Bees and blossoms

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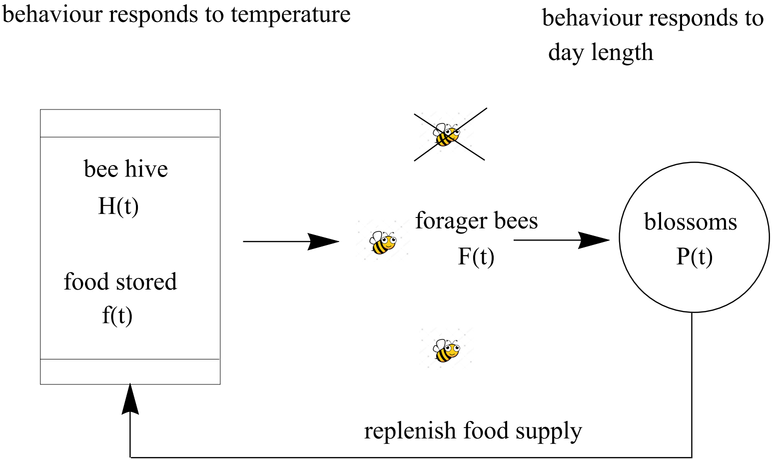
Outline

Problem statement

The goal is to:

- Study the effect of the mismatch in phenology between bees and blossoms.
- Understand the differing factors that drive rates of advance in blossoms and bee phenology, and develop models to quantify these and make predictions.
- Detect changes in bee population due to phenological mismatch.
- Develop a suitable model that applies to the Southern hemisphere.
- To counteract any negative impact on bee populations by implementing appropriate adaptation strategies when the mismatch date and location have been identified.

Problem statement



Objectives

1. A model for blossoms is developed.
   * The blossom model is unaffected by bee behaviour. This assumption may break down over longer times.
   * Flowers cannot align their behaviour with the bees.
2. A model describing bee population and blossom interaction is designed.
   * Bees require blossoms to replenish their food supply.
   * Bees have more freedom to align with blossom behaviour, but are stimulated by factors such as temperature as well.
3. We use phenology data to implement a machine learning algorithm to predict the future rates of blossoming and bee arrivals.

Blossom model

Ignoring biology, consider the following model for blossoms:

- For $0 \leq t \leq \tau_b + \tau$,

$$\frac{dP}{dt} = \alpha_1 P \left(1 - \frac{P}{P_{\max}}\right), \qquad P(0) = 1. \tag{1}$$

- For $\tau_b + \tau \leq t$,

$$\frac{dP}{dt} = -\alpha_2 P, \tag{2}$$

where τ_b is the day at which the first blossom is seen and τ is the number of days in which flowers are in blossom.

Parameter values

To find α_1, consider,

$$P(\tau_b) = 0.99 P_{\max}, \tag{3}$$

which gives

$$\alpha_1 = \frac{1}{\tau_b} \ln\left[100\left(P_{\max} - 1\right)\right]. \tag{4}$$

For α_2, let

$$P(\tau_d) = 0.1 P_{\max}, \tag{5}$$

$$\alpha_2 = \frac{1}{\tau_d} \ln 10. \tag{6}$$

where τ_d is the day when we expect that only 10% of the maximum blossoming population to remain. From Babojelić et al. 2014

- First flowering to peak flowering = 2 days.
- Full flowering period = 10 days.
- Full flowering to greatly diminished flowering = 1 day.

Blossom behaviour

Average number of blossoms on a Granny Smith apple tree (Palmer-Jones and Clinch 1967):

- 2200-7650
- 46 bees per 30 000 flowers

Defining dimensionless variables, $\overline{P} = P/P_{max}$, $\bar{t} = \alpha_1 t$, and omitting bars we obtain

- For $0 \leq t \leq \tau\alpha_1 + \tau_b\alpha_1$,

$$\frac{dP}{dt} = P\left(1 - P\right), \qquad P(0) = \frac{1}{P_{max}}. \tag{7}$$

