A proposal concerning laminar wakes behind bluff bodies at large Reynolds number

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Summary

This note advocates a model of the steady flow about a bluff body at large Reynolds number which is different from the classical free-streamline model of Helmholtz and Kirchhoff. It is suggested that, although the free-streamline model may be a proper solution of the Navier–Stokes equation with $\mu = 0$, it is unlikely to be the limit, as $\mu \to 0$, of the solution describing the steady flow due to the presence of a bluff body in an otherwise uniform stream. The limit solution proposed here is one which gives a closed wake.

A closed wake contains a standing eddy, or eddies, whose general features can be inferred from the results of an earlier investigation of steady flow in a closed region at large Reynolds number. In all cases, the drag (coefficient) on the body tends to zero as the Reynolds number tends to infinity. The procedure for finding the details of the closed wake behind two-dimensional and axisymmetrical bodies is described, although no particular case has yet been worked out.

1. Introduction

The determination of the steady flow about a bluff body placed in a uniform stream of incompressible viscous fluid at large Reynolds number (where the word 'steady' in this context implies that somehow turbulence has been suppressed) is an old problem, for which no completely satisfactory solution is available. Enough is known of solutions of the Navier–Stokes equation for us to anticipate that at large Reynolds number viscous forces will be small (when made non-dimensional in the usual way) everywhere except in the neighbourhood of a number of singular surfaces of the fluid. Away from these singular surfaces, or shear layers, the flow is essentially inviscid, and various analytical and numerical techniques are applicable; near the singular surfaces the flow has a boundary layer character, and again there are many relevant aids to analysis. However, these principles and techniques are of no avail unless the properties of the singular surfaces (viz. their position, and the velocity increments across them) in the fluid are known, and this is the principal stumbling-block of the problem.

In the case of fluid streaming steadily past a body of such streamline form that no separation of the boundary layer on the body surface occurs, the properties of the singular surfaces are evident enough, and a complete self-consistent description of the flow can be given. The body surface is one singular surface, and another is the surface formed from all the streamlines that extend downstream from the trailing edge in the corresponding irrotational flow. In the limit of infinite Reynolds number, the velocity of the fluid is everywhere the same as it would be for an inviscid fluid, except at the body surface and on the wake surface, and the difference between the solution for $\mu \to 0$ and that for $\mu = 0$ (where μ is the viscosity of the fluid) is here trivial.

In the case of bluff bodies immersed in a uniform stream, and in a number of other kinds of flow in which boundary-layer separation occurs, the properties of the singular surfaces are far from being evident intuitively, and cannot be calculated directly even for the simplest body shapes. Nor does experiment provide much guidance, since the steady flow in the wake of a bluff body is unstable, and becomes turbulent, at a Reynolds number which is not large enough to reveal clearly the character of the asymptotic steady flow. It is necessary, therefore, to make some guess about the properties of the singular surfaces before the character of the flow at large Reynolds number can be determined. The problem has no direct physical importance, of course, in view of the ever-present instability, but it has some mathematical interest for those concerned with properties of solutions of the Navier–Stokes equation. Moreover, it may well be that a knowledge of the steady flow in the limit of infinite Reynolds number would allow the determination, by some kind of asymptotic expansion, of the flow at the upper end of the range of Reynolds numbers at which the flow is stable, more readily than by an expansion valid in the neighbourhood of zero Reynolds number.

The model of the flow that is commonly used is the free-streamline model, based on Helmholtz's (1868) concept of vortex sheets, and worked out fully for the case of a (two-dimensional) plate set broadside-on to the stream by Kirchhoff (1869) and later by Rayleigh (1876). It is unlikely that these authors were concerned to find the complete form of the limit flow, and their aim may have been simply to use the observable phenomenon of vortex sheets shed from a body in order to provide a more realistic representation of the velocity distribution near the plate than can be obtained from a wholly irrotational flow. However, be that as it may, the free-streamline model of the limit flow is the only one available at the present time and is commonly regarded as a correct representation of the flow that would occur at large Reynolds number in the absence of turbulence. It is in this capacity alone that it is here being criticized and replaced by another model.

