Rogue Waves

Thama Duba, Colin Please, Graeme Hocking, Kendall Born, Meghan Kennealy

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- What is a rogue wave
- Mechanisms causing rogue waves
- Where rogue waves have been reported
- Modelling of oceanic rogue waves
- The model used and why?
- What would we modify?
- Interpretations
- Future work

What is a rogue wave?

- Rogue waves - also known as “freak”, “monster” or “abnormal” waves - are waves whose amplitude is unusually large for a given sea state.
- Unexpected and known to appear and disappear suddenly.
- Also occur in optical fibers, atmospheres and plasmas.

$$\frac{\eta_c}{H_s} > 1.2 \tag{1}$$

$$\frac{H}{H_s} > 2 \tag{2}$$

where η_c is the crest height, H is the wave height and H_s is the significant wave height as described in Bitner-Gregersen et al. (2014)

Why do we care?

Rogue waves are extremely destructive. The following are examples of rogue waves that left a wake of destruction.

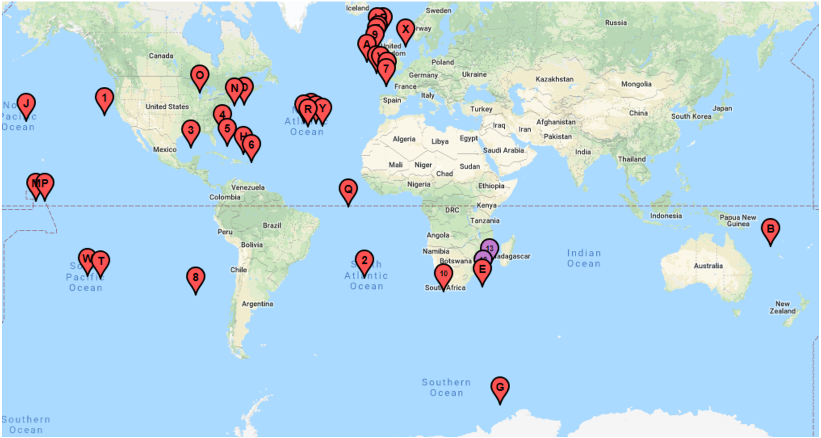
- The Draupner wave, New Year's Day 1995. Using a laser, the Draupner oil platform in the North Sea measured a wave with height of 25.6m
- In February 2000, an oceanographic research vessel recorded a wave of height 29m in Scotland
- 3-4 large oil tankers are badly damaged yearly when traveling the Agulhas current off the coast of South Africa.

These rogue waves threaten the lives of people aboard these ships, and a warning is needed.

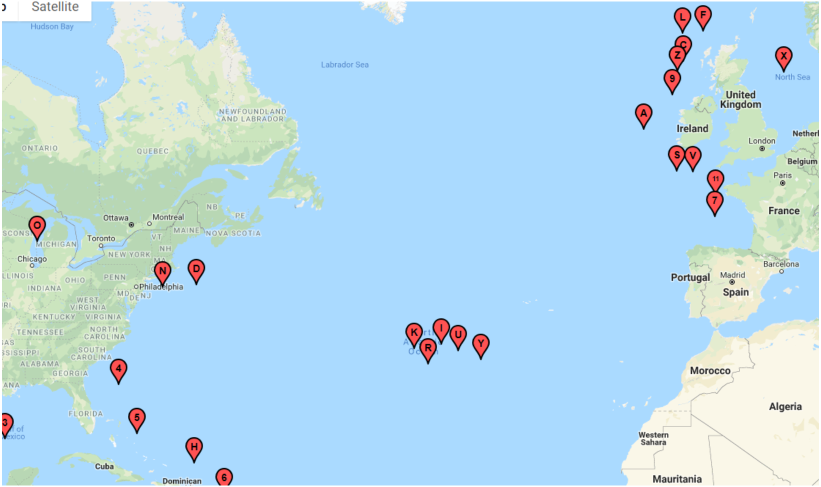
What possibly causes a rogue wave?

