MASK DESIGN FOR COVID-19

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Abstract

A simple Bernoulli model is used to estimate the pressure developed inside a rigid covid-19 mask and the proportion of air flow passing through and leaking around the edges of the mask due to a sneeze or cough. Rather speculatively the results are extended to deal with with the commonly used nonrigid cloth masks and masks with folds. This is done by introducing a constitutive law connecting the pressure developed within the mask and the volume within this space. If verified experimentally this model could be used to design masks that are more effective and comfortable.

1 Introduction

In the absence of a mask large droplets (defined as having a diameter greater than 1 mm) and intermediate (5 μm $\to$ 1 mm diameter droplets) are carried by the air stream generated by a sneeze a distance of the order of 70-90 cm [1]. Smaller aerosols travel further, remain longer in the atmosphere, and can be carried by the air conditioning system more broadly. It is not known which droplet sizes are most dangerous from the covid-19 virus spreading viewpoint. The larger droplets carry more virus particles and so are more likely to carry infection, however aerosols can more easily enter the lungs. Also droplets settling on solid surfaces can cause infection. Masks are used to both prevent the spread of the virus from an infected person and to help protect a possible victim from infection. Here our focus will be

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on the effect of a mask on the prevention of virus spreading. More specifically we will examine the air flow resulting from a sneeze or cough. The associated droplet dispersal away from the infected person will be addressed elsewhere.

Evidently an (almost) impermeable mask exactly fitting onto the individual's face will prevent the spread the virus but may kill the wearer. Also a comfortable, but very crude and poorly fitting mask, will be useless. It has been found that even with specially designed well fitting masks significant leakage occurs around the mask edges, [1]. One such commonly used surgical mask is N95. Such well designed masks, from the virus spreading point of view, are uncomfortable to wear, and wearers can have difficulty breathing. Of course individuals with lung conditions would be well advised to not to use such a mask. Evidently there is a compromise here: we want the best mask in a sneeze/cough virus spreading sense consistent with personal comfort needs. Of course the behaviour of the mask under normal breathing conditions could be also as important, or even more important.

There are many different mask designs using different filter types. Some are rigid, but most are made of un-woven or woven cloth often containing folds. Some masks have large enclosed space between the mask and face to enable freer breathing, whereas others closely hug the face but are more flexible to allow for more comfortable breathing. The aim here is not to determine the behaviour of a particular mask, as would be the case in engineering design, but to determine the design principles in a Covid-19 context. Given the complexity of the situation and the difficulty of experimentation, our aim is to produce the simplest model that incorporates major features of the problem. A complex/detailed model, even if available, would not (generally) provide useful 'design principles. One might anticipate/hope that the models we develop would be calibrated using a simple experimental setup. More specifically we ask: what permeability (fabric thickness, cloth type and structure) and mask fitting parameters (size, shape, support) will be best?

In section 2 we develop the flow model. We then discuss mask design using the rigid mask as a datum and then go on to discuss the use of a constitutive law model to describe the behaviour of cloth models with or without folds, see 3. In section 4 we determine the behaviour of masks of different quality and design under sneeze forcing. Finally, in section 5 we draw conclusions concerning the validity and usefulness of the proposed model.

2 The Defining equations

Masks fit reasonably snugly on the face but there will be a space (volume V (say)) between the face and the mask and there will be an effective separation distance δ around the mask edge. The effect of a cough or a sneeze (or indeed simply an out-breath) will be to cause an increase in air pressure p (above atmosphere) within this space. This in turn will cause an increase in volume V within this space, and also a flux through the face of the mask (mask through-flow flux Q_m) or a leakage flux Q_l from around the sides of the mask. Our concern is with the dependence of these fluxes on the physical parameters of the mask itself and parameters determining the

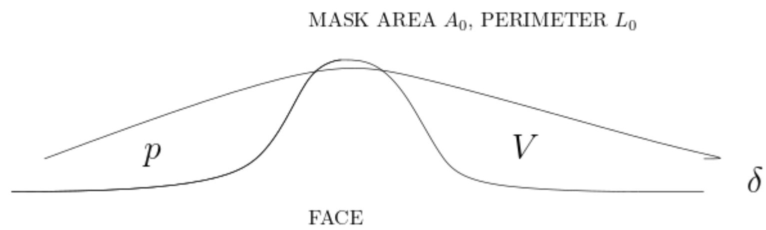

Figure 1: Mask geometry: The mask (area A_0) fits onto the face. The effective spacing around the edge of the mask with perimeter L_0 is δ. A volume V of air is trapped between the mask and the face. The effect of a sneeze is to increase the pressure inside the space to p above atmospheric pressure and to change V. Under normal breathing conditions there will be a periodic exchange of air between the lungs and the mask.

snugness of fit to the face.

