STOPE BOUNDARY OPTIMISATION FOR UNDERGROUOND MINES

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Abstract

After the exploration phase, data collected are analysed and interpreted using geostatistical modelling techniques to produce an orebody model. The orebody model is delineated into thousands of mining blocks in 3 dimesniosnal (3D) space with assigned grade values. The geological information will inform the type of mining to be adopted, whether surface or underground mining. Consequently, the appropriate mining method is selected depending on the type of deposit. It is at this stage that mine planners can commence with the generation of an optimal stope layout subject to economic and technical constraints. The selection of an optimum stoping layout is one of the important areas of mine planning; however, it is still relatively underdeveloped. There are several algorithms that have been developed to generate a stope layout for underground mining. However, none of these algorithms guarantees an optimal solution in 3D. This paper develops several methods of finding (sub)-optimal solutions in 2D and 3D. The problem is mathematically formulated, and solves using a deterministic Multi-Start algorithm, and a heuristic Particle Swarm Optimisation Algorithm.

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1 Introduction

Mining projects have long turnaround times and require large start-up capital to build and operate. The objective of mine production is to maximise return on investment, which is derived from the extraction and sale of the mineral. The return on investment will depend on the physical location of the ore, the mining layout and extraction sequence, technical factors associated with the orebody, grade of the orebody and the available mining methods [9]. It is in the early planning stages where a mine has the greatest level of flexibility to make decisions on the economic and technical criteria for operating a mine.

Thorough planning done in advance of constructing the mine lowers the risk of failure. Once the construction of the mine begins, the ability to alter the mine design diminishes exponentially as the mine matures [7]. Therefore, the mining engineer is required early on in a mining project to make long-term decisions that must optimise the cost efficiency and profitability of the mine operation.

The limited number of tested operations research (OR) techniques and the lack of tools and appropriate computer programs to address underground mine planning problems is an issue of concern to mining professionals[12]. This lack of software limits a company's capacity to develop underground mine plans that maximises the net present value (NPV) of the project [2, 9]. There is a recognised need by the mining industry for improved software tools to support the planning, design and operation of underground mines [3]. The strategic planning tools could help to minimise the potential for sub-optimal decisions being made at the outset of an operation by reviewing many different alternatives.

In the mine design process of underground mines, the mining engineer must first select a mining method that is amendable to extracting the orebody and then decide on a cut-off grade for extracting the orebody. The next step is to create a stope design that maximises the value of the mine. A stope is an underground production area from which ore is extracted from the surrounding rock mass [15]. The mine engineer will then design the access to the identified stopes. In addition, the mining engineer must sequence the extraction order of the stopes with the purpose to maximise economic ore recovery. Throughout this process, the mining engineer must consider the technical factors associated with the orebody and economic factors associated with the selected mining method [7]. Therefore, the design of the mine stopes, mainly their dimensions and location, is a critical aspect of the mine design process. One particular technical consideration to account for in the mining design is the level constraint. This constraints restricts the stopes to be selected in an ad-hoc query manner. In a 3D search space, a level would be defined to have the same dimensions as the stope for two of its dimensions, and where the third dimension is unrestricted. Mining a particular combination of dimensions and levels makes it easier to apply to the corresponding abstraction space. Moreover, applying level constraints reduces the search space [1].

The study of the best selection and configuration of a collection of objects adhering to an objective function defines Combinatorial Optimisation Problems (COP) [14]. Since the optimality of the mine design is determined by the selection and configuration of both the orebody and its stopes, this problem can be classified as a COP.

Historically, the mining engineer would design the stopes manually, which is a time consuming process. Furthermore, the use of rules-of-thumb in determining the dimensions and locations of the stopes would be common practice. However, rules-of-thumb calculations do not always produce optimised designs. Since the subsequent introduction and proliferation of computers, the use of software applications with built-in algorithms that can automatically design and optimise the stope layout has increased. While this has reduced the time required for the stope design process, the literature indicates that none of the current algorithms are able to guarantee the optimum stope design [13].

1.1 Objectives

The objectives of the research are as follows:

- Develop a mathematical model describing the Stope Boundary Optimisation Problem (SBOP).
- Apply multiple optimisation methods to obtain an optimal mining configuration for an underground mine.
- Test the methods on the available data and quantitatively express their performance.