- Various weather conditions and sea states, such as low pressures, hurricanes, cyclones.
- Linear and non-linear wave-wave interactions can influence the amplitude.
- Wave-current interactions, if waves and currents align.
- Topography of the sea bed.
- Wind, current and wave interactions.

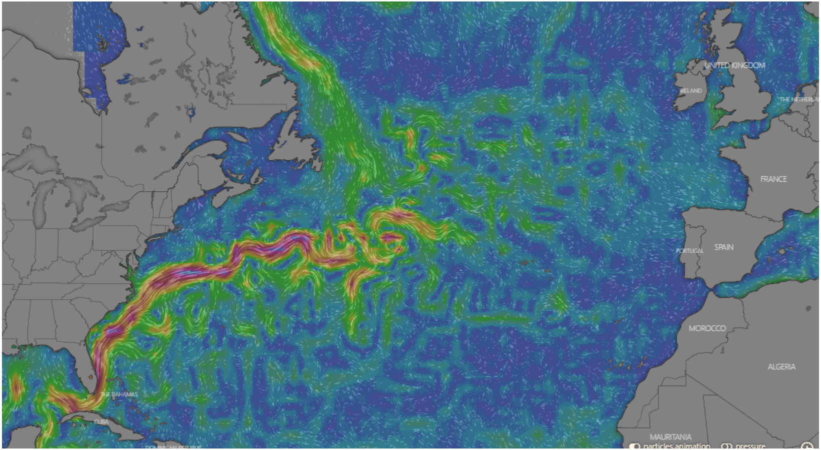
Where have rogue waves been reported?



In the North Atlantic

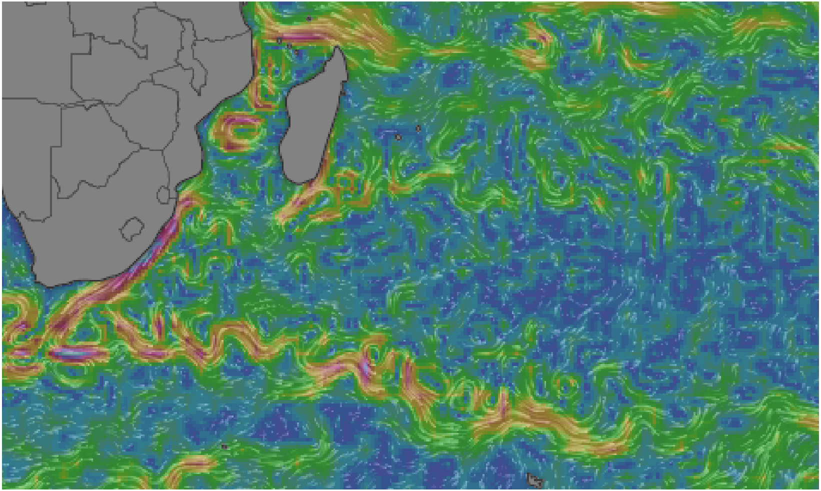


In the North Atlantic



In the Indian Ocean

In the Indian Ocean



Assumptions

- Irrotationality $\nabla^2 \phi = 0$
- Waves propagate in the x direction, uniform in the y direction.
- Bottom of the ocean is a impermeable.
- Incompressible fluid $\rho =$constant
- Inviscid fluid $\nu = 0$
- No slip boundary condition
- Vertical velocity at the bottom of the ocean is zero.

Modelling of oceanic rogue waves

Each model has a different level of approximation, which accounts for different interactions over longer timeframes. These are the long wave approximations of slow modulations.

- Non-linear Schrödinger equations
  Assumes steepness, $k_0 A << 1$ (k_0 is the initial wavelength), a narrow bandwidth $\Delta k/k$ (Δk is the modulation wavenumber) and is achieved by applying a Taylor series expansion to the dispersion relation for deep water waves.
- Dysthe Equations (Modified non-linear Schrödinger equations)
  Achieved by expanding the velocity potential ϕ and the surface displacement h.
- Korteweg–de Vries equations A similar derivation, but in shallow water and wont be considered.

