the wake

The drag on an object moving in the water has three components. The first is what is known as form drag and is due to the viscosity of the water causing it to "stick" to the body. The second is pressure drag due to the lower pressure in the wake region behind the swimmer. The final component is wave drag in which the energy used to create waves is taken away from forward progress.

In ships, the pressure or wake drag is due to the propellor and the motion of the ship, while for a swimmer it is due mainly to the kicking of the legs. This wake lies directly behind the object/swimmer and is very short. It is unlikely to affect other competitors or impact on the walls of the pool, and so can safely be neglected from consideration.

It is to be expected that each swimmer would generate approximately the same amount of form drag, but naively it might be expected that drag from the close proximity of the wall may affect the performance of the swimmers in the outside lane. The drag is generated within what is known as the boundary layer, which has a thickness, W given approximately by [1],

$$W(x) \approx 2\sqrt{\frac{\nu x}{U}}. \tag{1}$$

where ν is the kinematic viscosity, U is the speed of the swimmer and x is the distance along the swimmer. Thus for a $2m$ tall swimmer, the greatest width of the boundary layer, travelling at 2 m/s (the viscosity of water is 0.8 $\times 10^{-6}$ m^2/s at $30^o C$), would be no more than

$$W_{max} \approx 0.002m, \tag{2}$$

which is much less than a centimeter. The only way that the walls of the pool could have any impact on this swimmer is if they approached closer to the walls than this distance which would seem impossible while maintaining a reasonable swimming stroke. It is reasonable to conclude that form and pressure drag do not have any extra component due to the presence of the side wall, nor do they impact on other swimmers.

3 Wall effect

Another possible effect could be the sideways force generated by the proximity of the wall to the outside lane swimmer. This effect is known as "Ground effect" in aerodynamics and is important in generating the lift in aircraft to get off the ground at low speeds. If a significant sideways force is generated on the swimmer near the wall then they are effectively having to swim "into" the force to maintain a straight line down the pool. In fact ships operating in confined waters, such as near a wharf, may also feel this effect or indeed suction toward the wall, depending on the angle of the ship to the wall [7].

In order to consider this effect, we need to look at the Bernoulli equation in steady flow to estimate the pressure. The Bernoulli equation is determined from the integration of the Navier-Stokes equation in irrotational flow of an inviscid, incompressible fluid, and is

$$\frac{1}{\rho}P + \frac{1}{2}\left|\boldsymbol{q}\right|^2 + gz = \mathrm{C},$$

where z is the elevation, g is gravitational acceleration, $\boldsymbol{q}$ is the velocity vector and P and ρ are the pressure and density respectively. Thus, for our swimmer we can see that we might assume z is constant at the level of the water surface. The constant C must apply throughout the fluid, so assuming z is also constant, we find that the pressure difference between the two sides of a swimmer are

$$\Delta P \approx \frac{1}{2}\rho(|\boldsymbol{q}_w|^2 - |\boldsymbol{q}_o|^2)$$

where the subscripts w and o refer to the wall and outer sides of the swimmer's body.

If the flow about the swimmer is symmetric, that is, they are in the middle of the pool, then the velocities on either side will be the almost the same and there will be no side force. If the swimmer is close to the wall, then the velocities may differ in a manner that can be computed using the method of images.

Consider a "three-dimensional" swimmer located near a wall. For this situation we can define a velocity potential ϕ so that $\boldsymbol{q} = \nabla\phi$. The problem can be written as

$$\nabla^2\phi = 0, \tag{3}$$

subject to

$$\begin{aligned} \phi_y = 0 \text{ on } y = 0, \\ \text{and} \quad \nabla\phi \cdot \boldsymbol{n} = 0, \quad \boldsymbol{x} \in \partial\Omega, \end{aligned} \tag{4}$$

where $\boldsymbol{n}$ is normal to the body and $\partial\Omega$ is it's surface. The two conditions require that there is no flow through the wall of the pool and none through the surface of the swimmer. A fictitious swimmer can be approximately described by using a Rankine body (see [1]), for which the velocity potential can be written in three dimensions as

$$\phi_S(x, y, z) = Ux - \frac{m}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where m is the strength of the point source/sink,

