EXECUTIVE SUMMARIES

THE EFFECTS OF LANE POSITION IN A SWIMMING RACE

Industry Representative:
Ashleigh Hutchinson, School of Computer Science and Applied Mathematics, University of the Witwatersrand, Johannesburg

Moderator:
Graeme Hocking, Murdoch University, Perth, Western Australia

Student Moderator:
Johathan Julyan, University of the Witwatersrand

Study Group Members:
M Khalique, C Ruiters, K Prag, O E Alfred, P Thompson, T Motsepe, M Mahommed, H Linnet, M A Abdallah, and S Mitchell

Executive Summary

Swimmers in a race are subjected to a number of forces as they progress down the pool. Some of these are due to the water and some are due to the wave activity generated by themself and others. The problem was to consider the interaction of the various forces on the swimmers and study how that affects the "fairness" of the race given that the conditions in each lane are not identical.

A number of factors were considered including form drag and the turbulent wake, waves and reflections and the influence of the walls of the pool. After doing some preliminary scaling calculations and viewing video of some races, it was determined that the wake generated seemed to be restricted to the region directly behind the swimmer and remained in the same lane. Form drag due to the presence of the side wall was also shown to be of no relevance once several centimeters away from the wall. Preliminary calculations were conducted on the side force generated by the presence of the wall (lift) and these also seemed to indicate that the effect was miniscule.

The remaining factor that may play a role was therefore determined to be wave activity. The computation of solutions to the problem of waves on a water surface is surprisingly complicated. However, since the waves generated by a swimmer are relatively small, sensible approximations to the wave field can be obtained.

As a starting model, the waves generated by a single swimmer were computed. They show a wave train behind the first swimmer that, depending on the position of a swimmer directly behind, may either hinder them (if they are just behind a wave crest) or assist them (if they are just ahead and can surf down the wave front).

A review of the literature showed that in relatively deep water, the wave pattern behind a moving object has a characteristic angle (called the Kelvin angle) of around 19.5 degrees. It was also shown that the largest disturbance to the water is along this line.

Simple simulations were performed to show the interaction between a group of swimmers in a race with their wake-lines indicated. A swimmer just ahead of this line is essentially surfing, while one behind is swimming uphill and hence would be slightly retarded. It was demonstrated that although the lead swimmer is unlikely to be affected by the disturbance (except in a multiple lap race) those behind may be positively or negatively affected by the wave field generated by others. It also seems likely that the swimmer closest to the wall may suffer the greatest buffeting due to the wave reflections of all of the other swimmers.

While these affects are generally very small, in a close race they may have some impact. The conclusion is therefore that there is definitely some affect of lane position, but it is really dependent on where the swimmer will place in the race and which swimmer they are next to, rather than the lane position itself. Those in the lane nearest the wall may experience the greatest disturbance due to the waves interacting with the wall, but this was not considered in detail.

STOCKPILING OF BAGASSE: THE HAZARD OF SPONTANEOUS COMBUSTION

Industry Representative:
Richanr Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal

Moderator:
Tim Meyers, Centre de Recerca Matematica, Barcelona

Study Group Members:
Sarah Mitchell and Henri Laurie

Executive Summary

The mathematical model of Gray et al. (2002) describes the variation of temperature, water vapour, liquid water and oxygen in a one-dimensional bagasse pile. At MISGSA 2016 this model was analysed and preliminary numerical results obtained. At the 2017 meeting the work was extended to allow us to prescribe some simple guidelines on the storage of bagasse.

Firstly, during the 2017 meeting, the scaling of the governing equations was examined. This demonstrated that the system may be significantly simplified to a pair of equations describing temperature and oxygen variation throughout the pile. The water equations may be solved separately.

Due to the construction process, when a pile is first built the temperature and oxygen levels are approximately uniform. According to the mathematical model, within a few days, a quasi-steady state is reached. Since oxygen can enter through the top surface, which is open to the air, its level is highest there, moving deeper into the pile the oxygen level reduces as it is used in the reaction to generate heat. Due to heat loss at the surface the temperature is highest slightly below the surface.

Numerical simulations showed that for relatively small piles, up to around 4m, the maximum temperature very slowly declines over time. However, if the pile is too high, the maximum increases, very slowly at first (on the order of 50 days). This is followed by a very short phase where there is a rapid rise in temperature, leading to ignition. However, if a small pile is built and left for a few days so that it stabilises to a safe, quasi-steady state then another layer may be subsequently added (and more if necessary). This can permit much higher, stable piles than when using only a single layer.