2. The Helmholtz–Kirchhoff free-streamline model

The free-streamline model is undoubtedly right in its division of flow in the limit of infinite Reynolds number into regions of inviscid motion separated by singular surfaces across which the velocity and its derivatives may be discontinuous. In this respect we can improve on Helmholtz's and Kirchhoff's notions only by recognizing that vortex sheets need not always be uniform. Apart from this feature of vortex sheets, or detached boundary layers, which nowadays we should take for granted, the essence of the free-streamline model of flow past a bluff body is that it supposes the velocity of the fluid to be zero everywhere inside what may be called a wake bubble, the boundary of the wake bubble being a singular surface. Two other pieces of information must be supplied before the model determines the flow field completely; one is the location of the curve (which reduces to two points in cases of two-dimensional flow) of intersection of the body surface and the surface of the wake bubble, and the other is the (uniform) pressure in the wake bubble. It is not clear whether these two pieces of information are independent or not. So far as can be judged from work on two-dimensional flows, choice of the wake pressure seems to determine the position of intersection of the body surface and the wake bubble and *vice versa*; for instance, Southwell & Vaisey (1946) were able to find a definite shape of the wake bubble behind a circular cylinder, by a numerical solution of the equation for inviscid flow, for each of a number of different assumed wake pressures and with no assumption about where the free-streamlines met the surface of the body.

Kirchhoff found that for the two-dimensional flat plate that, when the wake pressure is equal to that far upstream, p_0 say, and the free-streamlines spring from the edges of the plate, the wake bubble is open, with a width which increases as $x^{1/2}$ at a large distance x from the body. Levi-Civita—see Birkhoff (1950)—found this same asymptotic parabolic form of the wake bubble for a family of two-dimensional bodies with the wake pressure taken as p_0. It appears that, for some choices of the pressure in the wake, the bubble is closed at the downstream end. Closed wake bubbles, which are necessarily characterized by a cusp at the downstream end in order to allow the velocity just outside the bubble boundary to be uniform, have been obtained for a circular cylinder (Southwell & Vaisey 1946) and for a truncated aerofoil (Lighthill 1949) by choosing wake pressures larger than p_0.

The present objection to the free-streamline model of steady flows in the limit of infinite Reynolds number takes one of two different forms according to whether the wake bubble is open or closed. If it is open, the objection is to precisely that property of the model—that is to say, the model appears (to this writer) as wrong in conception. If it is closed, the objection is that viscous stresses at the boundary of the bubble would build up an appreciable circulation in the wake bubble and that the model is not self-consistent.

The objection in the latter form rests on the results of a recent investigation (Batchelor 1956) into the properties of steady flow in a closed region at large Reynolds number R. It was argued there that viscous stresses acting in the shear layer at the boundary of the closed region have a small but persistent effect on the flow in the closed region, and that, provided the flow is truly steady, however large the Reynolds number may be (i.e.

provided the limit operations $t \to \infty$, $R \to \infty$, t being the time measured from an instant at which arbitrary initial conditions are prescribed, are carried out in that order), the flow in the closed region has a unique form. The form of the vorticity distribution could be determined explicitly in the cases of closed flows that are two-dimensional or axially symmetrical, although a multiplicative constant remained to be determined in each case from the condition of matching of the 'inviscid' flow in the core of the cavity with the viscous flow in the neighbourhood of the boundary. In one typical, but mathematically very simple case, this multiplicative constant was calculated, and it was established that, as expected, the general level of the velocity in the closed region was of the same order of magnitude as that of the velocity of the surrounding boundary. The implication of this work for truly steady flow past a bluff body at very large Reynolds number is that, if the wake bubble is closed, it will contain a standing eddy, or eddies, in which the velocity distribution has a unique and (in sufficiently simple cases) calculable form, and in which the velocity is of the same order of magnitude as (although no doubt smaller than, by a factor of, say, two or three) that outside the wake. This motion in the wake bubble will have an appreciable effect on the shape of the boundary of the wake bubble, and the notion of an equi-pressure boundary to a closed wake is not self-consistent, except perhaps as a rough approximation in suitable cases. The properties of a closed wake, with allowance for the internal circulation, are considered in the next section, this being just the model that is advocated here.