2.1 A Bernoulli model of leakage flow

The steady state Bernoulli's equation gives (many approximations here, see later)

$$\boxed{p + 1/2\rho v^2 = c,} \tag{1}$$

with c constant, which gives an expression for the leakage velocity v_l of air particles escaping from the enclosed space through the gap as a result of internal pressure (above atmospheric) p as

$$v_l = \sqrt{2p/\rho}, \tag{2}$$

with the associated total leakage flux given by

$$Q_l = \alpha\sqrt{2p/\rho}(L_0\delta), \tag{3}$$

where $\alpha \approx 0.6$ is a 'fitting parameter' which takes into account the (muliiple) inadequacies of the model.

The steady state Bernoulli approximation

The above Bernoulli equation approximation is central to the model setup and so requires justification. Firstly it should be noted that to improve on this approximation would require details about the flow within the mask, dependent as it is on the geometry (which changes in time) and the elastic components of mask components; a major, perhaps impossible and useless, task. One would need to solve the Navier-Stokes equations with the (mask) geometry unknown and to be determined as part of the calculation. Also it would be necessary to detail the inflow due to the sneeze; a crude model wouldn't suffice.

The steady state Bernoulli equation is derived from the momentum conservation equation under steady, incompressible and inviscid flow conditions and is an energy conservation statement for a particle of fluid leaving the mask space. In our present circumstances a fluid particle under high pressure within the mask is expelled into a lower pressure atmosphere, and in the process the potential energy it initially had by virtue of its pressure p is converted into kinetic energy. This determines the speed of particles exiting the mask through the sides. Now there are various assumptions underlying this, but this very simple result is has been found to determine the general features of the flow in many 'engineering' circumstances, and in these applications a correction factor α is applied to correct for inadequacies. In the present context the Reynolds number of the flow is about 4400 [1], so the flow is essentially inviscid. Also the pressure changes within the mask are small compared with atmospheric pressure, so the flow is essentially incompressible. Treating the flow as being steady is more concerning; this requires that the time span for air to circulate and mix within the mask space is small compared with the mask deflation time.

The primary error in the above calculation, however, is likely to be in the mass conservation result (5) which assumes that the gap size around the mask is uniform and unchanging under pressure forcing.

The approach used here is: what can't be dealt with with theoretical model-s should be dealt with using engineering models with experimentally determined fitting parameters.

2.2 Flow through the mask: Darcy's Law

We treat the mask material as a membrane with flow-through behaviour described by

$$\boxed{q = kp;} \tag{4}$$

where q is the flux per unit area out the mask, p the pressure difference across the mask face, and the 'bulk permeability' k as defined in (5) depends on the thickness of the material as well as its local permeability .[2] The total flux through the mask face is thus

$$Q_m = kpA_0. \tag{5}$$

2.3 Conservation of mass, sneeze/cough input

Assuming in-compressibility, mass conservation in the mask space requires

$$\boxed{\frac{\mathrm{d}V}{\mathrm{d}t} = Q_{in} - [Q_l + Q_m],} \tag{6}$$

[2] In an engineering context permeability is usually defined (locally) by $Q = (\kappa/\eta)p$ where η is the kinematic viscosity and Q is the flux/m through a 1 m^2 area driven by the pressure difference of p.

where $Q_{in}(t)$ is the volume flux input from the cough/sneeze (m^3/sec) and you'll recall $V = V(t)$ is the volume of air inside the mask. We assume this flux input is prescribed. Here $p = p(t)$, and Q_m, Q_l are functions of $p(t)$.

If we assume an equation of state $V(p)$ connecting p and V (see later) then this equation reduces to

$$\boxed{\left[\frac{\mathrm{d}V}{\mathrm{d}p}\right]\frac{\mathrm{d}p}{\mathrm{d}t} = Q_{in}(t) - \big[Q_l(p) + Q_m(p)\big],} \tag{7}$$

where we've explicitly noted the dependence of mask fluxes on the pressure $p(t)$ within the mask air gap.