$$R_1 = \left((x + L/2)^2 + (y - b)^2 + z^2)\right)^{1/2}$$

and

$$R_2 = \left((x - L/2)^2 + (y - b)^2 + z^2)\right)^{1/2} .$$

The radius of the swimmer is given approximately by [1]

$$R_S = \left(\frac{m}{\pi U}\right)^{1/2} .$$

This form represents a point source located at $(-L/2, b, 0)$ and a point sink at $(L/2, b, 0)$. The streamlines around this object are closed provided the source and sink are of equal strength, thus forming a closed body of length slightly greater than L. We can generate a flow with $\phi_y = 0$ on $y = 0$ by introducing an image "swimmer" with potential ϕ_I, so that

$$\phi(x, y, z) = \phi_S - \phi_I, \quad \phi_I = \frac{m}{4\pi}\left(\frac{1}{R_3} - \frac{1}{R_4}\right) \tag{5}$$

and

$$R_3 = \left((x + L/2)^2 + (y + b)^2 + z^2\right)^{1/2} ,$$

$$R_4 = \left((x - L/2)^2 + (y + b)^2 + z^2\right)^{1/2} .$$

This represents an image object reflected in the line $y = 0$. The presence of the image distorts the shape of the original slightly, but the effect is small and so we can use this potential to estimate the effective side force due to the presence of the wall. Figure 1 shows the side force as a function of distance from the wall for an object travelling at 1 m/s and 2 m/s for a swimmer of the same radius. It is clear that very close to the wall the resulting force will be very large. However, it drops quickly and within a body length is close to zero.

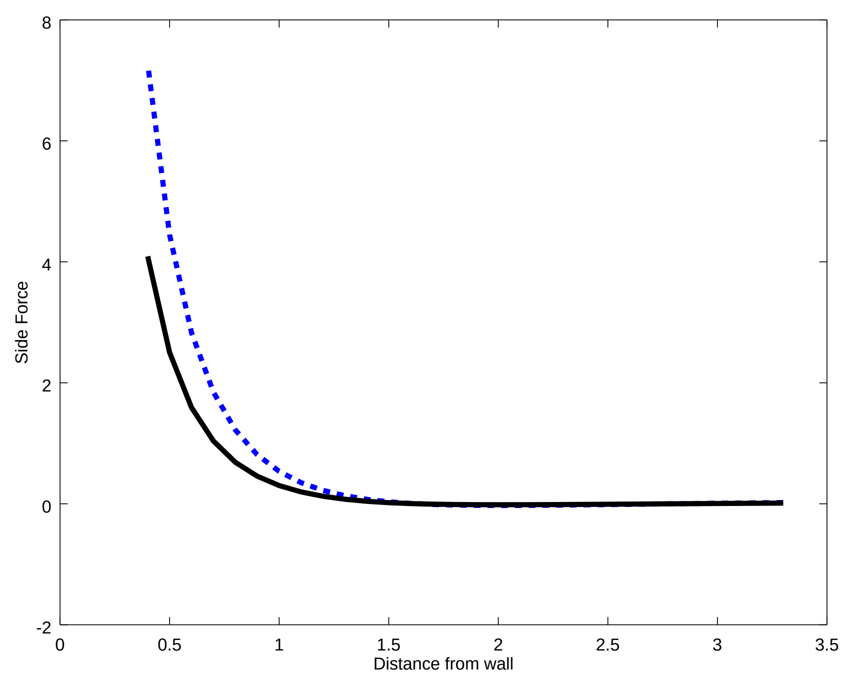

Figure 1: Side force (N) on the swimmer as a function of distance from the wall (m). Here the swimmer is approximately 35 cm "deep" and of length 1.7m swimming at about 2 m/s (dashed) and 1.5 m/s (solid).

Drag on a swimmer consists of pressure drag due to separation behind the swimmer, friction drag and wave drag. At speeds of around 2 m/s the breakdown is 40%, 3% and 57%. The drag coefficient is approximately $C_D \approx 1$ and so total drag is around $D \approx \frac{C_D}{2}\rho V^2 S$ where $S \approx 0.05$ is frontal area, and so is about 100 N. In Figure 1 the variation in side force can be seen as the swimmer is separated further from the wall. At a distance of about 50 cm the side force on the faster swimmer can be seen to be about 4 N, leading to a deflection angle of about 0.04. This is very small, and once the swimmer has moved to a distance of 1m this has dropped to less than 0.01. Thus unless the swimmer is closer than a quarter of their body length there is minimal effect.