2 Mathematical Model

2.1 Development of Stope Boundary Optimisation Model

For the sake of simplicity, the mathematical formulation is developed in 2D, and extended to 3D. The following assumptions are made in creating the 2D mathematical model:

- Let the mining area be represented by a grid with dimension, $n \times m$.
- The grid is made up of distinct blocks with predefined values.
- The stope dimension is fixed for 2D case, say $\alpha \times \beta$.
- The decision variable is binary.
- To ensure that the stopes are on the same level for easy mining, the following strategy is used: If $x_{ij} = 0$, then move to x_{ij+1}; If $x_{ij} = 1$, then move to $x_{ij+\beta}$; Once the level has been exhausted, move to $x_{i+\alpha j}$, and repeat the steps.

In 2D, the optimisation problem is specified as follows:

$$\text{maximise} \quad \sum_{i=1}^{n-p}\sum_{j=1}^{m-q} V_{ij}x_{ij}, \tag{1}$$

which models the SBOP as an optimisation problem subject to the constraints:

$$\sum_{i}^{i+p}\sum_{j}^{j+q} x_{ij} \leq 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\} \tag{2}$$

$$x_{ij} - \sum_{j'=j+1}^{j+q} x_{ij'} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\} \tag{3}$$

$$x_{ij} - \sum_{i'=i+1}^{i+p} x_{i'j} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\} \tag{4}$$

$$x_{ij} - \sum_{i'=i+1}^{i+p}\sum_{j'=j+1}^{j+q} x_{i'j'} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\} \tag{5}$$

where:

$$V_{ij} = \sum_{i}^{i+p}\sum_{j}^{j+q} u_{ij}, \quad p = \alpha - 1 \quad \text{and} \quad q = \beta - 1, \quad x_{ij} \in \{0, 1\}\,. \tag{6}$$

Extending the problem to 3D yields the following:

$$\text{maximise} \quad \sum_{i=1}^{n-p}\sum_{j=1}^{m-q}\sum_{k=1}^{s-r} V_{ijk}x_{ijk}, \tag{7}$$

subject to:

$$\sum_{i}^{i+p}\sum_{j}^{j+q}\sum_{k}^{k+r} x_{ijk} \leq 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\}, \forall\, k \in \{1, \cdots s-r\} \tag{8}$$

$$x_{ijk} - \sum_{j'=j+1}^{j+q} x_{ij'k} = 1 \quad \forall\, i \in \{1, \cdots n-p\} \forall\, j \in \{1, \cdots m-q\}, \forall\, k \in \{1, \cdots s-r\} \tag{9}$$

$$x_{ijk} - \sum_{i'=i+1}^{i+p} x_{i'jk} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\}, \forall\, k \in \{1, \cdots s-r\} \tag{10}$$

$$x_{ijk} - \sum_{k'=k+1}^{k+r} x_{ijk'} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\}, \forall\, k \in \{1, \cdots s-r\} \quad (11)$$

$$x_{ijk} - \sum_{i'=i+1}^{i+p} \sum_{j'=j+1}^{j+q} \sum_{k'=i+1}^{k+s} x_{i'j'k'} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\}, \forall\, k \in \{1, \cdots s-r\} \quad (12)$$

$$x_{ijk} - \sum_{i'=i+1}^{i+p} \sum_{j'=j+l}^{m-1} \sum_{k'=k+1}^{s-1} x_{i'j'k'} = 1 \quad \forall\, i \in \{1, \cdots n-p\}, \forall\, j \in \{1, \cdots m-q\}, \forall\, k \in \{1, \cdots s-r\} \quad (13)$$

where:

$$V_{ijk} = \sum_{i}^{i+p} \sum_{j}^{j+q} \sum_{k}^{k+r} u_{ijk}, \quad (14)$$

$$p = \alpha - a\ , q = \beta - b \quad \text{and} \quad r = \gamma - 1 \quad (15)$$

