Geometric design of effective fishing exclusion zones.

Proponent: Phil Broadbridge

LA TROBE UNIVERSITY

Moderator: Ashleigh Hutchinson

WITS UNIVERSITY

Acknowledgements to

J.M. Hill, D.J. Arrigo,
J.M. Goard, B. Hajek, R.J. Moitsheki, D. Triadis
P.A. Clarkson, E. Mansfield, M.C. Nucci and G.W. Bluman.

Reaction-diffusion equations.

$$\theta_t = \nabla \cdot [D(\theta)\nabla\theta] + R(\theta)$$

have many applications including population dynamics.
Dependent variable is population density of one or more species.
Diffusivity D represents mobility, reaction term R is net growth rate.

-Simplify by Kirchhoff transformation (1891) to get

$$u = \int D(\theta)d\theta$$

$$F(u)u_t = \nabla^2 u + Q(u) \quad ; \quad F(u) = 1/D(\theta), \quad Q(u) = R(\theta).$$

Population flux density is $-\nabla u$.

$$F(u)u_t = \nabla^2 u + Q(u); \quad F(u) = 1/D(\theta); \quad Q(u) = R(\theta).$$

Nonclassical symmetry analysis-
1D- Arrigo & Hill 1995
2D- Goard & Broadbridge 1996.

In 2+1-D, PDE has simple nonclassical symmetry

$$\bar{u} = e^{A\epsilon}u, \quad \bar{t} = t + \epsilon$$

whenever Q=Au F(u) + K^2 u. (1)

This gives a reduction to linear Helmholtz equation

$$u = \phi(x, y)e^{At} \; ; \quad \nabla^2\phi + K^2\phi = 0. \qquad (2)$$

This is true also in 3+1-D.

Sub (2) in governing PDE, get (1), equivalent to ODE

$$D(\theta) = u'(\theta) = \frac{Au}{R(\theta) - K^2 u} \qquad (3)$$

Now consider

$$u = \phi(x,y)e^{At} \ ; \quad \nabla^2\phi + K^2\phi = 0. \quad (2)$$

With $K^2>0$, the Helmholtz equation has positive radial solutions $J_0(Kr)$ between successive zeros of the Bessel function. Solution (2) then satisfies meaningful boundary conditions

