# PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

# 2016

# Mathematics in Industry Study Group South Africa
# MISGSA 2016

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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# PREFACE

The thirteenth Mathematics in Industry Study Group (MISG) in South Africa was held in the School of Computer Science and Applied Mathematics at the University of the Witwatersrand, Johannesburg, from Monday 11 January to Friday 15 January 2016.

The total number of registered participants at the MISG was seventy-four. There were seventeen academic staff, forty-three graduate students, nine industry representatives and five invited guests. The invited guests were:

| | |
|---|---|
| Graeme Hocking | Murdoch University, Western Australia, Australia |
| Neville Fowkes | University of Western Australia, Australia |
| Sarah Mitchell | University of Limerick, Ireland |
| Tim Myers | Centre de Recerca Matematica, Barcelona, Spain |
| Alfred Owoloko | Covenant University, Nigeria |

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Aciences
Council for Scientific and Industrial Research (CSIR)
Nelson Mandela Metropolitan University
North-West University
Rhodes University
University of Cape Town
University of Johannesburg
University of KwaZulu-Natal
University of Pretoria
University of Stellenbosch
University of the Witwatersrand

The MISG meeting was opened by Professor Helder Marques, Dean of the Faculty of Science at the University of the Witwatersrand

The MISG followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2015. Six problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by a senior moderator and one or more student moderators. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Workshop from Wednesday 6 January to Saturday 9 January 2016. The objective of the graduate Workshop is to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. Seven problems were presented to the graduate students. The problems and the presenters were:

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| E-Vota | Jeff Sanders,<br>African Institute for Mathematical Sciences and University of Stellenbosch |

| | |
|---|---|
| Safety barriers for dump trucks | Neville Fowkes<br>University of Western Australia |
| The design of a state change instrument | Neville Fowkes<br>University of Western Australia |
| Legalisation of Rhino horn trade | Ashleigh Hutchinson<br>University of the Witwatersrand |
| Linear least square problem and the bias-variance trade-off in machine learning | Montaz Ali<br>University to the Witwatersrand |
| Blood platelet production inventory problem | Michael Olusanya and<br>and Aderemi Adewumi |
| Interaction of birds with wind turbines ` | Craig Symes<br>University of the Witwatersrand |

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

**The sponsors of the Graduate Workshop and the MISG were:**

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- Centre of Excellence in Mathematical and Statistical Sciences

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

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| Fanucchi, Dario | Isazi Consulting, Johannesburg |
| Garber, Nicole | Isazi Consulting, Johannesburg |

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# PROBLEMS

# STORAGE OF SUGAR CANE BAGASSE

**Industry:** Sugar

## Industry Representative:

Richard Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal

## Problem Statement

Co-generation of electricity will provide an additional income source for the sugar industry, Energy for sugar processing is derived from burning bagasse, the fibrous residue that remains after the sugar has been extracted from the shredded sugar cane. Any excess bagasse can be burned to produce electricity which can be exported to the grid.

Sugar cane is not harvested for the whole year so factories shut down during the off-crop. It would not be attractive to only produce electricity for part of the year. Bagasse needs to be stored so that it can be burned during the off-crop to generate electricity all year round,

After the extraction process, the fibre from the cane is squeezed in a drying mill. This gives a final moisture of approximately 50%. There have been reports that if the bagasse is stacked in this state, the heating effect of microbial activity can lead to spontaneous combustion. The literature suggests that the likelihood of spontaneous ignition depends on factors such as moisture content, fermentable sugar content, oxygen availability, packing density and stack height.

Adjusting the moisture content has consequences in terms of energy loss. If the moisture content is increased prior to storage then this needs to be evaporated when the bagasse is burned in the boiler, that is, the net calorific value is reduced. Reduction of moisture requires energy to evaporate the water before the bagasse is stored,

Pelletising the bagasse has been suggested as an alternative to bulk stacking. To do this, the bagasse needs to be dried to 20% moisture, ground into smaller particles and then heated under pressure to cause it to bond into pellets. This process consumes some of the energy which would otherwise be available for electricity.

Research questions to consider:

- Is there an optimum/maximum height to avoid spontaneous combustion?
- Will there be an advantage in adjusting the moisture content in terms of usable energy stored per unit area per unit cane?
- Will there be an advantage in pelletising the bagasse in terms of usable energy stored per unit area per unit cane?