$$x_{ijk} \in \{0, 1\} \quad (16)$$

$$a = 1, ..., n,\ \ b = 1, ..., m. \quad (17)$$

2.2 Development of Multi-Start Algorithm

The Multi-Start (MS) algorithm aims to find a global optima or its close approximation to an objective function. The MS algorithm is a search technique which primarily consists of repeated application of constructive methods [11]. MS is a two-phase method; a global search followed by a local search. The global search entails the objective function being evaluated in a number of randomly sampled point (uniform distribution), and the local search manipulates points samples in the global search to yield candidate global optima. The MS algorithm is classified as deterministic or non-deterministic by the scheme used to make a selection during the search process. If a greedy selection is repeatedly made then the MS is classified as deterministic, and non-deterministic if a stochastic scheme is used. Stochastic schemes used include random selection or application of perturbation. Three key elements in MS methods that can be used for further classification purposes are: memory, randomisation and degree of rebuild.

The MS algorithm is commonly embedded to heuristic search schemes [10] to provide diversification in exploring a search space for solutions to COPs. These heuristics are based on local optimisation that aspire to find global optima. Hence, the MS algorithm is proposed to guide the search by repeatedly restarting the procedure from a new solution once a new solution has been found.

The MS algorithm's search space can be represented by binary decision variables (i.e., 0's and 1's). Starting from a null solution and selecting variables are then set to 1.

The application of the MS algorithm to the SBOP is now considered.

2.2.1 Application of Multi-Start Algorithm to the Stope Boundry Optimisation Problem

The first step of applying the MS algorithm to finding a solution to the mining problem is to project the selected orebody as a binary representation. The orebody is delineated into unit blocks which compose individual stopes. An example of a 2D representation is given in Figure 1.

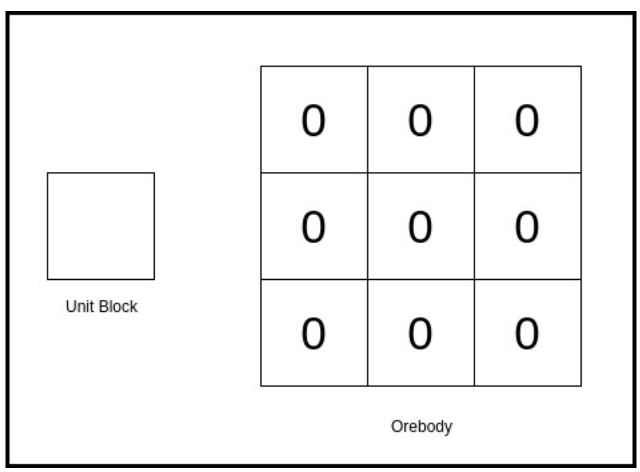


Figure 1: Representation of delineated orebody.

An example of stope configurations is shown in Figure 2. In this example stopes of size 2×2 are formed from the unit blocks into which the orebody was delineated. At initialisation a number of stope configurations are predefined which obey the level constraint.

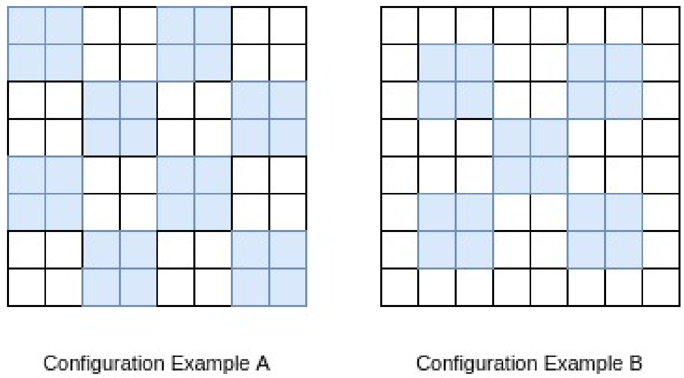


Figure 2: Examples of predefined stope configurations.

At initialisation each of the unit blocks is set to 0. Until some termination criterion is not met, the algorithm repeatedly selects the highest value stope from any of the predefined configurations which has not been previously selected and does not violate the level constraints. Next, the level in which the stope was selected is probed to find other feasible stopes to mine. Both the mine design and mine value are updated with the selected stopes. This procedure is summarised in Algorithm 1.