There is an additional effect that we have not considered. If the swimmer tilts toward or away from the wall a moment on their body may be induced. If they tilt toward the wall the flow speed will increase slightly near the head leading to a lower pressure, and decrease slightly near the rear end, leading to a higher pressure. As a consequence a moment will be induced to cause the deflection to increase in this direction. A similar (but opposite) effect will be felt if they tilt away from the wall, so that in addition to a slight increase in lift there will be a moment to increase the tilt. It is likely that these forces will be small but without more detailed calculations this can not be determined completely.

4 Formulation of the theory of "ship" waves

We have now considered all of the possible forces acting on the outside lane swimmer except wave interactions. To make some progress we will use a linearised theory of waves that assumes small wave amplitudes. While in some circumstances the waves may become quite large, it is quite reasonable to assume that the characteristics of the waves will be approximated by the linear theory.

Since we have discounted the effects of viscosity, we may assume the flow to be irrotational and the fluid to be inviscid and incompressible. If the x-coordinate is aligned with the direction of the swimmer and the z coordinate is vertical we can define a velocity potential $\phi(x, y, z)$ such that $\boldsymbol{q} = \nabla\phi$ provides the velocity $\boldsymbol{q} = (u, v, w)$. The full problem is to find the potential ϕ that satisfies Laplace's equation

$$\nabla^2\phi = 0. \tag{6}$$

Using this the integration of the Navier-Stokes equation [1] leads to the Bernoulli equation

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 = -\frac{1}{\rho}P - gz\ , \tag{7}$$

valid everywhere in the fluid. Once $\phi(x, y, z, t)$ is known the pressure everywhere in the fluid can be obtained. The problem with this is that the location of the free water surface is unknown, and so the conditions must be applied on an unknown boundary. Non-dimensionalizing and letting

$$x = L\overline{x}, \quad y = L\overline{y}, \quad z = L\overline{z}, \quad t = \frac{U}{g}\,\overline{t}, \quad \phi = UL\overline{\phi}, \quad h = L\overline{h}, \quad P = \rho g L\overline{P},$$

where U and L are the swimmer's speed and length respectively, the equations become (bars omitted)

$$\nabla^2\phi = 0, \quad z < h, \quad t > 0, \tag{8}$$

$$\frac{\partial\phi}{\partial t} + \frac{F^2}{2}\Big(|\nabla\phi|\Big)^2 + z = -P, \quad z < h, \tag{9}$$

where

$$F = \frac{U}{\sqrt{gL}}$$

is called the Froude number and is the main parameter in this problem.

To proceed we need conditions on the free surface $z = h(x, y, t)$. On the surface the pressure must be atmospheric, so that (9) becomes

$$\frac{\partial\phi}{\partial t} + \frac{F^2}{2}\Big(|\nabla\phi|\Big)^2 + z = 0, \quad z = h(x, y, t). \tag{10}$$

There is also the kinematic condition which states that a particle on the surface must remain so, that is,

$$\frac{D}{Dt}\,(z - h)|_{z=h} = 0\,,$$

which implies that

$$\frac{\partial h}{\partial t} + \frac{\partial\phi}{\partial x}\,\frac{\partial h}{\partial x} + \frac{\partial\phi}{\partial y}\,\frac{\partial h}{\partial y} - \frac{\partial\phi}{\partial z} = 0, \quad \text{on} \quad z = h\,. \tag{11}$$

The equation that dictates that there can be no flow through the surface of the swimmer is

$$\nabla\phi\cdot\boldsymbol{n} = 0 \tag{12}$$

on the body surface, with normal $\boldsymbol{n}$. These equations completely specify the flow and in principle it should be possible to compute the appropriate waves generated by the swimmer.

4.1 One swimmer behind another (2-D problem)

We begin by considering the simpler 2-dimensional problem. This problem is easier to solve but does illustrate the techniques required. It also serves as a model for the waves trailing behind a single swimmer.