The main preliminary guideline is therefore to *build the piles in thin layers, 2m seems reasonable, then wait a few days before adding subsequent layers.* Note, 2m is a conservative estimate, it is best to be safe. Events of spontaneous combustion are rare: the temperature build up is slow and occurs over a long period. It may therefore easily go unnoticed until disaster occurs.

More specific guidelines will be given in the full report. The exact thickness of these layers and the timing of adding them depend on factors such as the initial water content, ambient temperature and humidity. A detailed investigation into these factors could form a Masters project. Further, the effect of rain should resemble the addition of another layer, rain will bring in liquid water and oxygen as well as changing the temperature. This should also be examined.

MODELLING OF FRACTURE DEVELOPMENT (INITIATION AND PROPAGATION) IN ROCK BASED ON THE EXTENSION STRAIN FAILURE CRITERION

Industry Representative:
Richard Stacey, School of Mining Engineering, University of the Witwatersrand, Johannesburg

Moderators:
Colin Please, University of Oxford
Neville Fowkes, University of Western Australia
David Mason, University of the Witwatersrand

Student Moderators:
Keegan Anderson, University of Johannesburg
Jonathan Esteves, University of the Witwatersrnad

Executive Summary

The Mohr-Coulomb and Hoek-Brown material failure criteria normally used in civil and mechanical engineering fail to describe many mining circumstances. In particular failure can occur under much lower stress conditions than would be anticipated using these criteria in mining excavations. The behaviour is attributed to the presence of many fractures near the surface of the excavations. An alternative theory based on the idea that the maximum extension strain should not exceed a critical value for the material has been suggested and this has been shown to produce better results in specific cases. The Study Group was asked to investigate the extension strain criterion.

We first briefly summarise the extension strain criterion for fracture initiation proposed by Stacey in 1981. We will adopt the sign convention of rock mechanics that normal stresses are positive when compressive. The extension strain criterion states that fracture initiates when

$$\varepsilon_3 \geq \varepsilon_c \,, \tag{1}$$

where ε_c is the critical value of extension strain. The fractures will form in planes

at right angles to the direction of the extension strain which is the direction of the minimum principal stress, σ_3, that is the least compressive principal stress. The strain ε_3 is related to the three principal stresses, σ_1 , σ_2 and σ_3, by the component of the inverse Hooke's law

$$\varepsilon_3 = \frac{1}{E}\left[\nu(\sigma_1 + \sigma_2) - \sigma_3\right] , \tag{2}$$

where E is the Young's modulus and ν is the Poisson ratio of the rock mass. The three principal stresses are compressive and

$$\sigma_1 > \sigma_2 > \sigma_3 > 0 . \tag{3}$$

Note that if

$$\nu(\sigma_1 + \sigma_2) > \sigma_3 \tag{4}$$

then an extension strain will occur. This shows that an extension fracture can occur when all three principal stresses are compressive. Unlike the Mohr-Coulomb and Hoek-Brown failure criteria, the weakening effect of the intermediate stress, σ_2, is taken into account.

Since the fracture depends on an extension strain the criterion will apply in regions where the compressive stress σ_3 is low which will occur near excavation boundaries in the rock mass. In order to investigate the extension strain criterion the Study Group therefore considered a cylindrical excavation in the rock mass. The background vertical compressive stress at a depth must be sufficient to support the weight of the overburden rock mass. The background horizontal stress can be described using Heime's rule which equates the horizontal stress to k times the vertical stress, where the constant k depends on the local geology and the depth of the excavation. The stress in the rock surrounding the cylindrical excavation was calculated using plain strain theory of elasticity. The problem was formulated in terms of the Airy stress function which satisfies the biharmonic equation. Analytical solutions were obtained for the hoop stress, $\tau_{\theta\theta} = \sigma_1$, the axial stress, $\tau_{zz} = \sigma_2$, and the radial stress, $\tau_{rr} = \sigma_3$ and for the shear stress, $\tau_{r\theta}$. The radial stress had the smallest magnitude because it vanished on the boundary of the excavation. For $\theta = 0$, $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$, measured counter-clockwise form the horizontal, the normal stresses $\tau_{\theta\theta}$, τ_{zz} and τ_{rr} are principal stresses. The case $\theta = 0$ was studied in detail. It was found that for sufficiently large values of the Heime constant, k, there was extension strain in the radial direction even although all three normal principal stresses were compressive. At other values of θ a transformation to principal axes of stress was performed using Mathematica and computer plots of regions for extension strain in the rock mass near the excavation boundary were obtained for a range of values of k. These plots predict zones of fracturing which could lead to failure by mechanisms other than extension fracturing.