Algorithm 1 Multi-Start Algorithm

```
procedure MULTI-START ALGORITHM(κ, ζ)    ▷ Global Termination Criterion, Local Termination Criterion
    for i = 0 : 1 : κ do
        Find highest value stope    ▷ This stope is previously unselected and does not violate any level constraints
        for j = 0 : 1 : ζ do
            Probe level for any other feasible stopes
            Update mine value
        end for
    end for
    return Mine value
end procedure
```

The MS algorithm applied to this problem is classified as deterministic because the greedy selection scheme is utilised.

2.2.2 Parameters

The parameters selected for the MS algorithm were based on the delineation of the ore-body, the stope size and the restrictions enforced by the level constraint.

2.3 Development of Particle Swarm Optimisation Algorithm

The Particle Swarm Optimisation (PSO) algorithm optimises a problem by generating a population of particles, representing candidate solutions, and having each particle iteratively try to improve on its solution with regard to a given measure of quality. Each

Table 1: Selected MS parameters

Parameter	Description	Value
κ	Number of global restarts (equivalent to the maximum number of stopes in the mine.)	3200
ζ	Number of local searches (equivalent to the maximum number of stopes per level).	16
η	Number of predefined stope configurations.	8

particle will evaluate its current solution quality against the personal best solution it has achieved so far and also the global best solution found by any particle in the population. Each particle moves in search of better solutions throughout the search-space according to simple mathematical formulae that define the particle's position and velocity over time [8] . To search for the optimal solution, the velocity and positions of each particle are updated by the following equations:

$$\begin{cases} \mathbf{v_i^d}(k+1) & = \omega \mathbf{v_i^d}(k) + c_1 \mathbf{r_1} \left(\mathbf{P_{best}^d}(k) - \mathbf{P_i^d}(k) \right) + c_2 \mathbf{r_2} \left(\mathbf{g_{best}^d}(k) - \mathbf{P_i^d}(k) \right) \\ \mathbf{P_i^d}(k+1) & = \mathbf{P_i^d}(k) + \mathbf{v_i^d}(k+1) \ , \end{cases} \tag{18}$$

where c_1 and c_2 are acceleration constants regulating the relative velocities with respect to the personal best and global best positions respectively; $\mathbf{r_1}$ and $\mathbf{r_2}$ are $N \times 1$ vectors of random numbers drawn from a uniform distribution in the interval (0,1); and ω is an inertia parameter given by

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{K} k \ , \tag{19}$$

where ω_{max} and ω_{min} are the initial and final weights respectively, k is the iteration number and K is the total number of iterations.

One of the advantages of the PSO is that it is simple to code and it only requires the problem and a few parameters to solve. We will look at the problem definition then encoding strategy and the parameters and then apply the PSO to an orebody.

2.3.1 Parameter Selection

One of the advantages of the PSO compared to other algorithms is the relatively few number of parameters that have to be tuned in the algorithm [8]. The parameter values used for this research was based on a literature survey of the existing research on parameter selection for the PSO algorithm [4, 6]. These parameters are indicated in Table 2.

Table 2: Selected PSO parameters based on literature review

Parameter	Description	Value
ω_{max}	Maximum inertia coefficient	0.9
ω_{min}	Minimum inertia coefficient	0.4
K	Total number of iterations	100
$c_1; c_2$	Velocity coefficients	$c_1 = c_2 = 2$

The PSO is known to be very sensitive to the choice of parameters and parameter selection is one of the most important aspects in the PSO algorithm. It is accepted that generally the choice of the parameters will be problem dependent and that parameter hypertuning will often be required.

The encoding of the problem is specified in two dimensions with the intention of clarifying the strategy for modelling the problem. It was assumed that the sub-level open stoping method would be used to mine the deposit and three constraints associated with this mining method, namely overlap constraint, level constraint and uniqueness constraint, had to be considered in encoding the PSO algorithm. Figure 3 illustrates an example of a section of ore body, and how the selected mine configuration is encoded such that it can be passed to the PSO algorithm.

