THE EFFECT OF ALLOWING TAXIS TO USE BUS LANES ON BRT ROUTES II

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Abstract

In order to facilitate the flow of traffic in Gauteng Provence, South Africa, during peak hours the BRT is investigating the effect of allowing minibus taxis to use the lanes presently reserved for buses either throughout the day or just during peak hours. This work represents a continuation of the work commenced at the MISG 2014 meeting where it was shown that the traffic flow through the system was improved by allowing minibus taxis to switch across to the bus lane providing the density of the traffic exceeded a critical value determined by the flow characteristics of the lanes. In the present work we investigate further the disruptions that are likely to be caused by the movement of taxis into the bus lane. Additionally we present an economic pricing model that may be used to encourage commuters to travel by bus.

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1 Introduction

The Rea Vaya (or BRT) is Africa's first rapid transit system and has been designed to ensure fast, reliable and affordable public transport across Johannesburg. There are presently 3 trunk routes, 5 complementary routes as well as feeder routes. The system supplements the Metrobus system by providing good (4 or 6 lane) roads along major routes, with (2) dedicated lanes for buses. There are about 270 Rea Vaya buses in service. The Metrobus service has about 450 buses as well as rail transport. The buses on the BRT carry 90 to 117 passengers with 50% to 100% occupancy, and run at 5 minute intervals during peak hours and 10 minutes off peak.

Competing transport modes on the Rea Vaya are cars (including normal cabs) carrying about 1.5 passengers, and about 300 minibus taxis with carrying capacity 8 to 14 passengers. About 65% of passengers use the minibus taxis. The dedicated bus lanes are normally not congested whereas the normal lanes are usually congested especially during peak hours, and the authorities are investigating the effect of allowing minibus taxis to use the bus lanes. The proposal is that minibus taxis be allowed (or forced) to transfer across into the bus lane at locations adjacent to a bus stop as shown in Figure 1.

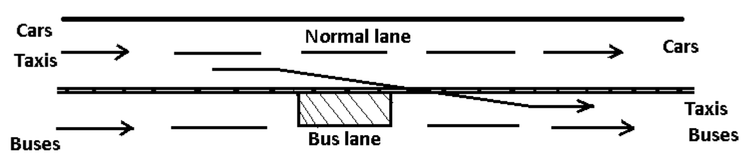

Figure 1: A simple lane changing scenario: minibus taxis travelling in the normal lane switch to the bus lane at a switching point, normally a bus stop. Part of the barrier near the bus stop would be removed to facilitate the movement of the minibus taxis into the bus lane. This Figure represents one half of the traffic system with vehicles in the other half travelling from right to left.

One would expect the shared bus lane arrangement to increase the net flow of all traffic (and also the average travel time of passengers) and at the MISG 2014 [3] this was shown to be the case for traffic densities greater than critical values which were identified in terms of the road traffic parameters and the relative proportion of cars and cabs, buses, and minibus taxis. The effect of the shared bus lane arrangement would be to increase flow in the bus lane which would normally reduce the average speed on this lane and this could potentially effect bus timetable reliability. Such reductions in average speed on the bus lane can be readily accounted for [3], whereas the unexpected disruptions and associated dynamic instabilities cannot be anticipated, and as such are more likely to be of concern. Such effects are more likely under lane switching conditions especially under congested traffic circumstances. These dynamic effects were examined in the MISG 2014 report [3] but will be further reported on here in Section 2, with more detailed lane transfer models described in Section 3. In Section 4 a simple model is introduced that may be used to determine an effective procedure to encourage passengers to use bus transport.