- For $\tau_b\alpha_1 + \tau\alpha_1 \leq t$,

$$\frac{dP}{dt} = -\frac{\alpha_2}{\alpha_1}P, = -\alpha P. \tag{8}$$

Blossom behaviour

The solutions are

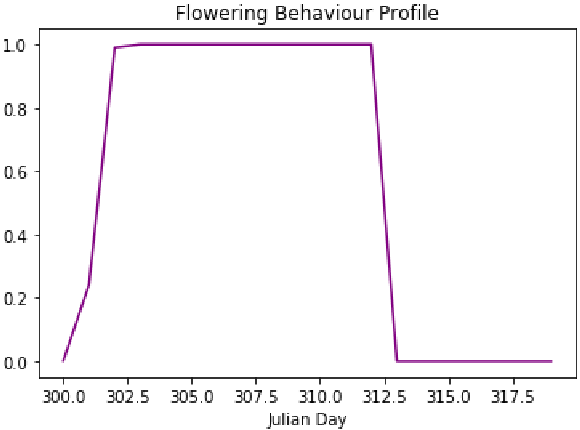
- For $0 \leq t \leq \tau_b \alpha_1 + \tau \alpha_1$,

$$P(t) = \frac{\exp(t)}{P_{max} - 1 + \exp(t)}. \tag{9}$$

- For $\tau_b \alpha_1 + \tau \alpha_1 \leq t$,

$$P(t) = \exp(-\alpha t). \tag{10}$$

Blossom behaviour



Blossom behaviour

- Plant flowering/blossom is determinate, and the growth stops when they reach physiological maturity.
- Determinate growth has three phases:
  1. acceleration phase.
  2. saturation phase (for ripening)
  3. decay phase.
- Growth rate has to be zero at the end point. This terminate growth pattern can be modeled by the beta distribution function (a family of asymmetrical uni-modal curves).

Blossom behaviour

- The beta function for growth rate is given as

$$\frac{dw}{dt} = c_m \left(\frac{t_e - t}{t_e - t_m} \right) \left(\frac{t}{t_m} \right)^{\frac{t_m}{t_e - t_m}}, \tag{11}$$

where

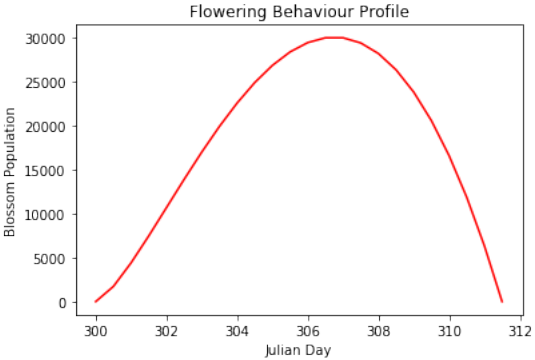
- c_m is the maximum growth rate.
- t_e is the end of growth.
- t_m is the point at which maximum growth rate is achieved.

The solution to equation (10) is

$$w = w_{max} \left(1 + \frac{t_e - t}{t_e - t_m} \right) \left(\frac{t}{t_e} \right)^{\frac{t_e}{t_e - t_m}}. \tag{12}$$

Blossom behaviour

The Determinate Model



Beehaviour

We now develop a compartment model to describe beehaviour.

- $H(t)$ = hive number.
- $F(t)$ = forager number.
- $f(t)$ = food stored.

Mathematical model

The problem is written mathematically as

$$\frac{dH}{dt} = LS - HR, \tag{13}$$

$$\frac{dF}{dt} = HR - mF, \tag{14}$$

$$\frac{df}{dt} = cFP - \gamma_A(F+H), \tag{15}$$

$$\tag{16}$$

where the coefficient functions are given by

$$R(H,F,f) = \alpha_{\min} + \alpha_{\max}\left(\frac{b^2}{b^2+f^2}\right) - \sigma\left(\frac{F}{F+H}\right),$$