This is an ordinary differential equation for determining $p(t)$. This can be solved for particular flux inputs $Q_{in}(t)$ (nose or mouth). The 'mask response' factor $[\frac{\mathrm{d}V}{\mathrm{d}p}]$ depends on the state equation for the mask; cases below.

Now the volume of space V under the mask will be determined by the geometry, structure and composition of the mask and the elasticity of the support, as well as the inflation pressure. A complete description of this mask (displacements etc) before, after and during a sneeze is not possible/appropriate for our purpose. As we have seen from the fluid mechanical point of view it is the pressure that drives the flow and the associated change in contained volume of air is the mask response so an 'equation of state for the mask V(p) would avoid a detailed specification of the mask geometry etc. Note however that such an equation of state is only possible/relevant when the mask 'is stretched. If the mask is 'loose (for example has unfolded folds) then there will be very little initial build up in pressure in the enclosed space, little elastic response in the mask or support and as a result only small flows either through the mask or out the sides. Under normal breathing this is probably the situation as required for comfort. The effect of the small increase is pressure would be to move the mask material locally and 'inertially; the folds etc. would unfold. Since the mass per unit area of the mask is normally very small there would be 'an instantanious adjustment until the space fills out, and then there will be resistance to further adjustment with accompanied through flow and leakage flow.

Whilst the description of the flow will be inadequate during this filling out period, this is not a major issue; there is little of interest happening during this period and the net effect of this input is 'correct' in real terms.

2.4 Scaling

We introduce scales so as to reduce the flux equation (12) to its simplest form and identify the important dimensionless groups. We write

$$Q_{in} = \bar{Q}Q'_{in}(t'), \bar{Q}Q'_m(t'), Q_l = \bar{Q}Q'_l(t'),\ t = t_0t', p = p_0p', v = V_0V', \tag{8}$$

where $\bar{Q}$ is a typical air flux (m^3/sec) from the nose or mouth, V_0 a typical mask space volume, and we choose

$$t_0 = \frac{V_0}{\bar{Q}},\ p_0 = \frac{\bar{Q}}{kA_0}, \tag{9}$$

the 'inflation' time scale, and the inflation pressure associated with the flux input $\bar{Q}$. With this choice the flux equation reduces to its simplest dimensionless form:

$$\left[\frac{\mathrm{d}V'}{\mathrm{d}p'}\right]\frac{\mathrm{d}p'}{\mathrm{d}t'} = Q'_{in} - \left[\xi_s\sqrt{p'} + p'\right] \; ; \tag{10}$$

where we will refer to the dimensionless group

$$\boxed{\xi = \left[\frac{\alpha}{k}\sqrt{\frac{2}{(\rho p_0)}}\right]\left[\frac{L_0\delta}{A_0}\right] \equiv [\mathcal{V}_r][\mathcal{A}_r]\,,} \tag{11}$$

as the *quality* parameter, and where we have separated out the velocity ratio and the area ratios (leakage to filter) components associated with this parameter. As suggested by the naming, ξ provides a measure for the ratio of the leakage flux to the mask flux; a well fitted mask with a fine filter corresponds to a *smaller* value of ξ. Dropping primes we get

$$\boxed{\left[\frac{\mathrm{d}V}{\mathrm{d}p}\right]\frac{\mathrm{d}p}{\mathrm{d}t} = Q_{in} - [\xi\sqrt{p} + p].} \tag{12}$$

It should be noted that with our simple model *just two factors*, the *quality parameter* ξ, and the *design function* $[\frac{\mathrm{d}V}{\mathrm{d}p}]$, are needed to characterise the air exchange behaviour of masks.

2.5 Data

Mask data

Much of this data is taken from Dbouk and Drikakis [1]. The typical mask covers the nose and mouth and is (about) of size 22 cm by 12 cm, with a gap around the sides of mask varying from a minimum of 4 to 6 mm to a maximum of 1.4 cm (the nose to eye corner). The thickness is of the mask filter is typically d_f=2 mm. Typically flow velocities of 5 m/sec are to be expected. The leakage model above indicates a typical velocity of $\alpha\sqrt{\frac{2p}{\rho}}$, which for a (typical) pressure difference of 100 Pa this gives 7.7 m/sec (perhaps a bit large).