2 The flux model: criticality

In the simplest steady state, single lane traffic flow models, the variables introduced are the traffic density N (vehicles/km), the vehicular speed V (km/hr), and the flux F (vehicles/hr), and the driver (or traffic) behaviour is described by

$$V(N) = V_{max}(1 - N/N_{max})^{\alpha}, \tag{1}$$

where (N_{max}, V_{max}) are the maximum density and speed on the lane and α is a parameter chosen to fit available data for the lane or lanes of interest. When combined with the flux verses density relationship

$$F(N) = N \cdot V(N) \tag{2}$$

this gives the steady state flux verses density relationship

$$F(N) = NV_{max}(1 - N/N_{max})^{\alpha}, \tag{3}$$

and after applying car conservation and assuming an instantaneous driver response to the local traffic state, one obtains the well-known continuum traffic model

$$N_t + F'(N)N_x = 0. \tag{4}$$

Boundary and initial conditions are appended to complete the 1-D dynamic model. Higher order models have been developed (with limited success) primarily to deal with the unstable flow that occurs for flux levels close to the maximum, see [4], [5].

In the classical Lighthill–Whitham model, $\alpha = 1$, so that the velocity profile is linear and the flux profile is quadratic with the maximum possible flux $N_{max}V_{max}/4$ occurring at half the maximum density $N_{max}/2$. The α fitted models differ primarily in that the maximum flux occurs at a larger traffic density and flux level in the $\alpha < 1$ case, and at a lower density with lower flux level for values of $\alpha > 1$, see Figure 2. The associated $V(N)$ profiles are also displayed.

The determination of the maximum flux and associated density for a road is of primary practical importance because if flux levels exceed the maximum possible steady state value (*the critical value*) then major flow instabilities result. Locally the flux levels can increase beyond the maximum steady state value but this cannot be sustained and will necessarily lead to an adjustment which will show itself as a disturbance that will propagate through the traffic system. Attempts have been made

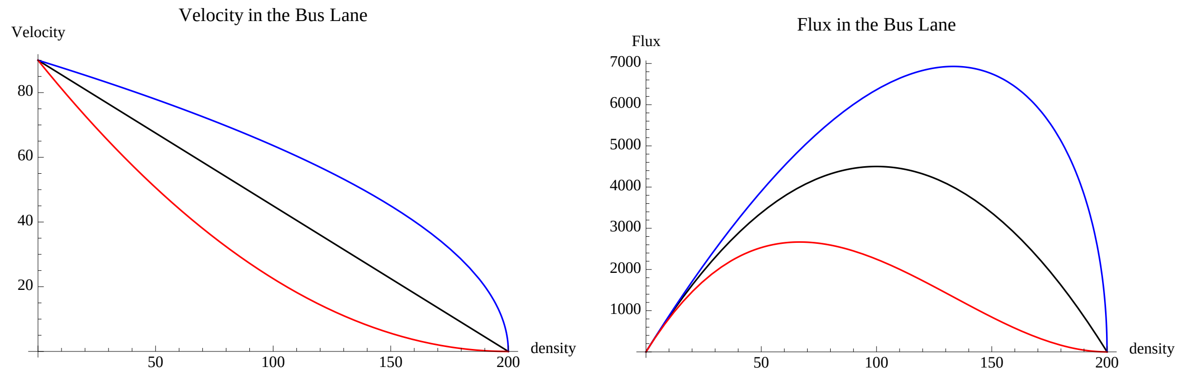

Figure 2: Traffic flow models: the upper curves corresponds to $\alpha = 1/2$, the lower curves to $\alpha = 2$, and the middle curves to $\alpha = 1$. For values of $\alpha < 1$ the maximum flux occurs at a higher traffic density, whereas for $\alpha > 1$ the maximum flux occurs at a lower density and flux level.

to quantify the resulting behaviour using an enhanced dynamical model, see [5], but the message is simple; design the traffic system to avoid this undesirable situation. Evidently this can be achieved by 'over designing' the system so that maximum flow will only occur under rare circumstances, and when these circumstances do occur steps should be taken to reduce the flux through the system, for example by controlling the traffic light timing.

The above single lane model is robust and is sensible for also describing the multilane flow in the normal flow lanes and flow in the (normally) single bus lane. Our focus here will be on the bus lane. Because of the size difference between buses and minibus taxis, the maximum density $N_{max} = N_{max}(\beta)$ (in vehicles per km) in the bus lane will depend on the proportion β of minibus taxis in this lane, and a sensible model is given by

$$N_{max}(\beta) = (1-\beta)N^b_{max} + \beta N^t_{max}, \text{ with } 0 < \beta \leq 1, \tag{5}$$

where N^b_{max} the maximum number of minibus taxis on the bus lane with buses absent, and N^t_{max} is the corresponding result if taxis are absent.