The extension strain criterion when first proposed was a fracture initiation criterion and not a rock failure criterion. Recent publications suggest that the extension strain criterion can be extended to predict rock failure under extension when the magnitudes of σ_1 and σ_2 are similar. Now, a basic assumption in the plain strain theory of elasticity is that the displacement vanishes in the z-direction along the axis of the cylindrical excavation. Thus the strain $\varepsilon_2 = 0$ and from the inverse Hooke's law,

$$\sigma_2 = \nu(\sigma_1 + \sigma_3) \; . \tag{5}$$

Since for the rock mass, $\nu \sim \frac{1}{4}$, the magnitudes of σ_1 and σ_2 are similar and the extension strain criterion can be applied as a measure of rock failure under extension in the cylindrical excavation considered.

The effect of the intermediate stress, σ_2, on the failure of rock when σ_1 is close in magnitude to σ_2 can be illustrated graphically in the $\sigma_2\,\sigma_1$-plane. The extension strain criterion for rock failure, given by (1) and (2), occupies the region

$$\sigma_1 \geq \sigma_2 \; , \qquad \sigma_1 \geq -\sigma_2 + \frac{1}{\nu}\left[E\varepsilon_c + \sigma_3\right] \; , \tag{6}$$

in the $\sigma_2\,\sigma_1$-plane. The limiting lines in (6) are orthogonal. This extends the rock failure criterion to a region in the σ_2, σ_1-plane below the limiting curves for the Mohr-Coulomb and Hoek-Brown failure criteria and demonstrates the weakening effect of the intermediate stress σ_2. For plain strain the Mohr-Coulomb and Hoek-Brown failure criteria can be expressed in terms of σ_1 and σ_2 by eliminating σ_3 using (5).

HARD-TO-BOIL MASSECUITE

Industry Representative:
Richard Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal.

Moderator:
Neville Fowkes, University of Western Australia

Student Moderator:
John Atherfold, University of the Witwatersrand

Executive Summary

The Problem

The pan boiling process in a sugar mill is one whereby sugar crystals are grown from a sucrose plus water syrup extracted from crushed cane. The syrup is boiled to remove excess moisture and seeded with small sugar crystals which then grow to the required size. This process requires careful tuning to ensure conditions are ideal for crystal growth but it normally works smoothly. However in the late harvest period in South Africa the process can 'get stuck' in that bubble formation is either inhibited or does not occur, and this means that crystallisation does not proceed normally. Various measures have been used to unstick the process but no generally effective cure has been found and the situation can be so bad that mills have been forced to close over several weeks.

The problem appears to be due to impurities that arise because there are harvesting delays due to rain in the late season.

The question: Why does the process stick and how is it best to deal with a 'stuck' batch?

The Work Done

The boiling of massecuite occurs within a calandria which consists of many open vertical tubes essentially filling a section of the boiler. The calandria is immersed in the massecuite syrup which fills up the tubes. The tubes are heated using steam and this causes massecuite to rise up and pass through the tubes setting up a cyclic exchange. Water vapour escapes out of the boiler. The effect of the impurities introduced is to increase the viscosity of the massecuite, decrease its conductivity, and also change the ability of the massecuite to release water vapour in the form of bubbles. There are other less dramatic effects. Such effects are magnified because the state change occurs within the small diameter calandria tubes and the flow regime changes dramatically if vapour flux levels are reduced. It appears that under normal conditions the flow is ‘slug like’ with a high heat exchange between the massecuite and the tube walls, whereas under hard-to-boil (HTB) circumstances bubbles are released slowly (if at all) and the flow is smooth with little heat transfer. At the meeting two tube flow models were produced to understand the heat exchange and flow process in the tubes; one simple, and the other more detailed. Both are aimed at understanding the effect of the changed circumstances on tube flow and evaporation. These models are in development.