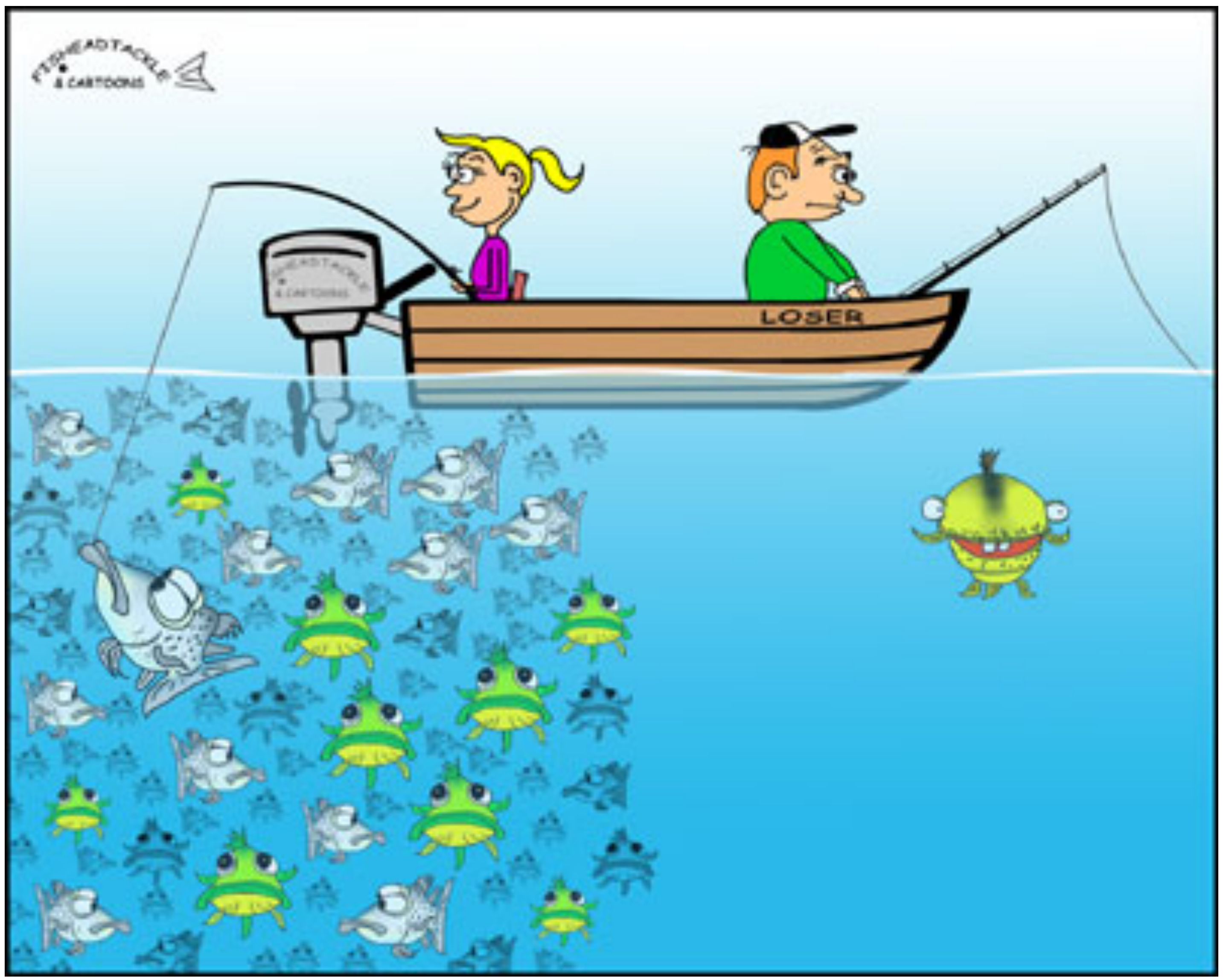
$$u_r = 0, \quad r = 0,$$

$$u = 0, \quad r = r_1; \quad J_0(Kr_1) = 0. \Longleftrightarrow \ u_r = -Bi\ u, \quad r = r_2 < r_1$$

Population model of a protected species.