The model used

The model considered was developed by Cousins and Sapsis (2015)

- The free surface elevation, η is defined as follows,

$$\eta = Re\{u(x,t)e^{i(kx-\omega t)}\} \tag{3}$$

ω is frequency, x, t are space and time respectively.

- The NLSE that describes the envelope of a slowly modulated carrier wave on the surface of deep water

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u}{\partial x} + \frac{i}{8}\frac{\partial^2 u}{\partial x^2} + \frac{i}{2}|u|^2 u = 0 \tag{4}$$

Where, u is the wave envelope.

The model used

- The wave envelope is described by,

$$u(x,t) = A(t)\,\mathrm{sech}\left(\frac{x - ct}{L(t)}\right) \tag{5}$$

where $c = \frac{1}{2}$ is the group velocity

- When $A_0 = 1/(\sqrt{2}L_0)$, the soliton wave group shape is constant in time. This is a special case.
- at $t = 0$,

$$u(x,0) \approx A(0)\,\mathrm{sech}\left(\frac{x}{L_0}\right) \tag{6}$$

The model used

Differentiating the NLSE, substituting, and integrating leaves the equation for amplitude, $A(t)$, the initial amplitude A_0 and the initial length, L_0,

$$\frac{d^2|A|^2}{d^2t} = \frac{K}{|A|^2}\left(\frac{d|A|^2}{dt}\right)^2 + \frac{3|A|^2(2|A|^2L^2 - 1)}{64L^2} \tag{7}$$

where $K = (3\pi^2 - 16)/8$

The length is described,

$$L(t) = L_0 \left|\frac{A_0}{A(t)}\right|^2 \tag{8}$$

Equations (8) and (7) are solved, subject to initial conditions

$$|A(0)|^2 = A_0^2 \tag{9}$$

$$L(0) = L_0 \tag{10}$$

$$\left.\frac{d|A|^2}{dt}\right|_{t=0} = 0 \tag{11}$$

The model used

Which results in

$$\frac{d^2|A|^2}{dt^2} = \frac{K}{|A|^2}\left(\frac{d|A|^2}{dt}\right)^2 + \frac{3|A|^2(2L_0^2A_0^4 - |A|^2)}{64L_0^2A_0^4} \qquad (12)$$

Phaseplane analysis

Reduced to a one dimensional ODE. Let $X = |A|^2$,

$$\frac{d^2X}{dt^2} = \frac{K}{X}\left(\frac{dX}{dt}\right)^2 + \frac{3X^2(2L_0^2A_0^4 - X)}{64L_0^2A_0^4} \qquad (13)$$

Rescaling,

$$X = L_0^2A_0^2x \qquad (14)$$

$$t = T\tau \qquad T^2 = (L_0^2A_0^4)^{-1} \qquad (15)$$

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{K}{x}\left(\frac{dx}{dt}\right)^2 + \frac{3}{64}x^2(2-x) \qquad (16)$$