2.1 Traffic parameter estimates

Typical vehicle lengths are: car or cab 3 to 3.5 m, minibus taxi 5-6 m and bus 10-12 m. Based on these values (and allowing an additional vehicle spacing of 0.5 m under maximum density conditions) we get approximately $N^b_{max} = 70$, $N^t_{max} = 150$, and assuming there are ten times more minibus taxis in the traffic stream than buses ($\beta = 0.9$) we obtain $N_{max}(0.9) = 140$. The resulting flux and velocity curves for a variety of values of β are shown in Figure 3 where a maximum velocity in the bus lane of 80 k/hr is assumed. Note that because of the relatively large numbers of minibus taxis in the traffic stream compared with buses the behaviour will change dramatically if minibus taxis transfer across to the bus lane.

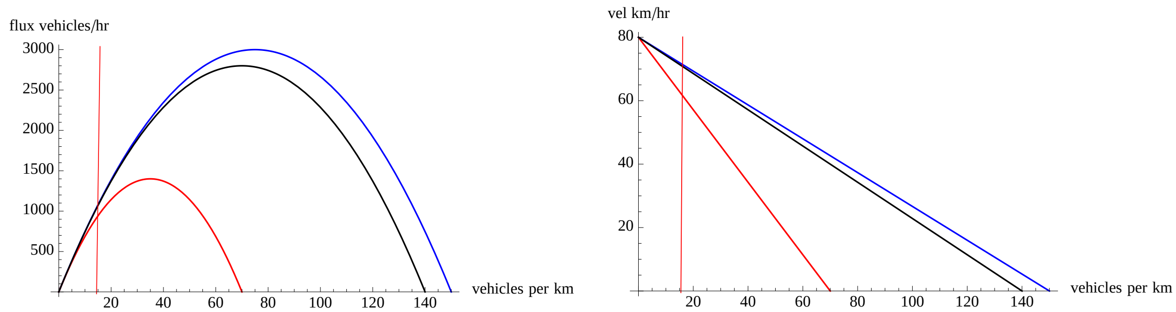

Figure 3: Changes in traffic parameters in the bus lane brought about by sharing. The lower curves corresponds to $\beta = 0$ and represents the present situation with no minibus taxis in the lane. The upper curves represent the situation if only minibus taxis occupied the lane. The middle curves correspond to $\beta = 0.9$ and represents the situation under present day circumstances if taxis were to share the bus lane. The vertical lines correspond to present density and flux levels. *Left:* Flux changes. *Right:* Velocity changes.

We now make estimates for present day flux/density levels at peak hours. At present buses for a particular service arrive in 5 minute intervals, so with about 3 services operating and delays we will assume a 1 minute interval between buses. With a maximum speed of 80 km/hr this gives a flux of 60 buses per hour and a density of about 1.3 buses per km. Assuming, as above, there are ten times more minibus taxis in the traffic stream that now join the buses in the bus lane there will now be about 15 vehicles per km sharing the lane, with the vehicular flux increasing to about 70 vehicles per hour, see Figure 3. These results suggest that present day flux/density levels are not close to the critical values, so major dynamic disruptions are not likely. Note however that the average vehicular speed would be reduced to about 73 k/hr; which would significantly affect the bus time table. These calculations were based on guessed data and on the classical parabolic flux density relation with $\alpha = 1$. If the $\alpha = 2$ model were more appropriate for the Rea Vaya bus lane, then critical flux levels would be reduced from 2700 vehicles per hour to 1700 vehicles per hour; still somewhat above present day values but now criticality issues may be of concern. Evidently no useful advice can be given until useful background data is supplied for the road system.