Bulk filter permeability

In the textile industry the 'bulk' permeability[3] is defined and measured as the flux (in $\mathrm{m}^3/\mathrm{m}^2$/sec, or $\mathrm{cm}^3/\mathrm{cm}^2$/sec) through a 1 m^2 area of cloth due to pressure drop of 10 mm of water (equivalent to 100 Pa) across the fabric. It is most strongly dependent on the pore size in the fabric and the fabric thickness.

The permeability of the filter is influenced by the fabric's material and structural properties, such as the raw material of the fabric, whether or not it is non-woven

[3] More conventionally in engineering the permeability is defined as the flux per unit area per unit length into the fabric divided by the kinematic viscosity or air in this case.

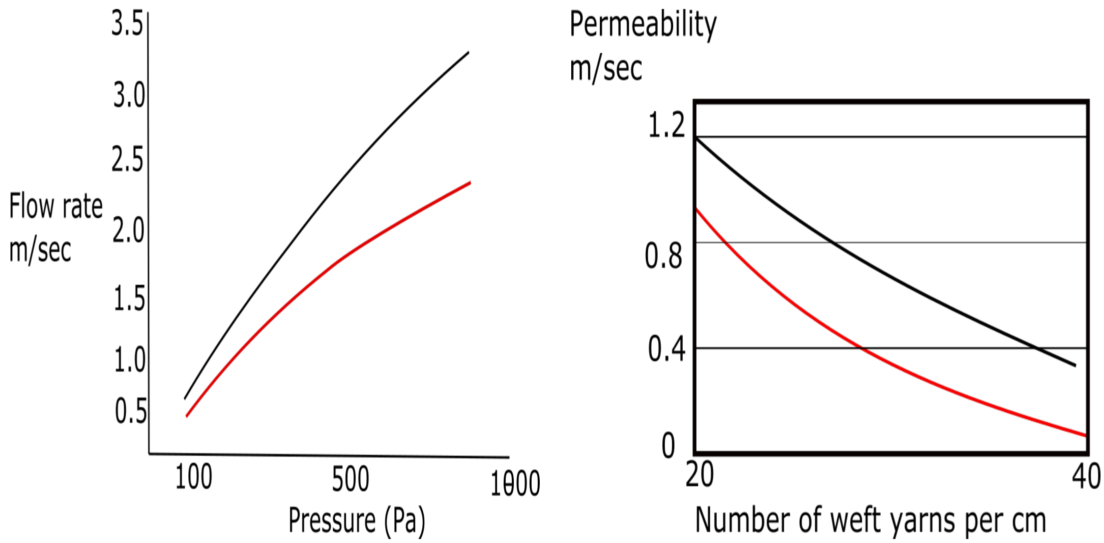

Figure 2: Permeability data: Left: Flow rate in m^3 per sec per square metre of two non-weave sample fabrics. Right: Changes in permeability in a woven fabric due to weave spacing. The upper fabrics curve has a weft number of to 60 20 threads per cm, the lower. The weft number varies from 20 to 60. In each case the applied pressure is the standard used for fabrics (100 Pa).

(wetlaid, melt or spun) or woven. If woven, the spacing between weaves, the particular weave, the set of yarns, yarn twist, cloth treatment and finishing, also effect the permeability, see [4], [5].

Typical values for non-woven cloth and woven cloth are displayed in Figure 2. Based on these figures we get values of bulk permeability (measured in more convenient units) of (2.6 to 3.8)$\times$ 10^{-3} m/(sec Pa) for non-weave materials, and (0.2 to 1.2) $10\times^{-3}$ m/(sec Pa) for the woven samples. The associated through-flow velocities due to a (typical) pressure drop of 100 Pa are (2.6 to 3.8) m/sec for non-woven materials (corresponding to sample 3 and sample 2 in the figure) and (0.2 to 1.2) m/sec for woven materials (bottom curve and top curve in the figure).

Recall that the quality parameter ξ is the product of the area ratio times the velocity ratio parameters $\mathcal{A}_r$, $\mathcal{V}_r$. The area ratio is typically small (δ being relatively small) and the velocity ratio typically large, again because δ is small, so the product can be either small or large depending on the mask filter and fit. Based of the above data we get:

- for woven filters ξ ranges from 0.64 to 7.6,
- for non-woven filters ξ ranges from 0.26 to 0.7.