Additionally at the meeting models were produced to understand why a bubble is released more slowly or not at all from the massecuite. A heat exchange model showed that the time taken for the boiling point to be reached is significantly effected by changes in the conductivity of the massecuite. A second model examined the effect of a change is rheology on the release of a bubble. This model suggests that polysachrides trap the bubbles within an elastic/plastic net so that an additional energy barrier needs to be overcome to release the bubbles. Whilst these models cannot do the basic chemistry they may lead to a better understanding of what needs to be done to release trapped bubbles.

POSITION CONTROL AND VIBRATION SUPPRESSION FOR A SINGLE LINK FLEXIBLE ROBOTIC MANIPULATOR USING OPTIMIZATION ALGORITHMS

Industry Representative:
Jimoh Pedro, University of the Witwatersrand

Moderators:
Montaz Ali, University of the Witwatersrand
Kailash Patidar, University of the Western Cape

Student Moderators:
Kazeem Dawodu, University of the Witwatersrand
Peter Popoola, University of KwaZulu-Natal

Executive Summary

In this work we considered a single link flexible robotic arm manipulator. A robotic arm manipulator is used to grip, pick and place an object after moving it a desired angular distance. It has numerous applications in industrial processes. We considered a flexible robotic manipulator over a rigid manipulator due to its overall efficiency. A flexible manipulator operates at a higher speed with a lighter weight for better maneuverability compared to the rigid manipulator that requires a larger motor to produce the required torque.

Several assumptions were made for the designing and mathematical modelling of the robotic manipulator. We assumed that the flexible single link manipulator is fixed at one end (hub) to allow for free rotation and flex along the horizontal plane (link). We also assumed that it is a single input and double output (SIMO) system with the *torque* as the input and the *tip deflections* as the outputs. To ensure the oscillation of the beam along the horizontal direction, its length was assumed to be greater than its thickness. It was finally assumed that the shear deformation of the link on the system was negligible since the beam rotation was considered as a rigid body motion. Sequel to these assumptions, the rotation and deflection of the beam were modelled using the Euler-Bernoulli beam equation

$$EI \, \frac{\partial^4 W(x,t)}{\partial x^4} + \rho \, Al^4 \, \frac{\partial^2 W(x,t)}{\partial t^2} = 0 \, ,$$

where W is the deflection of the beam, E the Young's modulus, I the moment of inertia, ρ the density, A the area of the beams cross-section and l the length of the beam.

We tested this problem both in an uncontrolled and a controlled environment. We found that in an uncontrolled environment the robotic arm had shortfalls which affected its efficiency. For the uncontrolled system, the angle of rotation of the robotic arm diverged with large deflection. It became pertinent to design an optimal control model for the controlled system. In the controlled environment, we designed a PID (Proportional Integral Derivative) feedback controller for the dynamical system (plant). The plant architecture was constructed using the MATLAB based *simulink*. This helped to minimize the performance index in such a manner that the error due to the rotation and displacement of the robotic manipulator reduced over time.

The optimal control model was formulated mathematically as a single objective optimization problem to help in the optimization of the controller gains (controller coefficients) using the *Pattern Search Optimization Algorithm* embedded on the Simulink-MATLAB subroutine. We later re-considered the *Multi-Objective Optimization Algorithm* as a strong tool for handling the control model to help realize faster our controller gains and the Linear Quadratic Regulator (LQR) controller to provide for better performance.

OPTIMAL MOVIE SCHEDULING

Industry Representative:
Obakeng Moepya, Isazi Consulting, Johannesburg

Moderator:
Dr Nick Hale, Stellenbosch University

Student Moderator:
Shane Josias, Stellenbosch University

Study Group Members: Abdulrahman, Abdulsalam, Bashirat , Bongani, Dineo, Evans, Gerhard, Ndamulelo, Ntakadzeni, E. Samuel, O. Samuel, Shireen, Oluwaseun, Tshephang, Willington

Executive Summary

The Problem

A television company has a portfolio of approximately 100 movies that may be played (with repeats allowed) during a given month and wishes to optimize its schedule so as to maximise the total number of viewers.

Obtaining actual viewership numbers is problematic (the TAMS data is sparse and unreliable) so instead they consider a model. The model they choose (which the group was not free to alter) is to associate with each movie a 'movie score' (like IMDB, but not precisely) and with each hour of the day a certain 'time score'. The objective is to maximize over all possible movies in all available time slots, the sum of the product of movie and time scores, subject to numerous constraints.