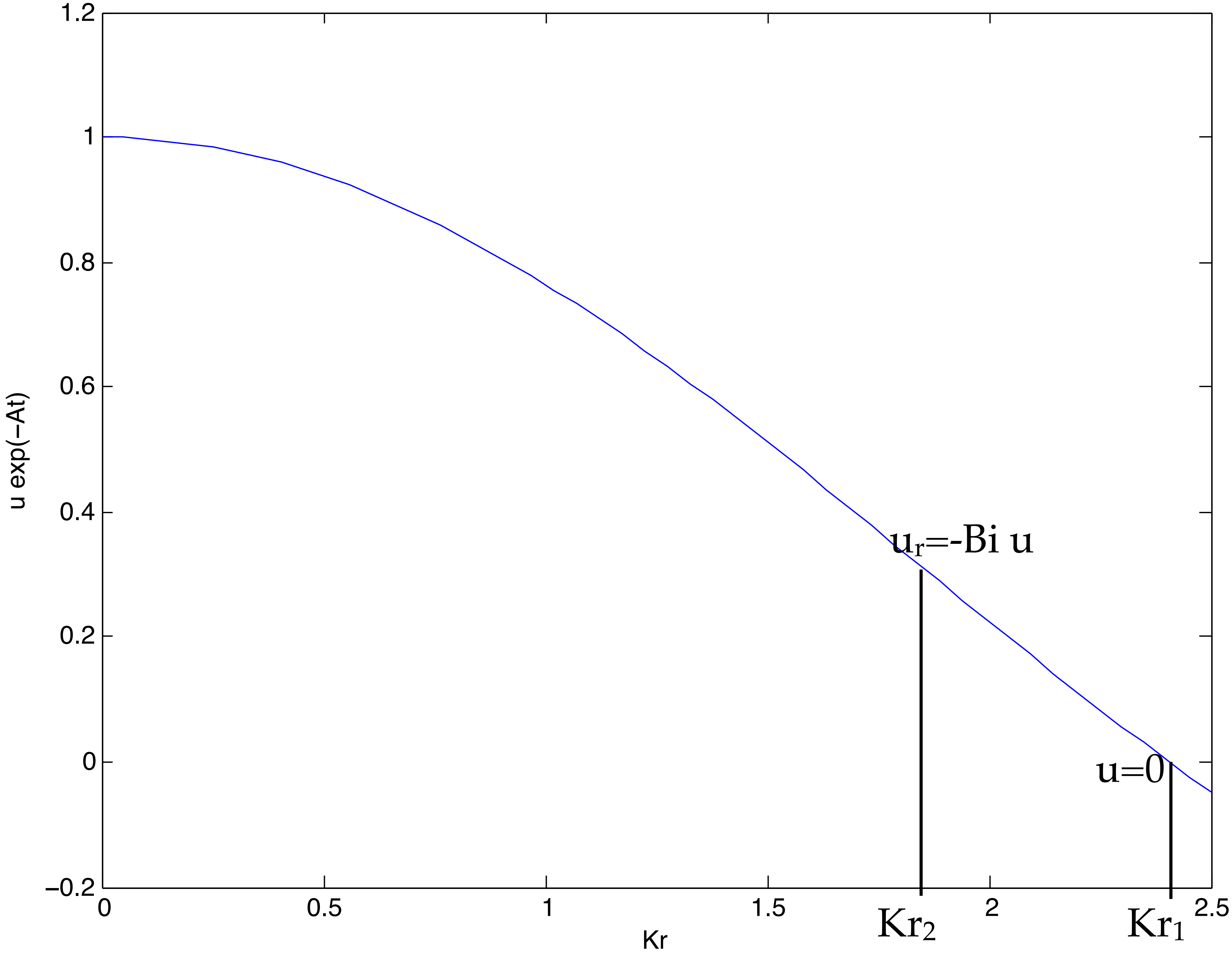
$$\theta_t = \nabla.(D(\theta)\nabla\theta) + s\theta(1-\theta)$$

Population density is scaled so carrying capacity is 1.