$$S(H,f) = \frac{f^2}{f^2+b^2}\frac{H}{H+\nu}.$$

Bee Model results

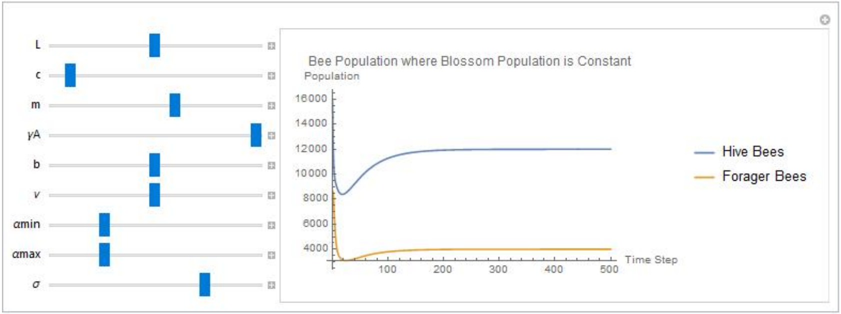


Figure: Bee Population where Blossom Population is Constant

Bee Model Results

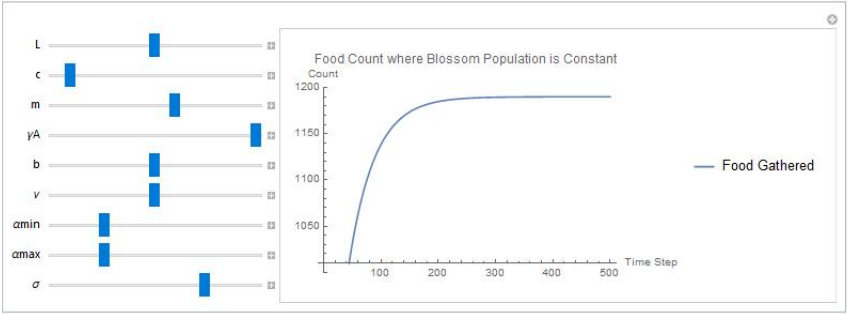


Figure: Food Count where Blossom Population is Constant

Bee Model Results

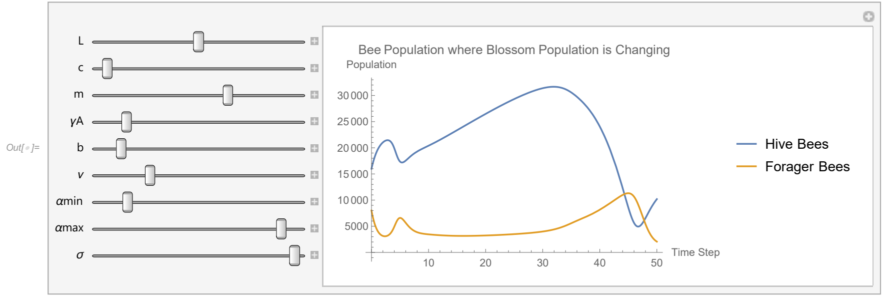


Figure: Bee Population where Blossom Population is Changing

Bee Model Results

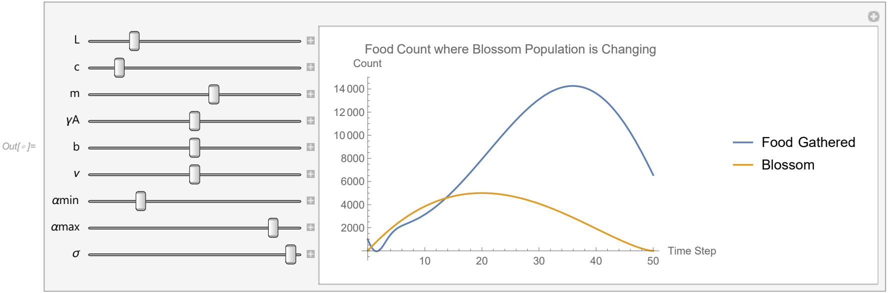


Figure: Food Count where Blossom Population is Changing

Forecasting blossom dates

The rate of change of flowering of Granny Smith and Golden Delicious Apples has already been quantified from observational data in the South Western Cape spanning 1973-2010.

- Granny Smith Apples: -0.11 d/yr
- Golden Delicious Apples: -0.18 d/yr

These provide linear models for the advance of the two species, assuming linear temperature increase. This assumption is reasonable in the short term (30 years), but not in the longer term (100 years) due to the intensification of climate change.