Phaseplane analysis

Let,

$$\frac{dx}{dt} = z \tag{17}$$

$$\frac{dz}{dt} = \frac{Kz^2}{x} + \frac{3}{64}x^2(2-x) \tag{18}$$

to obtain the ODE

$$\frac{dz}{dx} = \frac{Kz}{x} + \frac{3}{64}\frac{x^2(2-x)}{z} \tag{19}$$

With initial conditions,

$$x(0) = \frac{1}{L_0^2 A_0^2} \tag{20}$$

$$\dot{x}(0) = z(0) = 0 \tag{21}$$

Phaseplane analysis

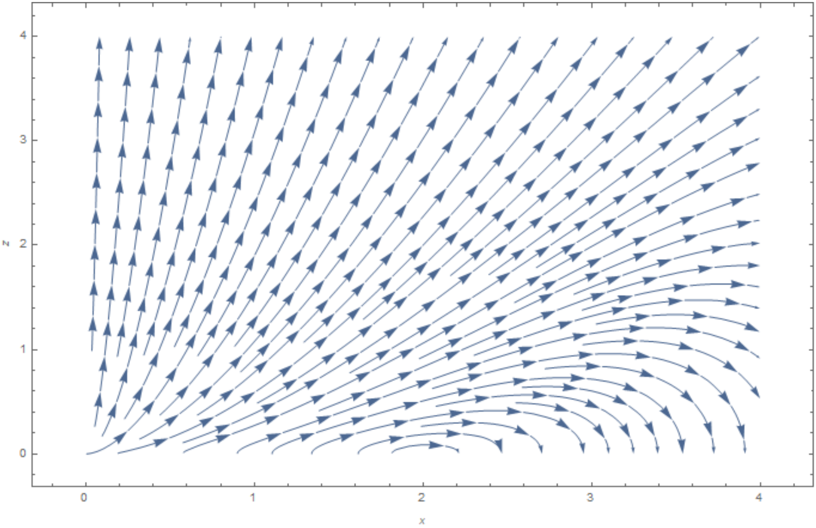


Figure 1: Phase plot of $\frac{dz}{dx}$

Phaseplane analysis

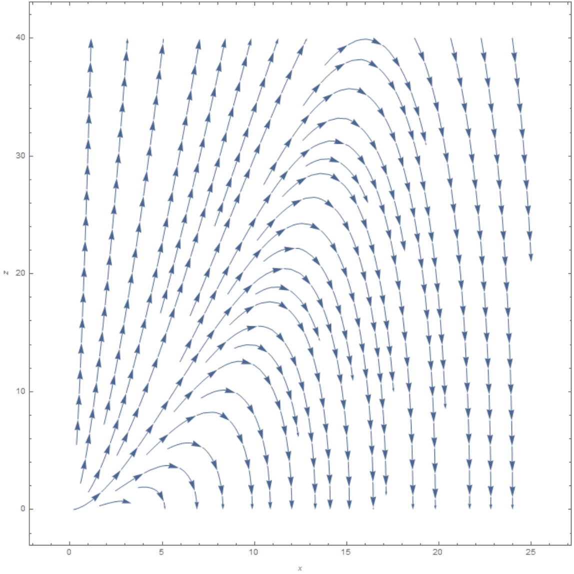


Figure 2: Phase plot of $\frac{dz}{dx}$

Phaseplane analysis

If the initial conditions fall within these ranges,

if $x(0) < 2$,

$$\frac{1}{L_0^2 A_0^2} < 2 \quad \Rightarrow A_0 > \frac{1}{\sqrt{2}L_0} \text{ The amplitude grows}$$

if $x(0) > 2$,

$$\frac{1}{L_0^2 A_0^2} > 2 \quad \Rightarrow A_0 < \frac{1}{\sqrt{2}L_0} \text{ The amplitude stays the same}$$

Phaseplane

- At values of $x(0) = \frac{1}{A_0^2 L_0^2}$ close to 0, the timescale is long and waves grow large very slowly.
- At values of $x(0) = \frac{1}{A_0^2 L_0^2}$ close to 2, the timescale is very small, but the amplitude does not get as large.
- A range between 0 and 2 could potentially be found such that large amplitudes are within a reasonable timeframe.

Future work

- Looking at modified NLSE, the Dysthe equations, it is expect that the Dysthe will have similar solutions to the NLSE, with possibly more stationary points.
- Looking at a larger time scale could give more insight into the problem.
- Determining the internal mechanisms of these waves.
- Determining whether these models can predict the breaking points of these waves.
- Compute the corresponding dimensional values for the quantities to estimate real sea states.
- Taking the same approach, other models could also be analysed in this way.

References

Bitner-Gregersen, E., Fernandez, L., Lefèvre, J., Monbaliu, J., and Toffoli, A. (2014). The north sea andrea storm and numerical simulations. *Natural Hazrds and Earth System Sciences*, 14:1407–1415.

Cousins, W. and Sapsis, T. P. (2015). Unsteady evolution of localized unidirectional deep-water wave groups. *Phys. Rev. E*, 91:063204.