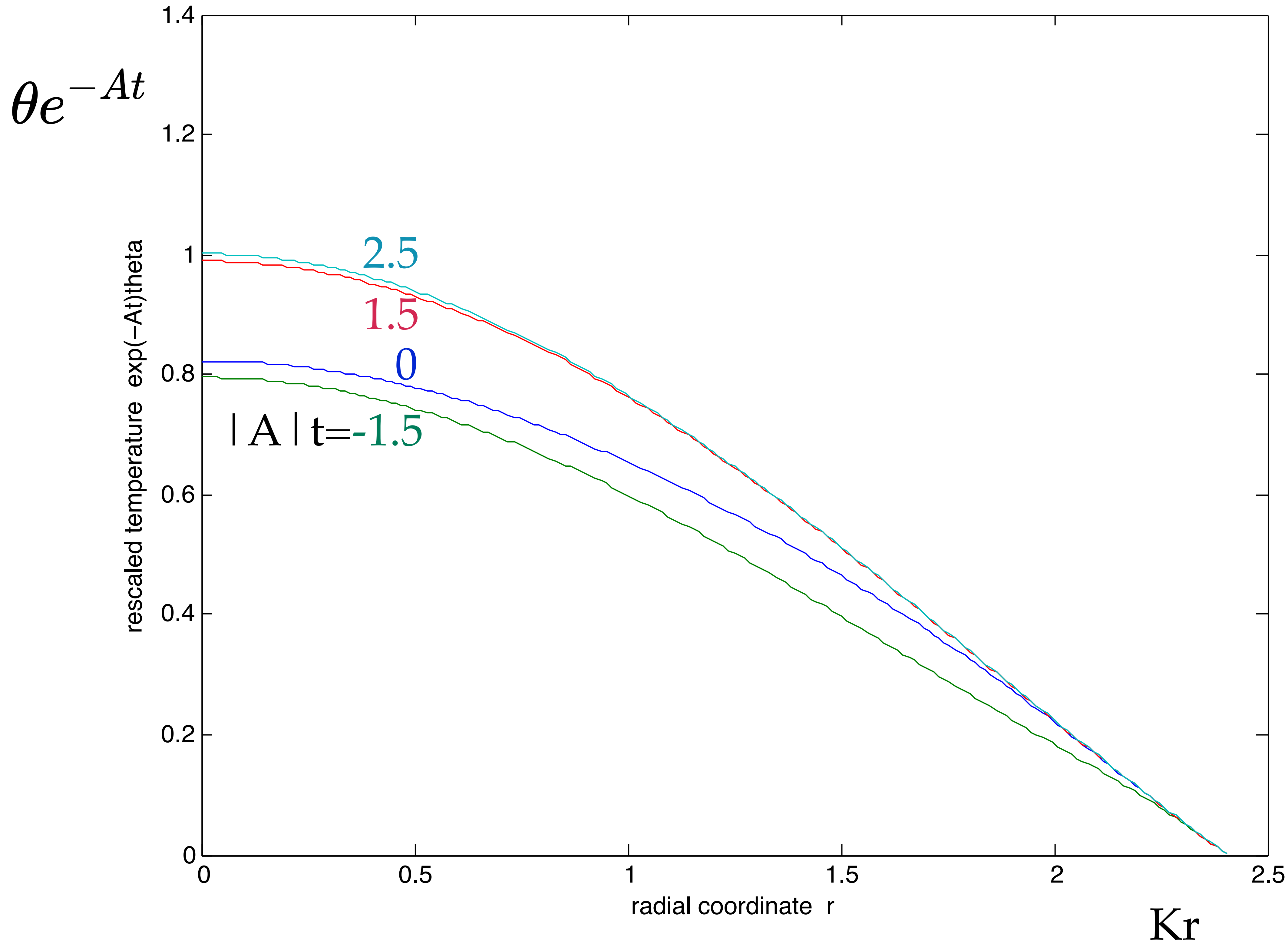


$$r = r_1$$

Similarity solution for Kirchhoff variable u vs r.