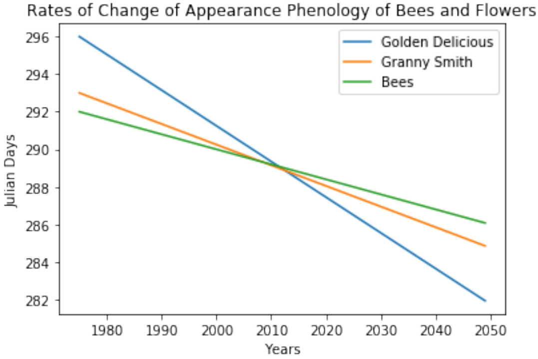
Forecasting blossom dates

To model the rate of change under intensifying climate change, the rate of change per degree Celsius is more accurate. This can also be obtained from the literature

- Granny Smith Apples: -2.4d/ °C
- Golden Delicious Apples: -4.2d/ °C

Temporal mismatch

The temporal mismatch is expressed as the diverging linear time-trends for the two species



Forecasting blossom dates

The Decomposable time series model (Ensemble)

$$y(t) = g(t) + s(t) + h(t) + e(t), \tag{17}$$

where

$g(t)$ is model's trend, i.e non periodic changes, for example, growth.

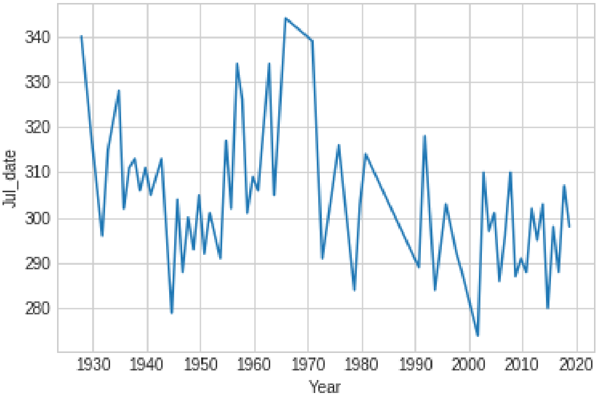
$s(t)$ is model's seasonality, presents periodic changes (i.e weekly, monthly, yearly).

$h(t)$ ties in effects of extremes (potentially irregular schedules ≤ 1 day).

$e(t)$ covers idiosyncratic changes not accommodated by model. by model.

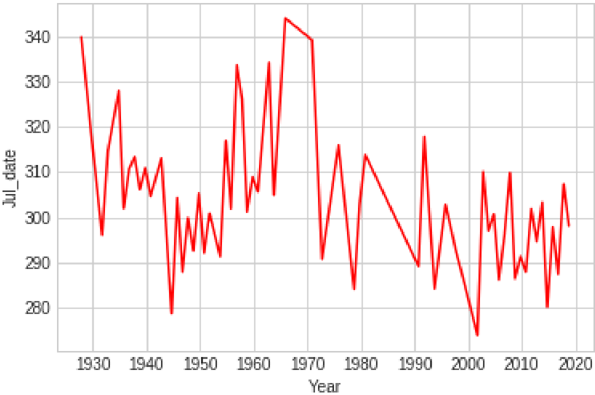
Forecasting blossom dates

Figure: Ground Truth



Forecasting blossom dates

Figure: Predicted (RMSE=0.2345)



Forecasting blossom dates

Date	Y	Y_hat	Y_hat_lower	Y_hat_upper
2016	288	287.451195	287.159760	287.762240
2017	307	307.314683	307.015869	307.607710
2018	298	298.149336	297.850908	298.46371
2019	NaN	299.13539	298.839122	299.45106
2020	NaN	300.119740	299.821715	300.419506

Change in bee arrival dates

- Bee arrival date is a function of latitude and temperature.
- recorded in the literature, averaged for 10 species in a range of locations in North America

- past 130 years: -0.08d/yr
- since 1970: -0.18 d/yr

- (Bartomeus et al., 2011)

Conclusions

Summarising, we have:

- The Blossom Model
- The Bee Model
- Used data to solve the problem using ML techniques

In future work, we hope to refine both models and their corresponding system of ODE's and see how this solution compares to the ML results.