# ROCK STRENGTH, ROCK BRITTLEMESS AND BLAST FRAGMENTATION

**Industry:** Mining

## Industry Representative:

Richard Staycey, School of Mining Engineering, University of the Witwatersrand, Johannesburg.

## Problem Statement

Rock fragmentation by blasting is usually ``tougher'' for stronger rocks, resulting in an increase in fragmentation size with rock strength. Contrary behaviour was recently experienced for an extraordinarily strong rock: A far greater percentage of fines occurred than was expected, and investigations were carried out in an attempt to explain this. It appeared that the fragmentation experienced could be related to both the strength and the brittleness of the rock. However, there was little information linking brittleness to fragmentation, and there are about 30 different definitions of brittleness. This perhaps explains the lack of a well understood link between fragmentation and blasting

The higher the rock strength, the greater the amount of energy contained in the rock at its peak strength, and therefore it could perhaps be expected that there should be a correlation between fragmentation resulting after failure of rorck specimens, whether in uniaxial compression strength (UCS) tests or in small scale blasting tests.

Research question to consider:

The empirical results show clear correlations between rock strength and fragmentation and between rock brittleness and fragmentation. The research question is whether these relationships can be supported by theory, so that brittleness, in combination with strength, can be taken into account in the prediction of fragmentation in blasting.

# AIR TRAFFIC FLOW MANAGEMENT REROUTING PROBLEM

**Industry:** Air transport

**Industry Representatives:**

Alex Alochukwu, University of KwaZulu-Natal

## Problem Statement

The air transportation industry has experienced rapid growth in recent times and the demand for airport and airspace usage increases exponentially as the number of users increase. Moreover, congestion problem abounds almost on a daily basis as a result of bad weather conditions and other unforeseen factors. This has a serious effect and impact on the Air Traffic Control System as well as on the nation's economy due to the significant costs incurred by the airlines and passengers arising from the flight delays. Hence, the need for an efficient and safe air traffic flow management that mitigates delays and congestion problems as well as minimize the cost of total delay.

Currently, the Central Airspace Management Unit (CAMU) is responsible for the management of air traffic flow and capacity management within South African airspace in collaboration with the South Africa Air Traffic Navigation Services (ATNS), the sole commercial provider of air traffic, navigation and associated services and responsible for air traffic control in approximately 10% of the world's airspace.

The problem posed to the study group is to find good and optimal scheduling air traffic flow management strategies that minimize delay costs as well as reducing the impact of congestion problems while satisfying the airport and en-route airspace capacity constraints.

# THE UTILITY PRICING DEATH SPIRAL

**Industry:** Energy (Electricity)

## Industry Representative:

Barend Erasmus, Global Change and Sustainability Research Institute, University of the Witwatersrand

## Problem Statement

Utility providers in large metros price electricity to cover the cost of infrastructure and generation. The electricity network – the wires and poles – have a fixed cost and consumers pay a charge for consumption to cover the cost of this infrastructure. The exact price is calculated to cover the cost of infrastructure that can cope with loads at peak usage. If the consumer client base shrinks, then the utility needs to increase prices to cover the fixed costs of using the network infrastructure. This means that a smaller client base has to start paying more, for the same service.

In South Africa, most of the metros rely on high-income clients for most of their income. However, this high-income group of clients, are also the most likely, and have the means, to move to an alternative source of electricity. This is especially likely if the higher prices are coupled with a perception of substandard service, as was experienced by recent bouts of load-shedding.

This puts the utility in a catch 22 position – not only do they need to increase prices, but at the same time, the customer base most likely to be able to afford these prices, are also capable of completely going off-grid. In turn, this puts additional pressure on the remaining users, pushing prices up yet again. Even if no-one goes off grid the increased awareness of the cost of electricity will also result in reduced demand.

Another complementary issue is the even larger effect of companies shutting down moving to solar photovoltaic energy generation.

Research questions to consider:

- At what point do price increases become non-viable as a strategy? Non-viable is defined as too low income for the utility to maintain the infrastructure.
- What other options exist for the utility provider to ensure a stable income?
- What are the implications for the deployment of renewable energy in metros?