A second swimmer trailing directly behind a single swimmer will experience the wave field of the swimmer ahead. In addition to the drafting effect of the second swimmer being in the wake region, there is a possibility that the second swimmer may be on the downslope of a trailing wave or the upslope. The pressure just below the surface of the upper part of a wave is slightly higher than atmospheric pressure, so that if the swimmer is slightly ahead they receive a small boost. Essentially, in

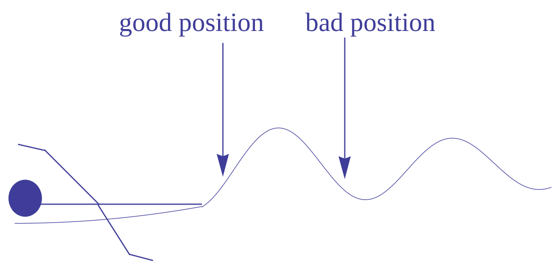

Figure 2: Waves behind a swimmer. A trailing swimmer can be assisted or hindered by their position in the wave train.

the former case the swimmer can surf down the wave, whereas in the latter they will feel as if they are swimming uphill, see Figure 2.

In two dimensions, suppose the variation from the undisturbed water surface is $y = \eta(x,t)$, then

$$\phi_{xx} + \phi_{yy} = 0 \quad \text{on } y < \eta(x,t)$$

with

$$\phi_t + \frac{1}{2}F^2\left(\phi_x^2 + \phi_y^2\right) + \eta = \frac{1}{2}F^2 - P(x), \qquad y = \eta(x,t)\ ,$$

$$\eta_t + \phi_x\eta_x - \phi_y = 0\,, \qquad y = \eta(x,t)\,.$$

An easy way to model a disturbance due to a craft or swimmer is to replace that disturbance by an equivalent pressure distribution $P(x-Vt)$ moving on the (undisturbed) surface where V is the speed of the swimmer; this avoids detailing the disturbance. This also avoids the necessity to include the interactions of the body with the water, but should produce a similar wave pattern.

These equations are very difficult to solve in full because of the nonlinear nature of the Bernoulli equation and the fact that the free surface is unknown a-priori. Therefore we assume the disturbance to the free surface is small and that the variation of ϕ from a uniform, horizontal stream, $\phi = x$, is small. We will also assume a steady solution, so in a frame of reference moving with the swimmer the water surface does not appear to change. Writing

$$\phi = x + \phi_0 + \ldots \quad \text{and} \quad \eta = 0 + \eta_1 + \ldots\ ,$$

substituting and dropping "small" terms, we find

$$\nabla^2\phi_0 = 0 \quad \text{on } y < 0\ ,$$

$$F^2\phi_{0xx} + \phi_{0y} = -P'(x) \ ,$$

with

$$\eta_1(x) = -P(x) - F^2\phi_{0x}(x,0) \ ,$$

where the surface conditions are all evaluated on $y = 0$.

It is possible to obtain a general solution using separation of variables as

$$\phi_0(x,y) = \int_0^\infty A(k)\ \mathrm{e}^{ky} \sin kx\ \mathrm{d}k$$

and substituting on $y = 0$ gives

$$F^2\phi_{0xx} + \phi_{0y} = -P'(x)$$

which implies that

$$\int_0^\infty (-F^2k^2 + k)A(k) \sin kx\ \mathrm{d}k = -P'(x) \ . \tag{13}$$

Letting

$$B(k) = (-F^2k^2 + k)A(k)$$

gives a Fourier sine transform for the given form of the pressure patch.

Suppose

$$P(x) = \kappa \mathrm{e}^{-x^2/\sigma^2};,,$$

so

$$P'(x) = -\frac{2x\kappa}{\sigma^2}\ \mathrm{e}^{-x^2/\sigma^2} \ ,$$

then

$$\int_0^\infty B(k) \sin kx\ \mathrm{d}k = -\frac{2x\kappa}{\sigma^2}\ \mathrm{e}^{-x^2/\sigma^2} \tag{14}$$

is a Fourier sine transform. The inverse is

$$B(k) = \frac{\kappa\sigma\ k\mathrm{e}^{-k^2\sigma^2/4}}{\sqrt{\pi}} \ , \tag{15}$$

so that

$$A(k) = \frac{\kappa\sigma \mathrm{e}^{-k^2\sigma^2/4}}{\sqrt{\pi}(1 - F^2k)} \tag{16}$$

and hence

$$\phi_0(x,0) = \frac{\kappa\sigma}{\sqrt{\pi}} \int_0^\infty \frac{\mathrm{e}^{-k^2\sigma^2/4}}{1 - F^2k} \sin kx\ \mathrm{d}k. \tag{17}$$