You will recall that small ξ values correspond to quality masks, so we can see that non-woven masks are generally of higher ‘quality’, basically because the pore size is smaller for these masks. These masks are better able to remove small particles but higher pressure differences are needed to drive the through-flow. Of course these masks are also less comfortable. Whilst the above values are representative and

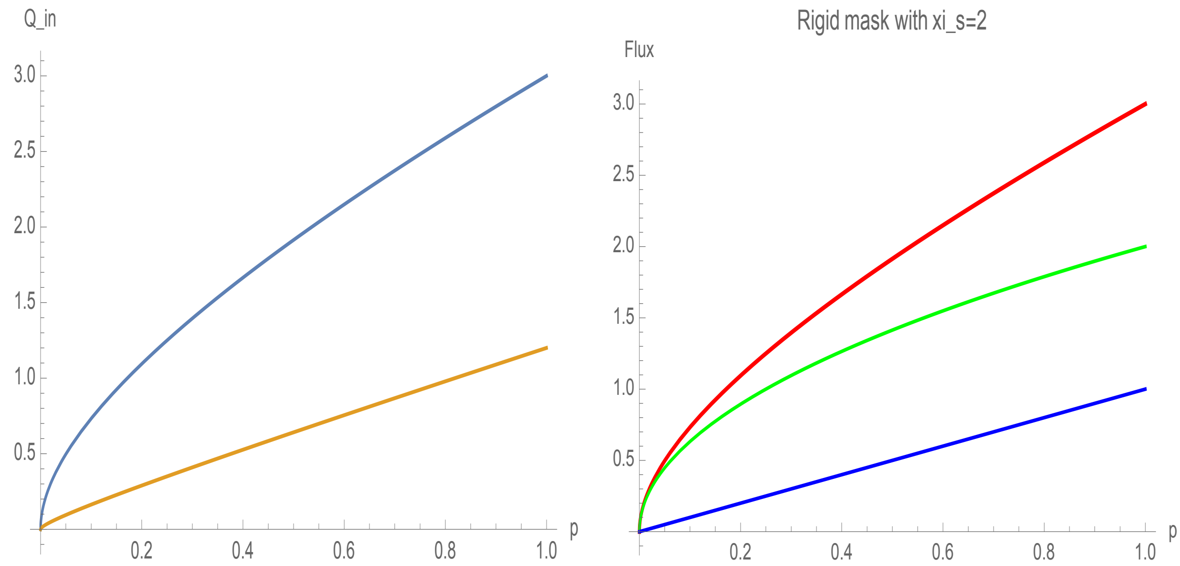

Figure 3: Rigid mask behaviour under inflation: *Left* Behaviour of solid masks with different ξ's under inflation: Top curve $\xi_s = 2$ (a leaky mask), Bottom curve $\xi = 0.2$ (a well fitted mask) *Right* Flux apportionment as a function of inflation pressure p: Flux input Q_{in} (red), leakage flux component $Q_l(p)$ (green), mask flux $Q_m(p)$ (blue)

seem reasonable, it should be pointed out that the face fitting gap can be 2 mm for a well fitted mask to 1.5 cm for a crude fit, and the permeability can also vary over several orders of magnitude, so that much smaller and larger values of the quality parameter are possible.

3 Mask design

The *mask design function* $[\frac{\mathrm{d}V}{\mathrm{d}p}]$ describes the mask behaviour under inflation; for example cloth masks with folds first inflate easily, and then with more difficulty as the folds unfold. We first look at a rigid mask design.

3.1 Rigid design masks

If the mask is rigid then no change in the mask volume space V occurs during a sneeze or cough, so that in this case $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$, so (12) gives

$$Q_{in} = \xi\sqrt{p} + p; \tag{13}$$

a quadratic in $\sqrt{p}$ which determines the pressure p due to any prescribed flux input $Q_{in}(t)$. The associated leakage and mask fluxes can be recovered using (3),(5).

Note that if p is small in (13) then the leakage flux term $\xi\sqrt{p}$ dominates (and increases rapidly with p), but for larger values of p the linear through-flow term takes over so filter efficiency increases, see Figure 3.

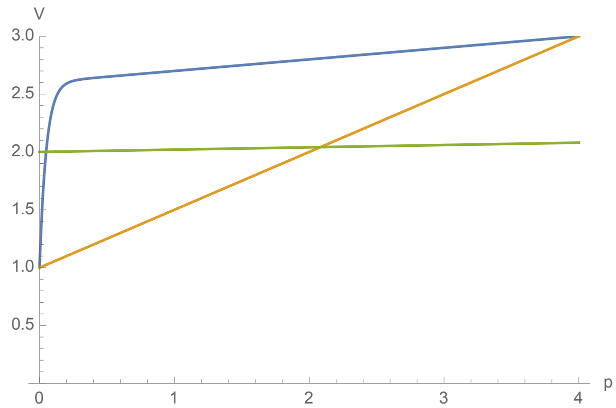

Figure 4: State equations for various masks: Upper (blue) curve a cloth mask with folds, a folded cloth mask. Middle (green) curve, a rigid mask. Lower (yellow) curve, a cloth mask.