# AUTOMATED DATA COLLECTION FROM A FOOTBALL VIDEO

## Industry: Sport

## Industry Representative:

Dario Fannucchi, Isazi Consulting

## Problem Statement

Data is playing an increasingly important role in sports. Good data assist a scout to choose the most promising candidates for potential recruitment, a coach to focus on the right player combinations and training schedule for his team, a player to better understand and improve his strengths and weaknesses, an advertiser to determine which players on the field generate more crowd and viewer activity and a betting house to fine tune the odds in advance of a game. Data driven decision making has revolutionized baseball in the USA and plays a significant role in many other sports. In football, the vital statistics and running distances of players are monitored by devices worn on their bodies, and most games generate several independent streams of video footage from different cameras, The German football team that won the 2014 World Cup used SAP software to analyse their games as well as those of their opponents. The software suggested combinations and moves that would be most likely to succeed against their upcoming opponents and helped monitor training sessions. Collecting and curating quality data from a football match is therefore clearly of immense value for a team and other stakeholders.

The data typically collected from a football game includes goals scored, assists, number of shots at goal, possession information, corners, off-sides, fouls, cards given, injuries, substitutions and sometimes the running distance of each of the players. There is scope for the collection of larger data sets, such as the position-per-time of the ball and each player on the field throughout the game, or detailed information on player posture throughout the game. Higher level information such as player sprint speeds, passing etc. could be derived from these data sets and all of the above `small' statistics could also be calculated.

Isazi Consulting has engaged with a number of local football clubs who have expressed interest in making use of detailed data to aid in decision making, with an aim to emplying sophisticated methods to mine this data and generate insights. There is therefor some appetite in the local market for the lower level detailed data of the kind described in the previous paragraph. We are interested in assessing the viability of collecting such data from the video feed of a match.

Given the video footage of a football game, what can be inferred about the position of the soccer ball and players on the field as a function of time. We are interested in the following outcomes from the Study Group

- What image processing and video processing methods are available in the literature for identifying and tracking objects in video?
- What methods are available in the literature for detecting a sudden switching of cameras?
- What existing research is available on mapping something from a video into real 3D coordinate space?
- Is it possible to identify and track the soccer ball consistently throughout a game simply from the published footage of that game?
- If several camera feeds are made available for a game, does this improve the ability to track the ball?
- Is it possible to map the coordinates of the ball into real 3D or 2D ``field'' space?
- Is it possible to track players in the game from the video feed alone?
- What methods are most successful at tracking players/balls in a football match?

Some match data will be made available for the Study Group.

# ASSESMENT OF THE PERFORMANCE OF COMPOSITE LINER: SHOTCRERE AND TSL

**Industry:** Mining

## Industry Representative:

Halil Yilmaz, School of Mining Engineering, University of the Witwatersrand

## Problem Statement

Shotcrete, typically 50 mm to 100 mm thick, has been a traditional surface support applied at the mines for many years. Thin Spray-on Liners (TSL's), on the other hand, have been an alternative to shotcrete with its reduced thickness of about 5 mm to 10 mm. The use and acceptance of TSLs however, has been problematic simply due to the wrong perception that ``a thin liner is a weak liner''. However, numerous physical property tests on both TSLs and shotcrete demonstrated that TSLs, in fact, match and in some cases over perform the strength of shotcrete.

It seems that the acceptance of TSLs would still take a while and shotcrete cannot be singled out in this process of acceptance. It is possible that the worth of TSLs could be shown by TSLs' influence in the performance enhancement of shotcrete. In this case, a TSL is applied over shotcrete to form a composite liner.

A number of laboratory tests have been performed to check the influence of TSLs on rock and shotcrete performance. Masethe (2015) found that tensile strength of shotcrete increased by 40% after 28 days when covered by 5 mm TSL. Mpunzi's (2011) laboratory tests on rocks coated with TSL material showed that the sprayed liners enhance the strength of the rock.

The research question that would be addressed during the workshop would therefore aim to address and quantify the positive contribution TSLs would impose over shotcrete performance. Would it be possible to match the performance of thicker shotcrete by reducing the thickness of shotcrete when it is used in combination with a TSL? If the outcome is positive, then one can, at least, motivate the application of TSL as surface support. The advantages in reducing the thickness of liner application are that the quantity of material transported to the excavation site would be reduced easing the logistical requirements, transport costs and spraying time.