The constraints included:

- A minimum and maximum number of runs of a movie in a month
- A minimum movie repeat time of 48 hours

- Acceptable playing hours due, for example, to age restrictions
- A minimum and maximum gap allowed between consecutive movies

There is an existing solution obtained by the company using simulated annealing, but our task was to see if we could improve upon this.

The Work Done

Our first step was to reformulate the problem in a more mathematical framework. Scheduling problems are well-understood, and there are numerous approaches for solving them. However, some facets of the given problem, namely the complicated constraints, the scoring metric, and the fact that not all tasks (i.e., movies) are the same length, make this situation more difficult to solve.

The group ultimately decided on an approach based on binary integer linear programming. In order to do this we made the assumption that all movies had a fixed 2 hour length (although we will explain shortly how this may be relaxed). With this assumption, one can block each day into 12 slots and the problem is reduced to deciding which movie to play in each slot so that (a) the constraints are satisfied and (b) the score metric is maximised.

One then considers the binary solution/state vector $\boldsymbol{x}$ corresponding to all possible movies and all possible time slots. That is, if a given movie is to be played at a given time, then the associated entry of the vector $\boldsymbol{x}$ is 1, otherwise it is 0. Each position in this vector $\boldsymbol{x}$ therefore has both a time and a movie score associated with it, so we take the product of these two and represent these in our score vector $\boldsymbol{c}$. The task then is reduced to maximising $\boldsymbol{c}^\top\boldsymbol{x}$.

However, we must also incorporate the constraints. If these can be expressed in the form $A\boldsymbol{x} \leq \boldsymbol{b}$ then we have reduced our given problem to a binary integer linear programming (BILP) problem

$$\begin{array}{ll} \text{Maximise} & \boldsymbol{c}^\top\boldsymbol{x} \\ \text{subject to} & A\boldsymbol{x} \leq \boldsymbol{b} \\ \text{and} & \boldsymbol{x} \in \{0,1\}^n \end{array}$$

where n is the total number of possible time slot-movie combinations (roughly 30*24/2*(number of movies)).

The group was indeed able to represent the first three constraints listed above (as well as the obvious constraints that only one movie may be played at a time) in this form, but we omit the details here.

The Outcomes

The group was able to take the data set given to us by Isazi Consulting and formulate the required BILP problem. Once this is obtained, the solution vector $\boldsymbol{x}$ can be obtained using off-the-shelf software. We used an academically-licensed version of the Gurobi software, as the resulting linear system was quite large.

We were able to produce a provisional schedule for the given data set with a score that was typically higher than obtained by the existing method, although we should remember that we our approach was violating the minimum/maximum gap constraints and was also based on the 2 hour length assumption.

However, we believe that the biggest outcome of the Study Group work was not the development of the algorithm for this problem, but rather showing that the optimal solution to the model problem resulted in a schedule which would be undesirable in practice. In particular, for the numerous data sets we ran the algorithm on, it would always play the highest-rated movies at peak time (6pm) on the first day, and then repeat them 48 hours later until the maximum number of plays had been reached. In the 6pm slot on the second day, it would play the second highest-rated movie and repeat this process. Clearly this is undesirable in the real world. This suggests that there are additional 'hidden constraints' on the real world problem that need to be addressed in the model. The mathematical formulation of the problem as a BILP was instrumental in bringing this observation to light.

Future work

The biggest problem with our approach was the patently false assumption that all movies are two hours long. Towards the end of the MISG week we realised one can rectify this by breaking each of the movies into 15 or 30 minute blocks and enforcing continuity between each of these (which can be expressed as $A\boldsymbol{x} = \boldsymbol{b}$). This would therefore remove the questionable assumption, but would significantly increase the size of the BILP problem and would affect computational time. Our suggestion would be to run a global algorithm on the monthly data set using something like the 2 hour assumption, and then to refine each day using this continuity approach.

POLLUTION FROM HAZARDOUS LANDFILL SITES

Industry Representative:
Viroshan Naicker, Department of Mathematics, Rhodes University

Moderator:
Colin Please, University of Oxford, Oxford, United Kingdom

Student Moderators:
Reem Elmahdi and Gerhard Kristen, University of Stellenbosch

Executive summary

Data has been collected by residents in a neighbourhood of Durban concerning noxious odours and health issues. This report examines how this data might be used to identify possible sources of the reported smells and their level of emission. The data consists of complaints made by individual residents in an area of Durban submitted using an online reporting system. In Figure 1 the region of interest is shown with key features including the approximate location for the complaints. The aim of this report is to consider the complaints data and to explore if the interpretation of the data can be enhanced by exploiting the known physics of the problem.