We turn now to a consideration of the other of the two alternatives, that is, the free-streamline model with a wake bubble which is not closed at the downstream end. The objections to this picture of the flow about a bluff body in the limit of infinite Reynolds number spring mainly from one's distaste for such a drastic interference with conditions at infinity far downstream. I cannot see any reason why a free-streamline flow pattern with an open wake should not be a valid solution of the Navier–Stokes equation in the limit of infinite Reynolds number (unlike a free-streamline flow with a closed wake, which is not self-consistent), but I think there is room for doubt about whether it is the solution that corresponds to the boundary conditions appropriate to a bluff body placed in an otherwise uniform stream.

One's suspicions about this are aroused by the sudden changes that appear to be necessary in the general character of the flow when the Reynolds number is changed continuously. At sufficiently small Reynolds numbers, the streamlines in contact with the two sides of a two-dimensional body certainly meet again further downstream, enclosing a finite region of fluid behind the body (see, for example, the calculations and observations of the flow past a circular cylinder, at Reynolds numbers between 20 and 30, described in Goldstein 1938, § 20). How then could the region bounded by the two branches of the surface streamline change from being closed to being open, as the Reynolds number increases? It is not sufficient to suppose that the point of closure moves further downstream, and ultimately to infinity, as the Reynolds number increases (nor is there definite evidence from calculations that this does happen, once the Reynolds number is above some small value, such as about 30 for a circular cylinder), because a closed wake bubble, however long, would develop an appreciable internal motion by viscous action and the bubble boundary would inevitably take up the one possible shape—which (so it is argued in the next section) is such that the length of the wake bubble is finite when the Reynolds number is infinite.

The boundary conditions that must be specified in order to make a solution of the Navier–Stokes equation unique are not known with certainty, so that it is not possible to give a precise statement of the mathematical problem under discussion. There is a fair presumption, however, that we are looking for the limit, as $R \to \infty$, of that steady solution of the Navier–Stokes equation for which the velocity of the fluid is zero at the surface of the given body and is uniform *everywhere* at infinity. It seems to be true of all the existing numerical solutions of the Navier–Stokes equation at finite Reynolds number that this set of boundary conditions is necessary and sufficient for uniqueness. If this presumption is correct, the free-streamline model with an open wake bubble gives an unwanted limit solution, in view of its property of non-uniformity of the velocity at infinity.

Pursuing another line of thought, when the Reynolds number is very large, but not infinite, the singular surface at the boundary of the open wake bubble is converted into a shear layer of small but finite thickness. The thickness of the shear layer will ultimately increase as $x^{1/2}$ (in a case of two-dimensional flow) by viscous entrainment, like any boundary layer with uniform external conditions, and the fluid in the wake bubble must supply part of the inflow to the shear layer. A small back-flow (from the region far downstream, and towards the body) in the wake bubble is thus necessary, and, since the width of the open wake bubble also varies as $x^{1/2}$ (in at least some cases), this backflow can be thought of as a *uniform* flow inside the wake bubble. The interference with the flow conditions at infinity that is implied in the free-streamline model now appears as even more drastic when the Reynolds number is not infinite, since the presence of the body must be supposed to be responsible for the generation of a small back-flow originating far downstream. Is it not likely that the solution represented by the free-streamline model with an open wake bubble is that which is obtained *only* by the imposition of suitable boundary conditions at infinity, i.e. by the external imposition of a small uniform backward velocity (which is of order $R^{-1/2}$ and hence becomes zero in the limit of infinite Reynolds number) over a region $y^2 < \text{const.} \times x$ at $x \to \infty$?