# EXECUTIVE SUMMARIES

# SPONTANEOUS COMBUSTION OF STORED SUGAR BAGASSE

**Industry Representative:**
Richard Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal

**Moderator:**
Rahab Kgatle, University of the Witwatersrand

**Student Moderators:**
Nothando Mhlongo, African Institute of Mathematical Sciences and Gediyon Weldegiyorgis, University of Pretoria

**Study Group Members:**
Tim Myers, Centre de Recerca Matem‘atica, Barcelona, Henri Laurie, University of Cape Town, Sarah Mitchell, University of Limerick, Gideon Fareo, University of the Witwatersrand

## Executive Summary

The fibre residue that results after sugar has been washed from shredded sugar cane is called bagasse. After the sugar concentrated liquid is extracted from the shredded cane, the remaining bagasse has a moisture content of 45-55%. The bagasse is a valuable resource for co-generation of electricity hence stockpiling is essential to using it when required. It is well known that bagasse stockpiles can spontaneously combust which amongst other issues leads to loss of a valuable resource. Consequently the Sugar Milling Reserch Institute are looking for guidelines for safe storage. Three issues raised by the Institute which are critical in finding safe methods for bagasse storage and avoiding spontaneous combustion are:

1. Calculating the maximum height of the bagasse heap required to avoid spontaneous combustion.

2. Investigating whether or not there are advantages in adjusting the moisture content (usable energy per unit area).

3. Investigating whether or not there is an advantage in pelletizing the bagasse (usable energy per unit area).

Self-heating experiments on stored bagasse were conducted in [1, 2] following the first recorded spontaneous combustion incident of the Mourilyan stockpile in March 1983. After two more bagasse ignition incidents between 1983 and 1988, an investigation was made by Dixon [3]. The main result of the latter study demonstrated the significant role that moisture content plays in the combustion process and concluded that it should never be neglected in the mathematical modelling of the problem. A paper by Gray *et al.* [4] took the moisture content into account and laid the groundwork for the discussions and mathematical models analysed during the study group. We attempted to address the three issues raised by industry. In the study, Gray considered a one dimensional model in which the temperature $U$, the vapour $Y$, the liquid $X$ and the oxygen $W$ in the stockpile all depend on the spatial variable $x$ along the height of the stockpile and the time $t$. The model constitutes the following four equations:

$$(\rho_b c_b + m_w X c_w)\frac{\partial U}{\partial t} = Q\rho_b ZW\exp(-E/RU) + Q_w\rho_b Z_w XW\exp(-E_w/RU)f(U) + L_v[Z_cY - Z_eX\exp(-L_v/RU)] + \kappa\nabla^2 U, \quad (1)$$

$$\frac{\partial Y}{\partial t} = Z_eX\exp(-L_v/RU) - Z_cY + D_Y\nabla^2 Y, \quad (2)$$

$$\frac{\partial X}{\partial t} = -Z_eX\exp(-L_v/RU) + Z_cY, \quad (3)$$

$$\frac{\partial W}{\partial t} = -F\rho_b ZW\exp(-E/RU) - F\rho_b Z_w XW\exp(-E_w/RU)f(U) + D_w\nabla^2 W, \quad (4)$$

where the values of all the remaining unknown parameters in (1) to (4) are given by Gray [4]. The function $f(U)$, which was obtained from experiments [5], is a switch. Below 58°C, $f(U)$ is approximately 1. There is a rapid transition to 0 as $U$ approaches 58°C. This implies that for temperatures $U < 58$°C the overall reaction is driven by the moisture dependent reaction while for temperatures $U > 58$°C the moisture dependent effect vanishes and the overall reaction is then driven by oxidation. The nonlinear diffusion equation for temperature, (1) is derived from the energy balance condition where generation of heat in the problem results from the oxidation reaction in the dry bagasse portion of the stockpile, the moisture dependent reaction in the wet bagasse portion of the stockpile and the latent heat required for the evaporation of liquid or condensation of steam processes to occur. The mass balance equations (2) and (3) result from the moisture content in the bagasse. From (3), the rate of change of the liquid content in the bagasse increases due to condensation of vapour and decreases due to evaporation. There are small quantities of liquid such that the diffusion process is neglected. From (2), the amount of vapour in the bagasse increases due to evaporation and decreases due to condensation and the diffusion of vapour is also considered. At temperatures less than 58°C, oxygen levels in the bagasse are low and they rapidly increase for temperatures greater than 58°C.