3.2 Non-rigid mask designs: the state equation $V(p)$

Simple cloth masks expand uniformly under increasing pressure, whereas cloth masks with folds expand rapidly (with folds unfolding) with increasing pressure until the mask unfolds and then the expansion rate is very slow. The associated state diagrams are displayed in the Figure 4. The rigid mask is also included.

3.3 Discussion

The introduction of a state equation assumes that such exits. It could be that the various mask components simply act 'inertially', that is that the effect of the impulsive sneeze jet is to cause the mask components to instantaneous (and independently) move with a speed determined by the local impulse. Subsequently the various components may come to rest due to local mechanical damping; for example fabric elements will be subjected to local frictional forces. Eventually, however, under inflation the global constraining (elastic) forces will take over and the movement will be more predictable. For example in the folded mask case the initial detailed unfolding process will be determined by air forcing details that are essentially unmeasurable and in any case the process is unstable. The associated volume change within the mask is more predictable, but still until global 'global elastic restoring' forces take effect the result is not 'determinable'. Fortunately from our point of view the determination of the initial mask movement is not of interest because there is little air exchange during this stage. The simple state law model will not accurately describe the initial movement but the end result will be correct, so the model should suffice for our purposes. In fact the design factor that matters here is the 'unstretched volume' of the mask/face space, see Figure 2.

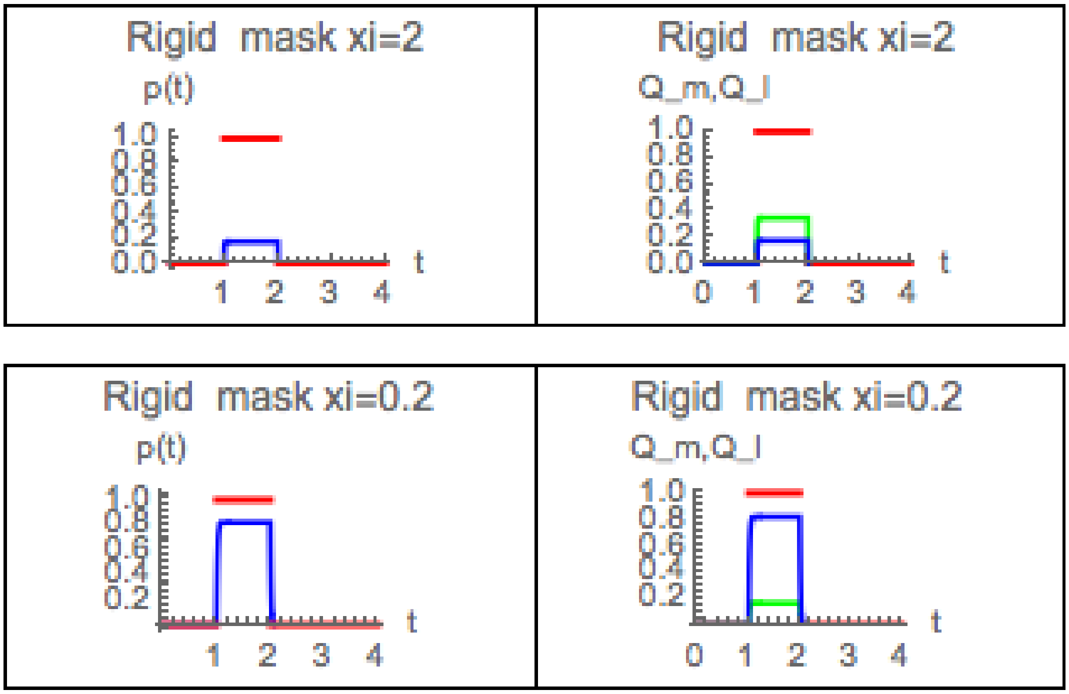

Figure 5: Rigid mask plots: The top figures correspond to a ($\xi = 2$) low quality mask, the bottom figures to a high quality ($\xi = 0.2$) mask. The left hand figures show $p(t)$ (blue), and for reference we have also displayed $Q_{in}(t)$ (red) (not to scale). The right hand figures display the input flux Q_{in} (red), the leakage flux Q_l (green) and the membrane flux Q_m (blue).