3. The proposed limit flow with a closed wake

The discussion of the Helmholtz–Kirchhoff free-streamline model given in the preceding section has already indicated the general features of the preferred alternative model of limit flow past a bluff body. The point of view taken here is that an open wake bubble is inadmissible for the problem at hand, although it may be a feature of the limit solution for some different set of boundary conditions at infinity. We should therefore seek a limit solution for which the wake bubble is closed, this being the essence of the present proposal. Opinions may vary about the admissibility of an open wake bubble, but I think there would be general agreement that, if a solution describing steady flow about a bluff body in the limit of infinite Reynolds number and exhibiting a closed wake bubble could be found, that solution would be regarded as the desired solution in preference to the free-streamline solution. I have not been able to find such a solution mathematically, but I believe that its existence is suggested by a number of arguments.

For the moment, let us accept the suggestion that the desired limit flow past a bluff body possesses a closed wake bubble and explore its implications. There is an immediate implication that the drag coefficient for the body is zero (in the limit, $R \to \infty$), since the flow everywhere outside the combined body and wake bubble is inviscid and irrotational. This is in contrast with the free-streamline model, which leads to a finite drag coefficient (the work done by the body appearing as an addition to the infinite amount of kinetic energy—using now axes fixed in the fluid at infinity upstream—possessed by the fluid). The prediction of finite drag coefficient by the free-streamline model has been greeted as a strong argument in its favour, since it avoids the conflict with experience usually described as the d'Alembert paradox*. My own view is that steady laminar flow in the limit of infinite Reynolds number is so far outside the range of 'experience' that the prediction of a finite value of the drag coefficient under these conditions is not more welcome physically than the prediction of a zero value. Indeed, in view of the indisputable result that the drag coefficient for a body of streamline form, for which the development of the laminar boundary layer over the entire boundary is calculable, is of order $R^{-1/2}$ when R is large, there is a slight advantage in the prediction of zero drag coefficient for a bluff body in the limit $R \to \infty$, since it reduces the distinction between bodies of different shape.

As mentioned already, an investigation of the character of steady flow in a closed region at large Reynolds number has been described in a previous paper (Batchelor 1956), and the results of this investigation can now be used in a description of the flow in the closed wake bubble for certain kinds of bluff body. It was found there that, given that the flow is truly steady however large the Reynolds number may be, the vorticity distribution in a closed region could be worked out in cases of (*a*) two-dimensional flow, (*b*) axisymmetric flow without azimuthal swirl, and (*c*) axisymmetric flow with swirl, subject to some restrictions. It is not necessary to describe all

*Inasmuch as in real flow past a bluff body the drag coefficient is finite—due, in all probability, to the existence of turbulence in the wake—the free-streamline model doubtless provides a closer representation of the general form of the pressure distribution on the body in the real flow than does the classical solution which assumes wholly irrotational motion. However, the representation of certain aspects of the real turbulent flow is an *ad hoc* objective, different from that pursued here.

the results here; suffice it to say that in case (*a*) the vorticity is uniform ($\omega = \omega_0$, say), and in case (*b*) the azimuthal component of vorticity (which is the only non-zero component) is equal to αr, where r denotes distance from the axis of symmetry and α is a constant. These vorticity distributions are valid throughout a closed region, except in the neighbourhood of the surrounding singular surface where viscous forces are not small. The constants ω_0 and α, which measure the strength of the inviscid 'standing eddy' in each case, are determined mathematically by the condition that steady motion should be possible in the surrounding viscous boundary layers.