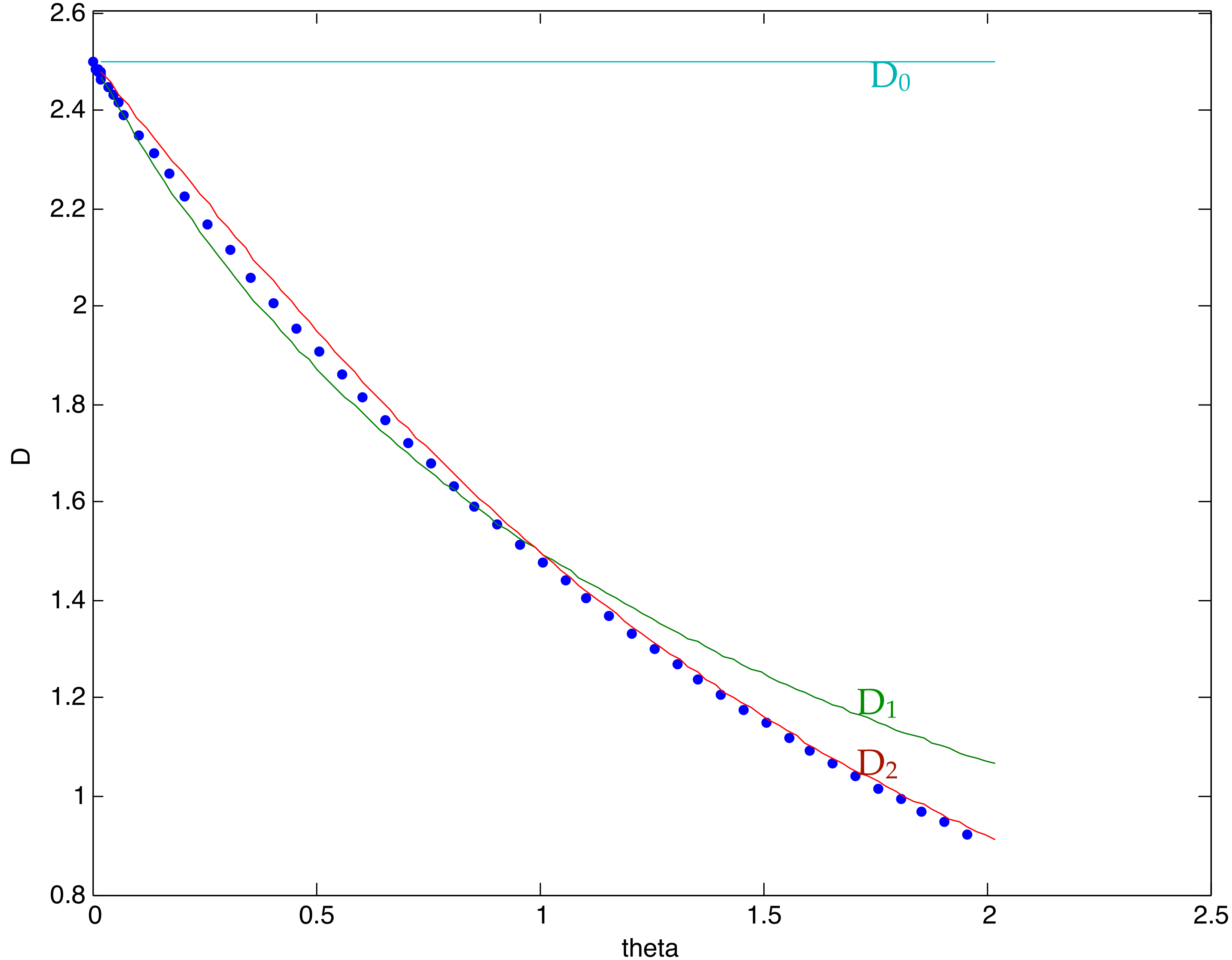


Population vs. r. Large-t approach to similarity solution.



K=1, A=-1.5, s=1, u0=0

...... ODE45 (MATLAB) numerical solution.



Similarity solution for Kirchhoff variable u(r,t) still applies to logistic source term