3 Lane transfer

The time and length scale required for the movement of minibus taxis from the normal lane to the bus lane and the associated disruption to the traffic flow would be very strongly dependent on the length of the transition zone and the approach to the crossing point. Relevant equations modelling the transition are given by

$$N_t^n + F_x^n = -Q(N^n, N^b), \tag{6}$$

$$N_t^b + F_x^b = Q(N^n, N^b), \tag{7}$$

where (N^n, F^n) and (N^b, F^b) are the number of vehicles and flux in the normal and bus lanes respectively, and Q is the flux of minibus taxis per unit length from the normal to the bus lane. A simple model for Q might be

$$Q = \gamma\left[(N^b_{max}(\beta) - N^b)N^n(1-\xi)\right] \quad \text{for } N^b < N^b_{max}, \tag{8}$$

$$Q = 0 \quad \text{for } N^b > N^b_{max}, \tag{9}$$

since no minibus taxis can transfer across to the bus lane if there are no gaps in the flow. Here ξ is the proportion of minibus taxis in the traffic stream, and the parameter γ would need to be fitted using real data from a comparable setup.

It should be noted that the flux equations (6, 7) can be combined to give a single equation describing the global flow

$$N^{tot}_t + F^{tot}_x = 0, \tag{10}$$

which has been examined in 2014 [3]. This suggests that the overall flow is unaffected by the transfer according to this simple model. There will however be dynamic disturbances propagating with speeds $(F^n_{N^n}, F^b_{N^b})$ in the individual streams. An analysis based on these observations is possible but more data is required to proceed further.

4 Modifying passenger behaviour

Problems of traffic flow can be dealt with in several ways. One is to make a large capital investment and build more roads, another is to change the traffic flow in other ways such as is being considered in this project. A third possibility is to reduce the number of vehicles on the road using some form of incentive model to change people's behaviour so that they no longer use their own car or so that they transfer to some more efficient means of transport.

In this section we propose a framework to consider an incentive scheme to do such a thing. It is clear that in the problem under consideration the reasons for the use of taxis over buses is partly cultural, partly convenience and partly financial. In that case it is likely that a more complicated system will be necessary than that which we present.

4.1 Simple model

Our goal is to encourage people into moving from taxis to buses to improve the flow of traffic. We consider transportation by bus and taxi, but also that the buses are unimpeded in travel. People travelling in cars are assumed to be unaffected but it would not be difficult to include them in the model if it became apparent that changes in behaviour were significant in that cohort. Instead of moving taxis to the bus lane, the goal is to move a significant number of people.

To do this, we need to come up with some simple premises on which to base the model. We assume that taxis take 50% longer to get from A to B to provide us with a parameter for travel time. If there are more people on the buses and less in taxis then this will manifest itself as a decrease in average travel time. We also need to define the number of taxis in some way, and so we assume that the number of people in each taxi takes the average value and hence the number of taxis is the total number of people using them divided by this quantity. We define the following variables/parameters;

- N_b = number of buses
- c_{bm} = total capacity of one bus
- C_{bm} = total capacity of all the buses ($= N_b c_{bm}$)
- k = stretching factor
- γ = relationship between taxi and bus journey price
- P_t = journey cost for one taxi
- N = total number of people on the road in one hour
- n_t = number of people travelling in one taxi
- T = time period (in hours)

Variable input parameters:

- p_b = price per person to travel by bus
- n_b = number of people travelling on one bus

Output parameters:

- N_t = number of taxis
- p_t = price per person to travel by taxi
- A_T = average travel time per passenger

The most important precept is that the price signal should have the effect that as the benefit of travelling on buses becomes large then the buses will fill to capacity, while if the benefit of travelling on the taxis becomes larger then everyone uses the taxis. This is represented diagramatically in Figure 4 which is the function

$$N_b n_b = \frac{C_{bm}}{2}\Big(1 - \tanh\big[k(p_b - \gamma p_t)\big]\Big) \tag{11}$$

plotted against the relative price of travel, $p_b - \gamma p_t$. The actual shape of this function would need to be determined by data but for the purpose of this demonstration it is adequate as it has the right behaviour. The parameter $p_b - \gamma p_t$ is pivotal in this whole process as it represents the weighting of the price signal. The price may be