4 Response curves for impulsive sneeze/cough flows

We model a sneeze as a fixed flux input over a small time interval (0.1 secs), and determine the response for different mask quality and designs. (We assume that the strength of the sneeze or cough is not effected by the presence of the mask; this accords with experience.)

We compare results for the fitting parameter $\xi = 2$, a poor quality leaky mask, and $\xi = 0.2$, a good quality and well fitted mask. We then repeat the exercise for masks of various designs as defined by the state equation $V(p)$: a rigid mask, a cloth mask, a cloth mask with folds.

4.1 The rigid mask response

Here we look at the rigid mask response to a sneeze input, see Figure 5. Because the mask is rigid and air is in-compressible, the pressure response to the input flux is immediate, and the apportionment of leakage flux to mask through-flow is as described earlier and determined by the quality parameter ξ. Also immediately after the sneeze the mask deflates (with this simple model). The pressure buildup

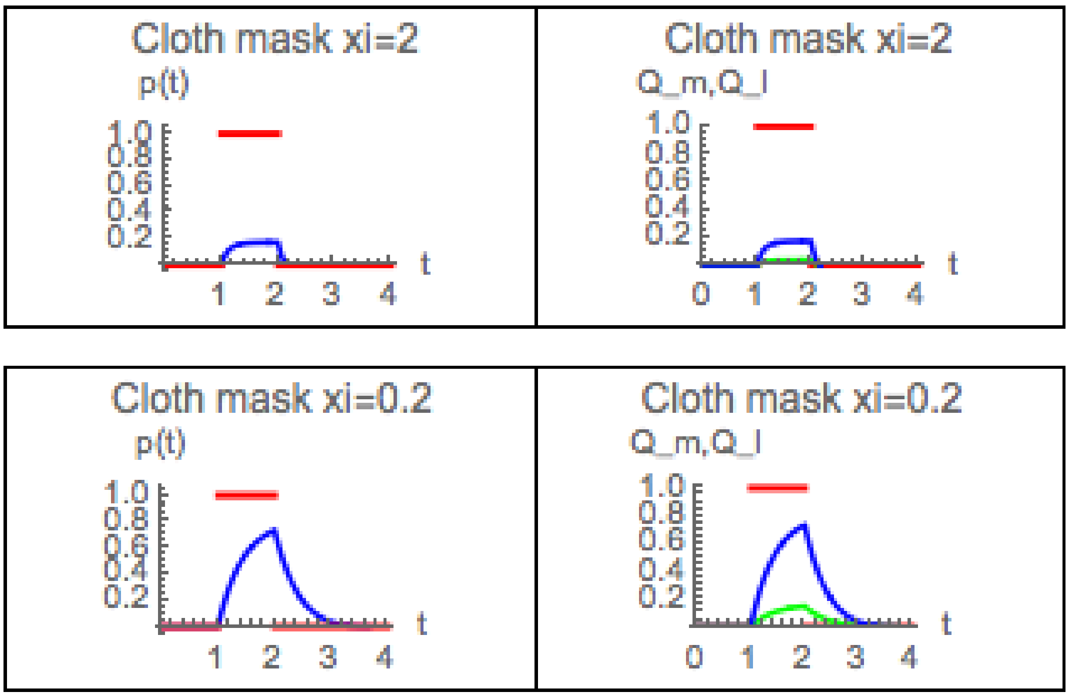

Figure 6: Cloth mask plots: The top figures correspond to a ($\xi = 2$) low quality mask, the bottom figures to a high quality ($\xi = 0.2$) mask. The left hand figures show $p(t)$ (blue), and for reference we have also displayed $Q_{in}(t)$ (red) (not to scale). The right hand figures display the input flux Q_{in} (red), the leakage flux Q_l (green) and the membrane flux Q_m (blue).