While Gray considered the Newton cooling boundary conditions on both ends of the bagasse, we only used the Newton cooling boundary condition at the top of the bagasse. The remaining boundary condition was obtained from the assumption that the bottom of the stockpile is completely insulated. The assumption is motivated by the fact that the stockpiles would be stored onto flat surfaces with negligible conductive properties. Gray used the Newton cooling boundary condition on both ends of the bagasse such that the problem was symmetrical about the axis corresponding to the insulation boundary condition defined in our problem. Therefore while Gray's problem was defined on a domain with two symmetrical halves, our problem was defined on one of those halves. This implies that solving our problem would be similar to solving half of Gray's problem. It follows that the results from our work are not expected to vary much from results obtained by Gray.

We began by considering the steady state solution since our interest was to find whether combustion would occur after a sufficiently long period of time, "the worst case scenario". Steady state was a sufficient assumption as the moisture reaction is quite rapid (of the order of days) while the entire storage time for bagasse can be up to approximately 9 months. This is substantiated by the time evolution of temperature profiles obtained from experiments in [5] which show that there is a sharp increase in temperature for the first ten days after which the temperature stabilizes over approximately the next 200 days. Following a gradual temperature drop, stability is reached again after 350 days.

We non-dimensionalised the system of equations (1) to (4) to determine the order of magnitude of various terms. This significantly simplified the problem.

Firstly, we assumed that the vapour variable is a constant at ambient conditions resulting in a steady state solution for the liquid content in the bagasse. We also assumed that the oxygen variable is a constant at ambient conditions. It remained to solve one equation, the non-linear diffusion equation for the temperature. We considered the steady state form of the non-linear diffusion equation for temperature and solved it using both analytical and numerical methods. The solution indicated that the assumptions made were collectively unrealistic as the problem yielded results with very small piles as compared to those in the literature.

Secondly, as before we assumed that the vapour variable is a constant at ambient conditions resulting in a steady state solution for the liquid content in the bagasse. However, unlike in the simplest case that was investigated earlier where the oxygen content was assumed constant, we relaxed this assumption by allowing the oxygen variable to evolve over time along the height of the pile. This led to a coupled system of equations for the temperature and the oxygen levels in the bagasse, (1) and (4). This problem was solved numerically and we found that the maximum height of the stockpile depends on the ambient moisture content. A safe limit for the stockpile height could therefore be recommended to the Sugar Milling Research Institute. An increase in humidity (which leads to an increase in moisture content in the stockpile) can cause a previously stable stockpile to ignite. The height of the stockpile (avoiding ignition) is inversely proportional to the humidity levels in the atmosphere wherein the bagasse is stored. It therefore follows that knowing the general humidity of the environment in which the bagasse is stored assists greatly in choosing an appropriate height of the stockpile which does not lead to combustion. Another model that was discussed was that of pseudo steady state conditions resulting from assuming that the conductivities of the oxygen and the vapour in the bagasse were negligible and allowing all other variables to depend on time. An interesting result that was reflected in our analysis, and was obtained by Gray, is that in a previously stable stockpile, the direct increase of the liquid content (due to rainfall) can cause the stockpile to ignite.

Lastly, we considered the case whereby the density of the bagasse varies linearly from the bottom (where it is the most dense) to the top (where it is the least dense) of the stockpile. We found that the temperature distribution from the bottom of the stockpile to the top of the stockpile is more gradual than when the density of the bagasse along the height of the pile is assumed constant.

# References


[1] B.F. Gray. Progress report to Sugar Research Institute, University of KwaZulu-Natal, Durban, July 1984.

[2] B.F. Gray, J.F. Griffiths and S.M. Hasko, Spontaneous ignition hazads in stockpiles of cellulosic materials: criteria for safe storage. J. Chem. Tech. and Biotech.A (1984) 34A, 453.

[3] T.F. Dixon. Spontaneous combustion in bagasse stockpiles. Proceedings of the 1988 Conference of the Australian Society of Sugar Cane Technologists, Cairns, Queensland, Australia. Edited by B.T. Egan, 1988, pp. 53 – 61.

[4] B.F. Gray, M.J. Sexton, B. Halliburton and C. Macaskill. Wetting-induced ignition in cellulosic materials. Fire Safety J. (2002), 37, 465 – 479.