Now, since

$$\eta_1 = -P(x) - F^2\phi_{0x} \ ,$$

h

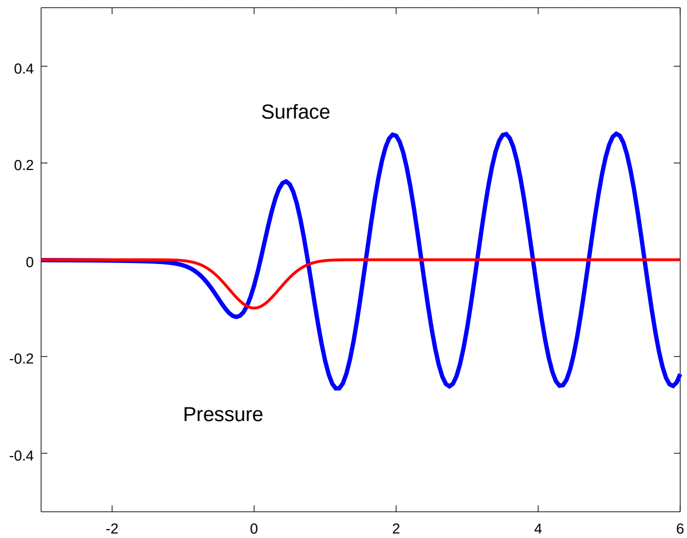

Figure 3: Free surface waves for a single pressure patch "swimmer" with $F = 0.5$ and surface pressure $P(x) = P_0 \mathrm{e}^{-4x^2}$ and $\kappa = 0.25$ (blue line). The red line is the pressure function.

not quite finally

$$\eta_1 = -\kappa \mathrm{e}^{-x^2/\sigma^2} + \frac{F^2 \kappa \sigma}{\sqrt{\pi}} \int_0^\infty \frac{k\mathrm{e}^{-k^2\sigma^2/4}}{1 - F^2 k} \cos kx \, \mathrm{d}k + a_1 \cos\left((x - \phi)/F^2\right) \ , \qquad (18)$$

where the last term is a solution to the homogeneous equation and the integral should be evaluated as a Cauchy-Principal value. The value of a_1 must be chosen so that upstream waves are eliminated. The integral term contains waves that travel outward in both directions, so a_1 must be chosen to exactly cancel those upstream. It turns out that this can be done by taking the real part of the residue of the complex integral along the real axis as $F^2 k \to 1$. In that case, we find

$$a_1 = \frac{\kappa \sigma \sqrt{\pi}}{F^2} \, \mathrm{e}^{-\sigma^2/4F^4} \qquad (19)$$

cancels the upstream waves perfectly. An example solution is shown in Figure 3, in which $F = 0.5, \kappa = 0.25$ and $\sigma = 0.5$. The pressure is shown as a red line and the resulting free surface is blue. In this scenario, whether the trailing swimmer is helped or hindered can be as little as a difference in distance of half a body length.

4.2 A single swimmer (3-D problem)

In order to consider the effects of each swimmer on the others and the effect of the wall we need to move to a 3 dimensional analysis. The problem becomes much more complicated, but we can follow a similar argument. The equations in that case are

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad \text{on } z < h(x,y,t) \tag{20}$$

with

$$\phi_t + \frac{F^2}{2}\left(\phi_x^2 + \phi_y^2 + \phi_z^2\right) + h = \frac{F^2}{2} - P(x,y) , \qquad z = h(x,y,t) , \tag{21}$$

and

$$h_t + \phi_x h_x + \phi_y h_y - \phi_z = 0, \qquad z = h(x,y,t) , \tag{22}$$

where $z = h(x,y,t)$ is the equation of the water surface. There is not space to include the full derivation of the solution to these equations here, but to gain an indication of the wave field we can go back to the source of original work by Kelvin [5] and subsequent works by Havelock [3, 4]. This early work derived the so-called Kelvin angle; the angle of waves in the trailing wake subtends an angle of approximately 39 degrees in infinitely deep water. Michell [2] was the first to compute the linearized wave pattern due to a moving object that included the shape of the ship (or swimmer). The resulting solution involves a quadruple integral. Even computing solutions to this integral can be very difficult.