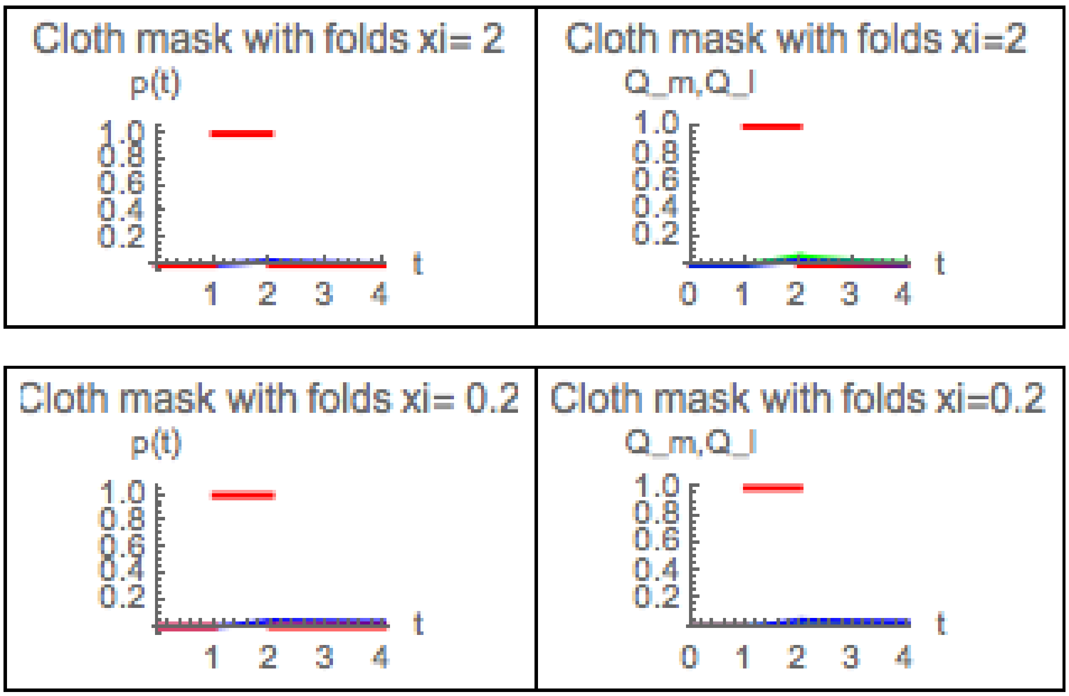

Figure 7: Cloth mask with folds: The top figures correspond to a ($\xi = 2$) low quality mask, the bottom figures to a high quality ($\xi = 0.2$) mask. The left hand figures show $p(t)$ (blue), and for reference we have also displayed $Q_{in}(t)$ (red) (not to scale). The right hand figures display the input flux Q_{in} (red), the leakage flux Q_l (green) and the membrane flux Q_m (blue).

within the mask is greater for the high quality mask ($\xi = 0.2$), and the mask flow is also greater.

4.2 The cloth mask response

Note that the mask space pressure increases rapidly initially and after the sneeze finishes there is an exponential reduction of pressure to zero (atmospheric pressure). As indicated earlier much of the initial flux leaks out the sides of the mask for both the low and high quality masks. At higher pressures much of the flux from the sneeze leaks out the sides for a low quality mask, whereas it passes through the mask if ξ is small. As one would expect for the high quality mask it takes significant time for the mask to deflate after a sneeze. This is especially important if there are repeated sneezes/coughs.

4.3 Cloth mask with folds

In this case the pressure build up within the mask and associated fluxes are small until the folds unfold, and the deflation of the mask is slow. Essentially this mask behaves like an almost rigid mask with greater initial mask space. From the comfort point of view this initial space should be chosen to 'accommodate' the expelled volume from a sneeze.

5 Summary

The simplicity of the model developed above is compelling in that a very few simple experiments are required to determine the quality and design of a mask. Any further 'sophistication' in either the fluids description or the mask response to forcing would require **much** more, hard to collect, and less universally useful data. Of course there are many underlying assumptions; real data will be required to determine the usefulness of the model.

In the above work we have only looked at the flow induced by a sneeze. The above model determines the velocity of flow through the mask and can be used to estimate the droplet capture by the mask. The work can be extended in an obvious way to deal with the periodic input due to a cough spasm. Additionally one can model a 'comfort index' associated with a mask; some combination of the pressure level within the mask and the duration time of the sneeze; experimental input essential here.

In general terms the above work suggests that a rigid mask with an initial mask volume compatible with the volume of air released with a sneeze or a cloth mask with the same unfolded volume are sensible designs for comfort. None of this is unexpected but the above model provides a practical means for quantifying the quality of different masks.

Work is continuing.

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