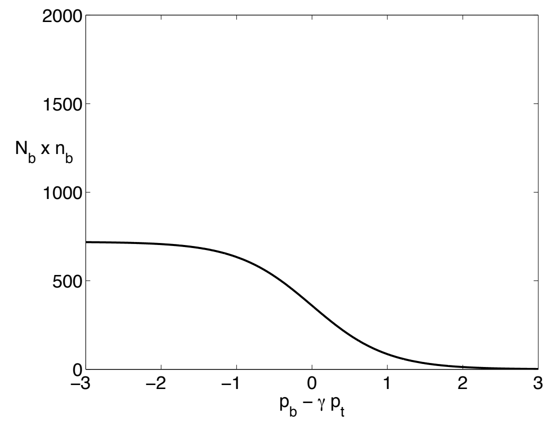

Figure 4: Surface plot of the total number of people using the bus, $N_b n_b$, against $p_b - \gamma p_t$, the relative price of travel on buses and taxis. This function is synthetic to illustrate the process - in reality some data would need to be collected to define the shape of this curve.

money, but may equally be convenience or some other desirable outcome. Equally, other variables with the same general form could be incorporated using different measures of "price" once data had been collected on motivation for travel choices. This is the number of people travelling on all the buses during the time period. Therefore, if p_b is large, such that $(p_b - \gamma p_t)$ is large and positive, then $N_b n_b = 0$, that is no-one travels by bus because the cost is too high. If p_t is large, such that $(p_b - \gamma p_t)$ is large and negative, then $N_b n_b = C_{bm}$, that is the buses are at capacity because the taxis are too expensive.

Rearranging (11) gives

$$p_t = \frac{1}{\gamma}\left[p_b - \frac{1}{k}\tanh^{-1}\left(1 - \frac{2N_b n_b}{C_{bm}}\right)\right]. \tag{12}$$

This enforces the requirement (since the domain of the inverse hyperbolic tan function is $(-1, 1)$),

$$-1 < 1 - \frac{2N_b n_b}{C_{bm}} < 1\ , \tag{13}$$

which implies that

$$0 < n_b < \frac{C_{bm}}{N_b}\ , \tag{14}$$

or simply $0 < n_b < c_{bm}$, which, of course, makes sense physically.

The fare charged for passengers using taxis has to be sufficient to cover the cost for running the taxi (including the profit margin) which requires that

$$p_t = \frac{P_t N_t n_t}{N - N_b n_b}, \tag{15}$$

and rearranging this gives an explicit expression for the number of taxis required

$$N_t = \frac{p_t(N - N_b n_b)}{P_t n_t} = \frac{N - N_b n_b}{\gamma P_t n_t}\left[p_b - \frac{1}{k}\tanh^{-1}\left(1 - \frac{2N_b n_b}{C_{bm}}\right)\right], \tag{16}$$

which is a function of p_b and n_b. This assumes that all taxis travel with the average number of passengers, again something that can undoubtedly be improved.

Finally, the average travel time per passenger is then

$$A_T = \frac{T(N_b n_b + 1.5 N_t n_t)}{N}. \tag{17}$$

4.2 Results

We used the following values for the input parameters: $N_b = 12$, $c_{bm} = 60$, $C_{bm} = 720$, $k = 1$, $\gamma = 0.5$, $P_t = 5$, $N = 2000$, $n_t = 10$, $T = 1$. These are estimates only of a situation in which there are buses on a single route every 5 minutes, for which the buses hold a maximum of 60 people each, there are 2000 passengers on buses and taxis travelling in an hour and that on average each taxi carries 10 people. The behaviour is indicated in Figures 5, 6 and 7.

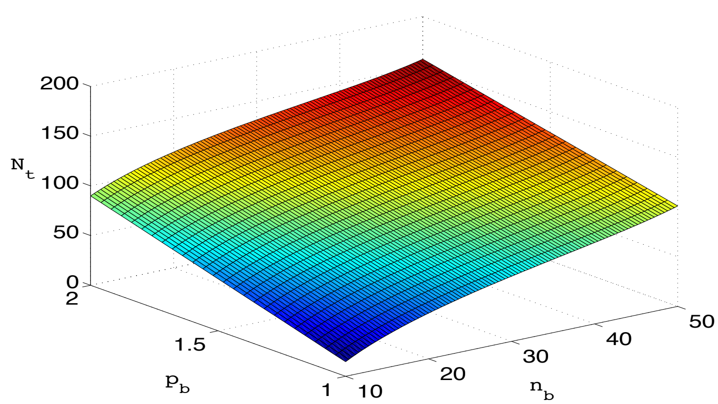

Figure 5: Surface plot of number of people using taxis, N_t, against the bus price, p_b, and people per bus, n_b.