There are two components to the solution, a particular solution which gives no waves and a free wave solution for which the upstream radiation condition determines the solution, [8]. Here we consider the flow past a pressure patch with form

$$p(x,y) = P_0 \exp\left(-\pi^2(x^2+y^2)\right) . \tag{23}$$

The exact solution for the linearized free surface $h(x,y)$ in the infinite depth case Wehausen and Laitone [9], or Pethiyagoda et al [6] is

$$\begin{aligned} h(x,y) = \quad & -p(x,y) + \frac{P_0}{2\pi^2}\int_{-\pi/2}^{\pi/2}\int_0^\infty \frac{k^2\hat{p}(k,\theta)\cos\Big(k\big[|x|\cos\theta + y\sin\theta\big]\Big)}{k-k_0}\,\mathrm{d}k\ \mathrm{d}\theta \\ & + \frac{P_0 H(x)}{\pi}\int_{-\pi/2}^{\pi/2} k_0^2\ \hat{p}(k_0,\theta)\sin\big(k_0[x\cos\theta + y\sin\theta]\big)\ \mathrm{d}\theta , \end{aligned} \tag{24}$$

where $H(x)$ is the Heaviside step function and

$$k_0 = 1/\left(F^2\cos^2\theta\right)$$

and

$$\hat{p}(k,\theta) = \frac{1}{\pi}\exp\left(\frac{-k^2}{4\pi^2}\right)$$

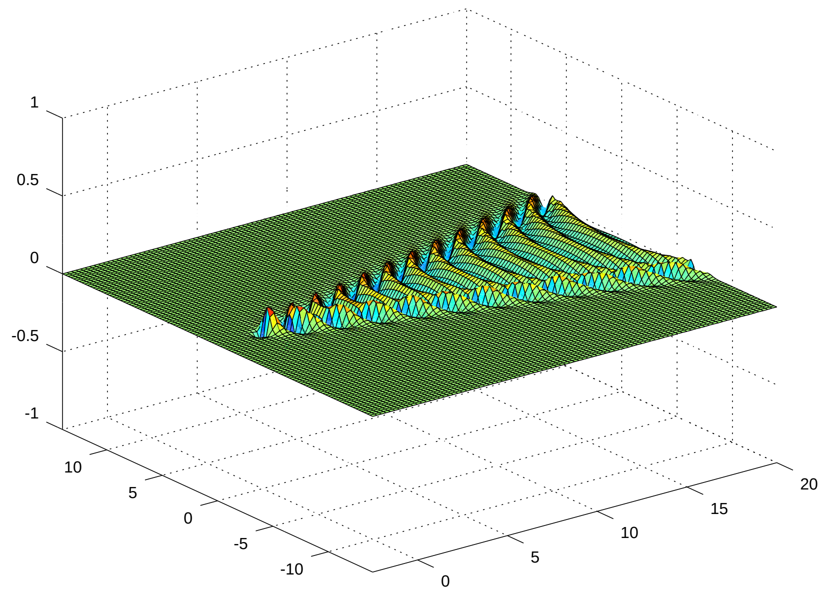

Figure 4: Wave pattern generated from the pressure patch given in equation (23). The Kelvin angle can be clearly seen, as can the fact that waves along this line are the highest.

is the Fourier transform of the pressure distribution given by (23). It is the final term that produces the free waves generated by the pressure patch in the flowing stream, and so this is the term that produces the wave field downstream. A single such pattern is shown in Figure 4. This integral can be evaluated by brute force quadrature, but not many years ago even calculating this linearized solution proved quite difficult due to computational limitations and the highly oscillatory nature of the integrand. Pethiyagoda et al [6] have presented some nice work on how to compute such wave fields efficiently using some clever numerical techniques. They then extended their work to solve efficiently for the full nonlinear problem. At the MISG, the group also computed the far-field solution for this problem (see Appendix).