The approach taken is to exploit a well known simple mathematical model of dispersion of chemicals, called the "Gaussian Dispersion Model". This model accounts for the direction of the wind and the turbulent dispersion caused by the stability conditions of the atmosphere. Some of the parameters in this model can be assessed independently using weather data. However, what is unknown is the strength of any sources of odours and their positions. Hence the mathematical problem is one of considering the inverse problem where the values of the parameters (particularly the position and strength of any sources) are determined by requiring that the mathematical model predict the observed data. A critical aspect in making such an inverse problem computationally feasible is having a simple mathematical model of the problem.

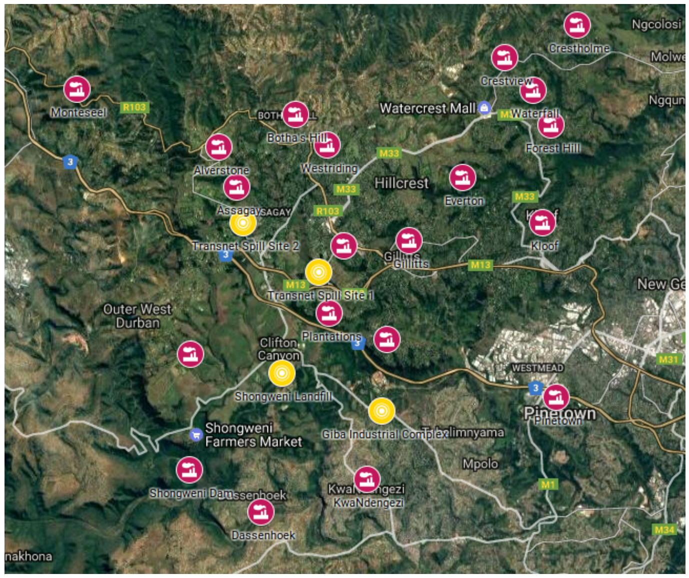

Figure 1: Plan view of region of interest. The red dots represent areas where complaints have been lodged, and the yellow dots possible sources of pollution.

Inverse problems are usually posed as optimisation problems where the values of the unknown parameters are found by minimising some measure of the difference between the predictions of the mathematical model and the physically observed data. In this case there are two significant complicating issues that make the approach needed different from that of most previous work detecting sources of odours. These issues are:

- There is a lack of concentration data. The complainants only record that an odour has been detected. They indicate the type of smell but give no indication of concentration
- There is missing data. There is no data indicating when low concentrations occur and residents do not report all high levels.

These make the inverse problem slightly non-standard. In particular there are two extreme situations that could explain all the data. Firstly the "many sources" case where every complaint is due to a local source such as burning toast in each kitchen, and secondly the "huge source" case where there is an enormous source that gets everywhere and everyone is affected all the time so that complaints are made independent of where the source is. The optimisation problem is therefore multi-objective since there is a need to include practical considerations about the number and size of the sources.

The report sets out the mathematical framework in which the data can be analysed. The existing data has been cleaned up (eg. typographical errors of location are corrected) to enable it to be easily incorporated into the methodology. Critical in the mathematical model is the direction of the wind, the wind speed and the atmospheric conditions. The South African Weather Service has installed a continuous monitoring station in the area which became functional in November 2016. This can provide an independent source of accurate data regarding weather conditions. Future work is to include this to extend and validate current complaint data.

The key step in the methodology is to realise that for a given complaint the mathematical model determines the probability of the source being in a certain area subject to some assessment of the ratio of the sensitivity of the complainant to odour and the strength of the emitting source. By considering the probability of the position of possible sources as predicted by all the complaints for a given sensitivity/emission-strength ratio areas of high probability can be identified. By exploring a full range of reasonable ratios probable sources can be identified. The approach should give an independent assessment, based on the best available physical model, of where the noxious fumes are being emitted.

The next step is to use the data, both from complaints and from the weather service in the mathematical model and explore the resulting probabilities to determine probable emission sources.