As an example of the application of these results to the proposed limit flow with a closed wake bubble, consider the case of a two-dimensional flat plate set broadside-on to the stream. The surface of the plate is one of the singular curves, in the neighbourhood of which viscous forces are appreciable however large the Reynolds number may be, and there will be other such singular curves in the fluid coincident with the streamlines passing downstream from the edges of the plate. These streamlines mark the boundary of the wake bubble, which we are supposing to be closed at the downstream end, and their shape is shown schematically in figure 1. The

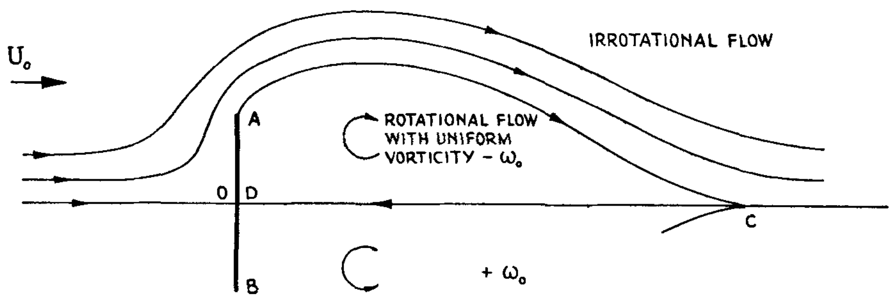

Figure 1. Flow past a two-dimensional flat plate in the limit of infinite Reynolds number, as proposed herein.

motion inside the wake bubble is driven by the action of viscous stresses at the curve AC, so that the streamlines just outside AC have greater total head than those just inside AC. In the limit $R \to \infty$, the streamline AC will thus be a vortex sheet, whose strength will vary along its length according to the requirements of continuity of pressure across AC, viz.

$$(U/U_0)^2 - (V/U_0)^2 = \text{constant, } h \text{ say,}$$

where U and V are the velocities just outside and inside the vortex sheet respectively and U_0 is the speed of the uniform stream. At the closure point C, V is necessarily zero, and this relation shows that U is then finite at C. This requires the wake bubble to have a cusp at C.

The property of symmetry about the line bisecting the plate shows that the wake bubble must be divided into two halves, the vorticity taking opposite signs in the two halves. This suggests that the line of symmetry DC is a singular curve, as can also be seen to be the case from a consideration of the viscous shear layer in the neighbourhood of the curve AC. The streamline that intersects the plate boundary at A divides the unclosed streamlines outside the wake bubble from the closed streamlines inside the bubble, and, since viscous forces are significant in a thin layer extending on both sides of the curve AC, it follows that at C part of the boundary layer about AC passes downstream and part returns to the plate along the line of symmetry CD. Thus the wake slit extending downstream from C is a singular curve, so too is the portion of the line of symmetry CD, and finally so too is the portion DA of the boundary in view of the need to satisfy the no-slip condition there. We are led to a simple picture of the wake bubble as a pear-shaped domain in which viscous forces are negligible everywhere except near the boundary and the line of symmetry, the inviscid flow not near these curves being characterized by uniform vorticity, of strength $-\omega_0$ in the upper half of the bubble and $+\omega_0$ in the lower*. (*A propos* of the point made earlier, about the possible value of a knowledge of the limit flow as a means to the calculation of the flow at finite Reynolds numbers, it will be noticed that the general form of the flow past a circular cylinder at Reynolds numbers between 20 and 30 (see Goldstein 1938, §20), and that at Reynolds number 40 (as found numerically by Kawaguti 1953), both resemble that of the corresponding limit flow proposed here.)

The closed line $ACDA$ is a streamline lying within a continuous boundary layer surrounding one half of the wake bubble. Over the part AC of this closed curve the viscous stress exerted by the external flow tends to produce a clockwise rotation of the upper standing eddy, on the part DA the stress opposes circulation in any direction, and on the part CD the stress is zero owing to the symmetry. The standing eddy takes up an equilibrium rate of rotation, measured by the vorticity $-\omega_0$, in response to these driving forces; the larger is the ratio of the length of boundary over which the eddy is driven to that over which there is retardation, the larger will be ω_0, and the smaller will be h. An equivalent statement is that ω_0 takes such a value that the relative momentum of the boundary layer about $ACDA$ is augmented over part of its path and depleted over other parts, the net effect being to allow the boundary layer to return to its starting point without change.