[5] Halliburton B. Ph.D. thesis, Macquarie University, Sydney, Australia, 2001.

# ROCK STRENGTH, ROCK BRITTLENESS AND BLAST FRAGMENTATION

**Industry Representative:**
Richard Stacey, School of Mining Engineering, University of the Witwatersrand, Johannesburg

**Moderator:**
Neville Fowkes, University of Western Australia

**Student Moderator:**
Argaz Ibrahim, African Institute for Mathematical Sciences

## Executive Summary

The Kuz-Ram fragmentation model is an experimental model that has been used for the past twenty-five years for estimating the mean fragment size and distribution after quarry blasting. It is well known that the model is inadequate in that the formula does not directly account for rock properties such as the yield strength, Young's modulus and brittleness. The Study Group was asked to investigate the possibility of improving the model to take into account rock brittleness and rock strength in the prediction of fragmentation in blasting. It was suggested that the Tarasov-Potvin (2013) brittleness index be considered. This brittleness index is based on determining the amount of energy expended in static stress/strain laboratory tests and can distinguish between brittle rocks such as granite and ductile rocks such as sandstone.

In response the Study Group developed three models.

The method of dimensional analysis was applied to derive a new empirical fragmentation model. The formula obtained for the mean fragment size contains what are seen to be the most important parameters of the problem including rock properties such as brittleness and yield stress and the speed of propagation of elastic primary waves as well as the explosive energy input per unit volume per unit time due to the charge. Observations will be necessary to determine if this model describes rock fragmentation accurately.

A breaking springs model was developed the aim of which was to determine the dynamics of fragmentation. As the external stress is increased some springs break and

the remaining springs bear the load. Intact rock corresponds to intact springs and cracks correspond to broken springs. The distribution of spring breakages should correlate with fragment sizes. Correspondences between the spring characteristics and the rock characteristics have been identified.

A continuum model is also under development. The face of a semi-infinite rock mass is impulsively hit which models the effect of an explosive charge. If the stress levels that are generated are less than fracture levels then a longitudinal pressure pulse travels from the face into the rock mass. If the stress levels exceed fracture levels then the rock will partially crush and crack. The aim of the analysis is to determine the speed of propagation of the moving fragmentation front, the extent of propagation and the expected fragment size.

# AIR TRAFFIC FLOW MANAGEMENT

**Industry Representative:**
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## Executive Summary

The Air Traffic Flow Management Problem (ATFMP) is designed to model and optimise air traffic over a defined geographical area, and without exceeding airport or route capacity. It is a planning activity designed to address overcapacity problems which occur either when airspace capacity is reduced or when demand is high. The ATFMP manages air traffic to ensure safe and efficient flow of aircraft throughout the airspace at the lowest cost.

The ATFMP prevents over capacity of airports and flight routes by modifying the departure times and trajectories of flights, either by assigning ground holding delay, airborne holding delay or various other control actions, including rerouting of flights, flight cancellation and speed control.

A fundamental challenge for air traffic management arises when there is a system disruption, due to weather conditions, equipment outages or air traffic demand surges. These disruptions can be unpredictable and cause significant capacity problems. For instance, there are temporary and substantial reductions in airspace and airport capacity whenever there are adverse weather conditions.

Air traffic managers are faced with challenges when the number of flights departing or arriving from a certain airport as well as the number of aircraft traversing a particular sector of airspace exceeds the usual capacity. This may be as a result of a change in the number of runways available, air traffic control (ATC) capacity, airspace restrictions and restrictions as to which aircraft can follow an aircraft of a given class. In such cases, the air traffic managers must find optimal scheduling strategies that mitigate congestion as well as minimise delay costs.

Since disruption will always occur at some time, the question that arises is how to formulate the problem to account for these uncertainties in the system, as well as how to re-optimise the schedule after disruptions using rerouting or departure delay

options (ATFMRP). The ATFM can be large and hence difficult to solve in terms of CPU time, hence a computationally efficient model is needed for the ATFMRP.

The study group investigated methods of formulating and solving the ATFMRP. Artificially constructed data sets were used.

# UTILITY PRICING DEATH SPIRAL

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## Executive Summary

Utility companies generate and supply electricity to customers for both commercial and household usage. In recent years, concerns have been raised about the possibility of a "utility death spiral", which could result in dramatic increases in the price of electricity and the closure of many affected utilities. In this work we developed a mathematical model in order to investigate the conditions under which a utility death spiral is likely to occur.