$$\theta_t = \nabla.(D(\theta)\nabla\theta) + s\theta(1-\theta)$$

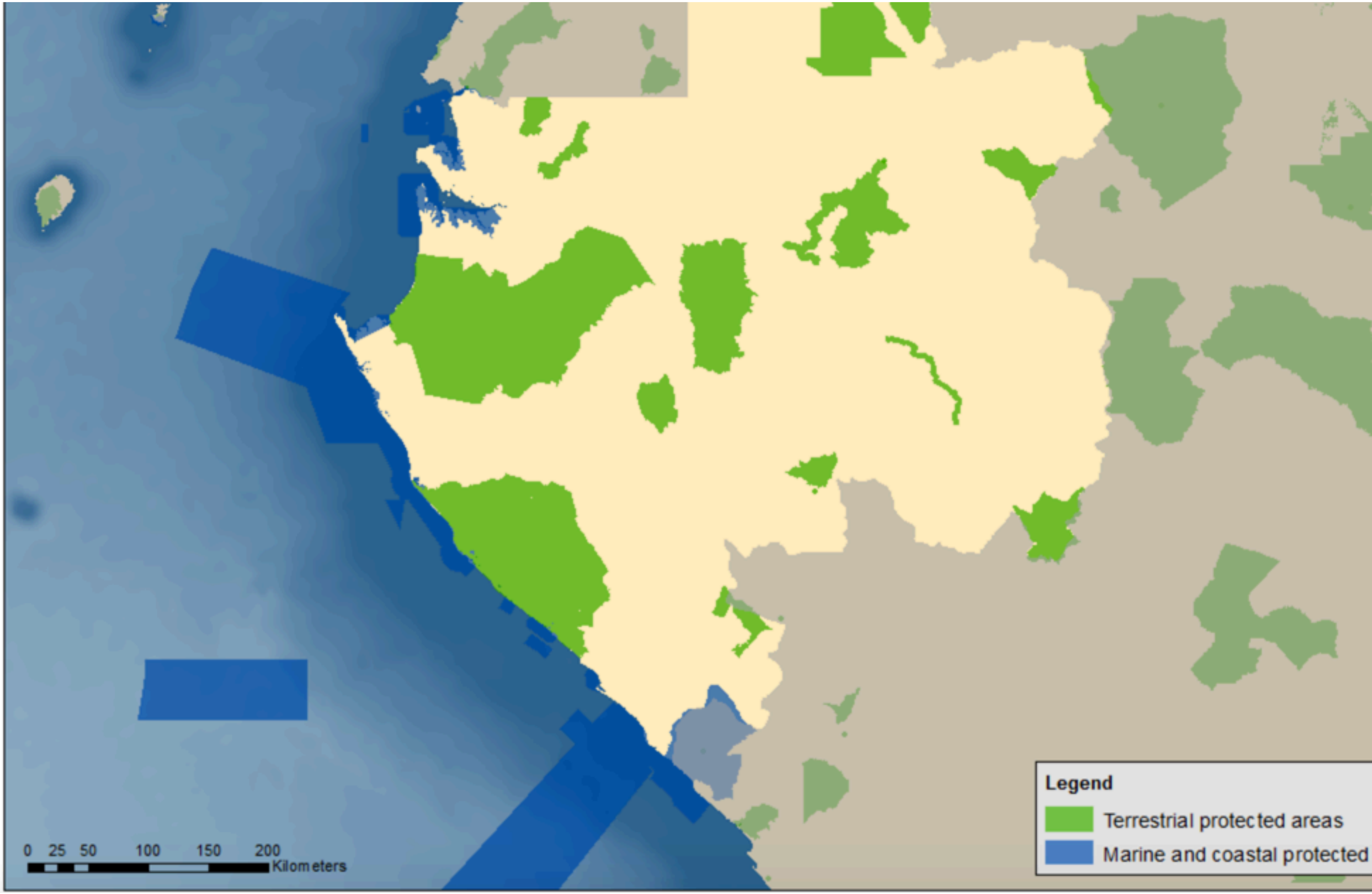
However, exponentially decaying solution does not exist if radius of region satisfies

$$r_1 > \lambda_1\sqrt{D(0)/s} \qquad (\text{use} \quad Kr_1 = \lambda_1)$$

where λ_1 is first zero of Bessel J_0.

When θ is marine population density, this is the minimum radius of a successful fishing exclusion zone.

e.g. D(0)=100 km^2/yr and exponential growth time 1/s =5 yr
$2r_1 > 100$km
Same answer follows from linear stability analysis, without solving nonlinear problem.



Southern Gabon

How robust is this estimate ? -Extensions to the model

0. System parameters matched from biological data; sensitivity analysis.
1. Rectangular and linear geometry (easy, maybe interesting).
2. Softer boundary conditions - fraction is lost through boundary (analytic solution is easy in approximate form).
3. Heterogeneous environment (e.g. more growth in centre).
4. Anisotropic mobility (pelagic fish don't approach the shelf).
5. Network of separated reserves (as in Gabon). (adapt land-based work on koalas etc)
6. two-species predator-prey with different mobilities- steady states and stability analysis.