It is not difficult to see how this picture of the flow should be changed to suit bodies of different shape. If a two-dimensional plate is not broadside-on to the stream, the flow has the same general features as for the broadside-on position, except for the symmetry; in particular, the dividing free viscous layer CD is no longer straight, and the vorticities of opposite sign in

* Actually, this picture may be a little too simple since it is possible that the fluid at the centre of the boundary layer on CD is brought to rest, before reaching D, by the adverse pressure gradient near the stagnation point at D, and that the free boundary layer cuts across the corner at D. In this event, another singular surface would exist to divide the main body of fluid in the wake bubble from a secondary standing eddy in the corner at D, and indeed there may even be a whole sequence of such singular surfaces and standing eddies of diminishing size as D is approached. These are refinements to the picture and I propose to ignore them in order not to obscure the main argument.

the two sections of the wake bubble are no longer of equal magnitude (but note that the loss of head across the bubble boundary, represented by h, must be the same for the two sections of the bubble in view of the need for continuity of the pressure across the dividing free layer CD at position C where the velocity is zero on both sides). In the case of two-dimensional bodies without sharp edges, there is the additional complication that the two positions of separation of the boundary layer on the forward portion of the body are no longer determined by the geometry alone, but in other respects the picture is unchanged. In the case of bluff axisymmetric bodies placed symmetrically in the stream, the singular surface in the interior of the wake bubble degenerates to a line, and the free viscous layer returning from the closure point along the axis of symmetry in the form of a thin cylinder no longer separates two regions of inviscid motion. The distribution of vorticity within the wake bubble is now given by $\omega = \alpha r$, and the bubble boundary has the same general form (schematically) as that in figure 1. Again the constant α is determined by the condition that the motion in the viscous layer bounding the wake bubble should be steady. For a three-dimensional body without axial symmetry, the same general principles apply, but the form of the distribution of vorticity within the wake bubble is not known.

The proposed model of steady flow at very large Reynolds number has now been described, and there remains the question, does such a flow satisfy the Navier–Stokes equation? There are two parts to this question, one concerning the flow of the thin layers where viscous forces are significant, and one concerning the regions in which the flow is essentially inviscid. Consider first the viscous layers. One viscous layer forms on the forward portion of the body, separates from it and passes downstream in contact with the wake bubble, leaves the wake bubble at the closure point, and passes to infinity downstream as a wake. The development of this viscous layer is not subject to any restrictions, and will proceed according to whatever inviscid velocity distribution exists just outside the layer. Another viscous layer has a closed path, which includes the back of the body and the inner side of the boundary of the wake bubble (being in contact with the other viscous layer over this part of its path). This viscous layer is subject to the restriction imposed by the need to return back on itself after one circuit of its path, and steady viscous flow will be possible only if the inviscid velocity distribution just outside the layer has a suitable form. I think it is reasonably certain that this restriction can be satisfied by an appropriate choice of the intensity of the inviscid motion within the wake bubble, the latter being measured by the constants ω_0 and α in the simple cases described above. If the boundary of the wake bubble did not change its shape with change of ω_0 or α, this proposition would be beyond question, and it seems likely to remain valid—although no longer readily provable—when the changes of shape in the bubble are admitted.