- Figure 5: this shows the number of taxis, N_t, plotted against the price of travelling by bus (per person), p_b, and the number of people travelling on one bus, n_b. We see that if we decrease p_b then the number of taxis reduces, because travelling by bus has become more popular. Similarly, if we decrease n_b.

- Figure 6: this shows the price of travelling by taxi (per person) against p_b and n_b. Here, if we decrease the number of people travelling on one bus then p_t reduces, as expected.

- Figure 7: this shows the average travel time, A_T, plotted against p_b and n_b. Here, if we increase p_b, so that more people will travel in the taxis, then the average travel time increases.

If this information is to be useful, it needs to be based on real data and real information about people's reasons for travelling in different modes and so it is essential that further information be garnered. Once this is obtained, the basic framework here could be fleshed out to provide a good model of the effects of different policies.

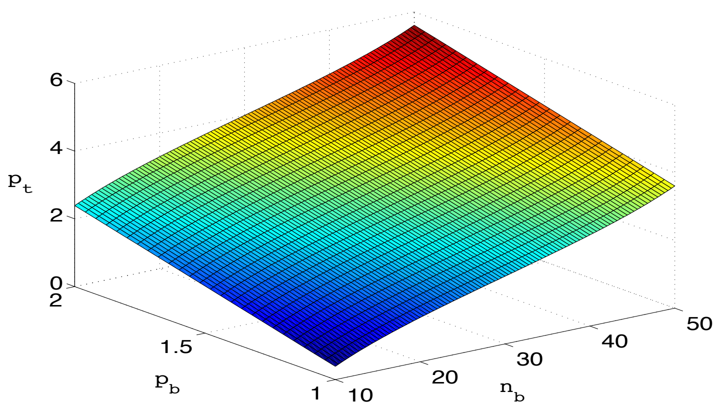

Figure 6: Surface plot of taxi price, p_t, against bus price, p_b, and people per bus, n_b.

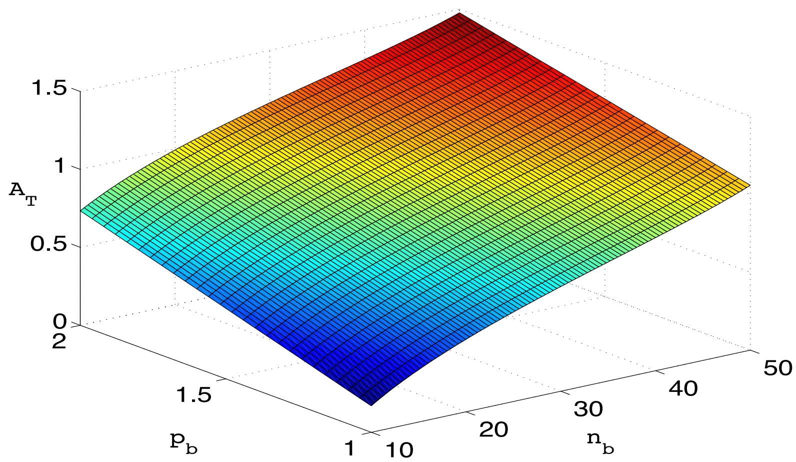

Figure 7: Surface plot of average travel time, A_T, against bus price, p_b, and the number of people per bus, n_b.

4.3 Concluding remarks

This simple model provides a surface of parameters that display the interactions between them. Here the variations are approximately linear, but with the incorporation of more detail on passenger behaviour they may become more complicated. Since we do not know exactly where the real situation lies on these surfaces it is only an indicator of general outcomes. However, it is clear that the model provides a framework which after the inclusion of extra information may form the basis of further work.

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