Utilities generate most of their revenue through electricity tariffs and their income is dependent on a consistent demand for electricity. However, as the cost of renewable energy technology such as solar panels drops, many customers may begin to generate their own energy. This transition to renewable energy sources decreases the overall electricity demand from the utility and lowers their total revenue.

Costs for the maintenance and repair of infrastructure form a large part of the utility's budget and need to be covered regardless of the drop in demand for electricity. In order to maintain their budget, tariff prices are increased. This creates further incentive for customers still connected to the grid to transition to renewable sources. A feedback loop is created as more and more users are tempted into using renewable energy and the utility must rely on a smaller pool of users to cover its costs.

In South Africa, utilities are even more sensitive to this problem because they provide cheap or free electricity to consumers that otherwise cannot afford to pay for it. The utilities are heavily dependent on high tariff customers, not only to subsidize the provision of free basic electricity but also to cover the cost of maintaining and expanding the national energy grid. Unfortunately, the high tariff customers are most likely to have the means and motivation to switch to some form of alternative energy.

In this work we provided a mathematical model that explains the number of users in the different tariff groups and predicts the effects of possible strategies to maintain the utility's income. Factors such as inconsistent electricity supply and high tariff prices are taken into account in order to model the movement of users from high tariff groups to a group that supplements its electricity with renewable energy. Other relevant factors, such as the rate of economic growth, are taken into account and discussed.

# AUTOMATED DATA COLLECTION FROM A FOOTBALL VIDEO

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## Executive Summary

Data driven analysis of sports is becoming increasingly valuable to many stakeholders for understanding and improving many aspects of team performance. One major difficulty is in acquiring this data, particularly if information about opposing teams is required, which they are unlikely to willingly provide. A solution to this is to build computational models to automatically extract relevant information from video footage. To this end, the Study Group was asked to investigate methods for performing this analysis in the context of football games. There were many desired outputs, but the group chose to focus on methods for identifying the position and orientation of the cameras which recorded the games, as well as tracking the position of the players and ball on the field.

The problem was thus divided into two main components: camera localisation and player tracking. Camera localisation was viewed as a critical problem, without which player positions could not be mapped into absolute coordinates on the field. This was successfully addressed by solving for the projection, rotation, and translation matrices of each image/frame, by finding correspondences between the field lines on the image and those of a ground-truth football field (using a Hough transform). This allowed each frame to be projected back into the absolute coordinates of the field.

Players and the ball on the field could then be identified through a combination of background subtraction and edge detection (in particular the Sobel edge detector). Computing the optical flow between successive frames then allowed for the moving objects to be tracked.

Although this represented a first-pass solution to the problem, the Study Group was able to convincingly show that it is possible to identify the position and orientation of a single camera recording a football game, project its view into absolute field coordinates, and finally track players and the ball as they moved around the field.

# ASSESSMENT OF THE PERFORMANCE OF COMPOSITE LINER: SHOTCRETE AND THIN SPRAY-ON LINER (TSL)

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## Executive Summary

Shotcrete is used in mines to provide structural support to tunnels, It is structurally strong but it is expensive to apply because layers are typically 50 mm to 100 mm thick. Thin spray-on liners (TSLs) have reduced thickness of 50 mm to 10 mm. They are inexpensive to apply but do not provide structural support. The aim of the investigation was to determine if TSLs can be used in combination with shotcrete to impose structural performance. Is it possible to reduce shotcrete thickness, but maintain the same performance, by a TSL application over or under shotcrete.

A two-dimensional model was formulated using the plane stress theory of elasticity. The stress tensor was expressed in terms of the Airy stress function which satisfies the biharmonic equation. A layer of rock in the form of a beam supported at its ends was considered. A layer of shotcrete with TSL over the shotcrete and TSL under the shotcrete was applied to give support to the beam. Boundary and matching conditions at the interfaces were imposed. If the shotcrete is applied first it will bond strongly to the rough surface of the rock. If the TSL is applied first it will penetrate fractures in the rock mass and repair fractures while in other circumstances we expect the TSL not to contribute. The shotcrete will bond less strongly to the smooth surface to the TSL which may lead to debonding that can be modelled by the interface conditions. The analysis continued after the meeting. We await confirmation from the analysis.