4.3 Multiple swimmers

Since the solution presented here is linear, we can add more swimmers to the picture by simply superimposing them onto the same "pool" in different lanes and positions. Figure 5 shows the pattern from five swimmers in lanes at different locations. It is clear from this picture that the lead swimmers are in clear water, but those trailing can be in the middle of quite a wavy region due to the waves of the other, leading swimmers. A swimmer in the outside lane is exposed to the waves of those swimmers ahead and also the reflections of their waves. However, these reflections are no worse

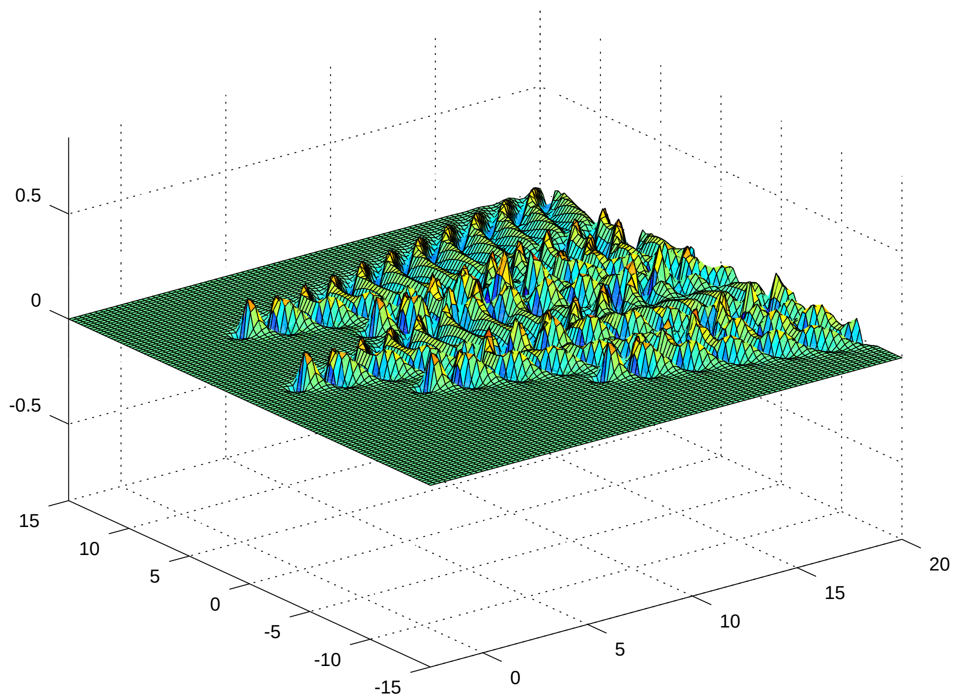

Figure 5: Wave pattern generated by five pressure patch “swimmers” superimposed in different lane positions. The swimmers have been separated a little to make the picture clearer.

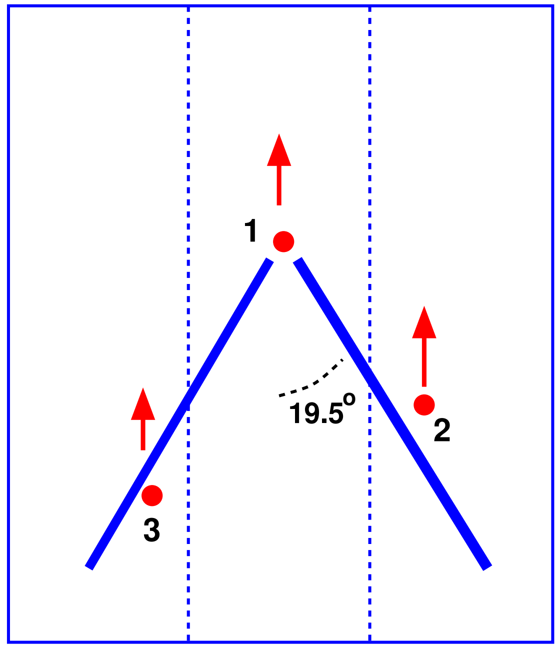

Figure 6: Three swimmers where 2 and 3 are ahead and behind the wake of swimmer 1.

than the superposition of two swimmers one on each side in the same position (by the method of images), that is, a swimmer in a middle lane may experience the same effect from two swimmers ahead the same distance in adjacent lanes. This linear theory can be used to generate a number of different scenarios for all swimmers.