The flow in the regions of inviscid flow must now be shown to be self-consistent, bearing in mind that, for a given shape of the bubble boundary, the inviscid flow inside the wake bubble is already prescribed—as to distribution, by the requirements of steady flow in a closed region, as worked out in detail for certain simple cases in the earlier paper, and as to intensity, by the need for steadiness in the surrounding viscous layer. Moreover, the flow outside the combined body and wake bubble is prescribed by the irrotationality, as soon as the shape of the bubble boundary is given. Thus the crux of the inquiry lies in the possibility of finding a bubble shape, for each of a number of arbitrarily chosen values of ω_0 or α (speaking of simple symmetrical bodies for the sake of definiteness), and for a given position of intersection of the bubble boundary with the body surface, such that the inviscid motions in the two sides of the bubble boundary satisfy the condition

$$(U/U_0)^2-(V/U_0)^2 = h,$$

where h is an unprescribed constant (which may vary with ω_0 or α). If such a family of bubble shapes for different values of ω_0 or α within some continuous range can be found, then presumably for one of them the condition for steadiness of the closed viscous layer will be satisfied and the Navier–Stokes equation will be satisfied everywhere.

So far as the regions of inviscid flow are concerned, the proposed model of the limit flow can be regarded as differing from the free-streamline model by its allowance for motion inside the wake bubble (as well as by its rejection of open bubbles). As stated in § 2, it is generally believed that the free-streamline model for a given body is determinate when either the position of intersection of the free-streamlines with the body surface or the wake pressure is given, the required condition coming presumably from a consideration of the boundary layer on the forward portion of the body surface. The model proposed here replaces the wake pressure parameter by the parameter h (the two being uniquely related when there is no motion in the wake bubble), and introduces an additional parameter ω_0 or α, characterizing the motion inside the wake bubble, which must be determined from a consideration of the closed boundary layer surrounding the wake bubble. It seems a reasonable inference from experience with the free-streamline model that, for a given body shape and position of intersection of the bubble boundary with the body surface, the value of h is determined by the choice of ω_0 or α. A 'relaxation' solution of the inviscid equations for the two-dimensional flow down a finite step in an otherwise plane boundary, the region in the lee of the step being occupied by a standing eddy with uniform vorticity ω_0, has been obtained by Miss Ann Hawk at Cambridge for one arbitrarily chosen value of ω_0, and the value of h was here found to emerge as one of the results of the calculation.

Problems of inviscid flow with finite vorticity and boundaries of unknown shape are very difficult to handle, and I have not been able to show in general that it is in fact possible to find a bubble shape for a given body shape and position of the intersection of the wake bubble with the body surface, and an arbitrary value of ω_0 or α. A little support for the belief that an appropriate bubble shape exists for each value of w_0 or α (within a certain range) can be found in the fact that a Hill's spherical vortex (see Lamb 1932, ch. 7) has a vorticity distribution of the required form, viz. $\omega = \alpha r$, and satisfies the above boundary condition with $h = 0$ when $\alpha = 15U_0/4a^2$ (a being the radius of the sphere containing the vorticity). A Hill's spherical vortex may be regarded as a possible wake bubble for a body in the form of a sector of a spherical shell subtending an angle θ_0, say, at the centre of the sphere, so far as conditions outside the viscous layers are concerned, α having the maximum possible value and h being zero. At the other extreme there is the free-streamline solution for the same spherical cap (the details of which are not known mathematically, although such a flow has been observed about a cavitating sphere in water), corresponding to $\alpha = 0$ and to whatever is the value of h that corresponds to the intersection of the free-streamlines and the body being at the edge of the spherical cap. For all intermediate values of α, and consequently of h, there should exist an inviscid wake bubble for the same spherical cap, the bubble boundary having a finite length (at any rate, for $\alpha \neq 0$) and a cusp at the rearmost point. The value of α for which the viscous layer bounding the wake bubble is in steady motion seems likely to be one corresponding to a bubble whose length is not many times larger than the linear dimensions of the spherical cap—for if it *were* much larger, the area of bubble boundary over which the irrotational stream accelerates the standing eddy would be much larger than the area over which the cap retards it, and the vorticity in the standing eddy would take up a value close to its maximum, and this in turn corresponds to a *short* bubble.

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