Now that we have established the general wave patterns of a single swimmer, we can consider their interactions in a simplifed manner by looking at how a race may proceed. Ignoring the amplitude of waves, the dominant components lie on the Kelvin angle and so a sketch of a typical race situation can be considered as in Figure 6. Swimmer 2 is clearly in an advantageous position and working less than swimmer 3 who is swimming "into" the dominant wave of the lead swimmer. Although 2 and 3 may only be a short distance behind the leader, swimmer 2 will have a significant advantage as the race proceeds if they can remain in this location.

5 Conclusions

The problem under consideration was whether the swimmer in the outside lane of a race was disadvantaged by their position in the pool.

Analysis shows that we can discount form drag, side boundary layer, turbulent wake and most likely "wall effect". The largest interference to a swimmer in the outside lane is from the waves generated by leading swimmers. In this outer lane there may be a little extra buffeting as waves reflect off the wall, but this will be no worse than the interaction of the waves from two swimmers in adjacent lanes to a slower swimmer.

Modern high level swimming events have lane ropes that assist in the damping of the waves generated by the swimmers. It seems fairly clear from these computations that a swimmer in the outside lane is no more disdavantaged than a swimmer in the central lanes trailing behind. In other words, while the leading swimmers are unlikely to be affected by the waves of others, those behind may be either assisted or hindered.

For example, a swimmer may "ride" on the waves of a swimmer ahead of them, but if they slip behind the leading waves then they may be hindered. One can imagine a situation where two equally matched swimmers coming second and third may be differently affected by the waves of the leader, causing a different race placing.

The winner probably will not change (except in a long race), but positions below can be altered. A good strategy (if possible) is to try to get in the lane NEXT to the fastest swimmer and then seek to position yourself on the upstream side of the transverse wave, leaving potentially more energy for a stronger finish.

Further work might consist of a more detailed analysis of the "wall effect" to compute forces and moments for more realistic body shapes at different angles and also a more accurate derivation of the nonlinear wave field, although the latter is a very difficult problem to solve.

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APPENDIX: Far downstream approximation for 3 D model

The group did some computations to find the far-field solution due to a disturbance caused by a swimmer. Let

$$\phi = \epsilon\phi^*, \quad h = \epsilon h^*, \quad P = \epsilon P^*. \tag{A·1}$$

The linearised equations are (* omitted)

$$\nabla^2\phi = 0, \quad z < 0, \tag{A·2}$$

$$\frac{\partial \phi}{\partial t} + gh = -\frac{1}{\rho}P, \quad z = 0, \tag{A·3}$$

$$-\frac{\partial h}{\partial t} + \frac{\partial \phi}{\partial z} = 0, \quad z = 0. \tag{A·4}$$

Without lane ropes we also have

$$|\nabla\phi| \to 0 \text{ as } x^2 + y^2 + z^2 \to \infty. \tag{A·5}$$

We assume the swimmer travels in the x direction at constant speed U. We transform the coordinates as follows:

$$x - t = \frac{X}{\epsilon}, \quad y = \frac{Y}{\epsilon}, \quad t = \frac{T}{\epsilon}. \tag{A·6}$$

The equations are now

$$\epsilon^2\frac{\partial^2\phi}{\partial X^2} + \epsilon^2\frac{\partial^2\phi}{\partial Y^2} + \frac{\partial^2\phi}{\partial z^2} = 0, \quad z < 0, \tag{A·7}$$

$$\epsilon^2\frac{\partial^2\phi}{\partial X^2} + \frac{\partial\phi}{\partial z} = 0, \quad z = 0, \tag{A·8}$$

where P is zero because we are far downstream. To solve, let

$$\phi = A(X, Y, z)\exp\left[iu(X, Y, z, t)/\epsilon\right]. \tag{A·9}$$

We obtain

$$A = a(X, Y)\exp\left[z\left(p^2 + q^2\right)^{1/2}\right], \tag{A·10}$$

$$p^4 = (p^2 + q^2), \tag{A·11}$$

where

$$p = \frac{\partial u}{\partial X}, \quad q = \frac{\partial u}{\partial Y}. \tag{A·12}$$

Using the Lagrange-Charpit method we obtain

$$u(x,y) = cx + \sqrt{c^4 - c^2}\,y + b. \tag{A·13}$$

Therefore,

$$\phi = a(X,Y)\exp\left[c^2 z\right]\exp\left[i(cx + \sqrt{c^4 - c^2}\,y + b)/\epsilon\right]. \tag{A·14}$$