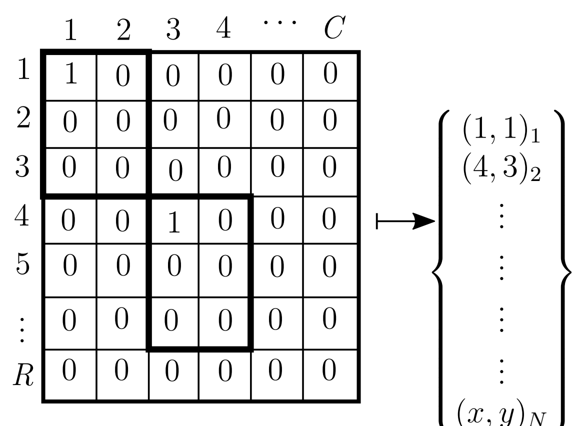


Figure 3: Encoding of a Mine Configuration.

In this trivial example, there are R rows and C columns in the orebody, representing the orebody extent. Each block represents a block in the ore body. The stope size is fixed at 3×2 blocks and there are N possible stopes. The stopes selected in this particular configuration are marked in bold. The starting corner of each selected stope is marked with the number one and the rest of the ore body is padded with zeros. The co-ordinates of all the ones in the ore body are then found and stored in a set, as illustrated in Figure 3. This set of co-ordinates forms one member of the population. Each member of the population is therefore an entire mine configuration. Figure 3 is an illustration of the encoding of the 2D SBOP. In 3D, the dimensionality of the problem is $N \times 3$.

2.3.2 Fitness Evaluation

The PSO is initialised by generating random solutions, that is, particles which represent a specific mine layout. The number of stopes that may be used in the mine layout is pre-defined. Therefore, each particle will consist of the pre-defined number of stopes randomly selected from the set of all possible stopes.

The fitness of a particle is a direct function of the final value of the mine, that is, the sum of the values of the selected stopes in the mine layout. The Net Smelter Return (NSR) was used as the measure of value. Incorporated into the fitness evaluation are three important constraints; the level constraint, uniquness constraint, and overlap constraint. The nature of these constraints is illustrated in Figure 4.

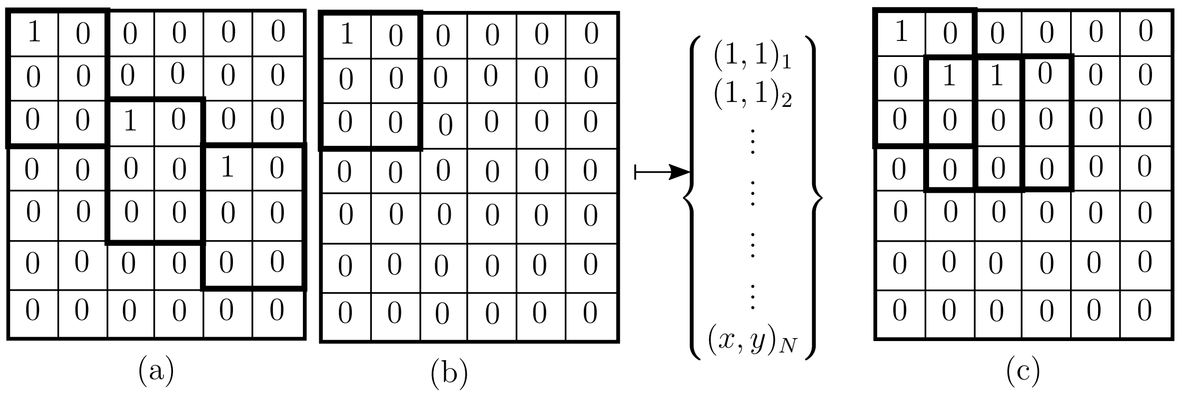


Figure 4: Visual representation of (a) Level constraint violation, (b) Uniqueness constraint violation, (c) Overlap constraint violation.

The level violation indicates multiple stopes which have blocks on different mining levels. This is not allowed according to the design of the mine as each stope must lie within the defined level spacing. Uniqueness violation occurs when two stopes occur at the same location. Since a stope cannot be mined twice, the number or stopes needs to be explicitly specified in the model. Overlap violation occurs when stopes are overlapping. These stopes may or may not be on the same level. This incurs a penalty because stopes may not overlap, again because once a stope or a portion of it has been mined, it cannot physically be mined again. The fitness of the configuration is therefore taken to be the linear combination of the mine value and the three penalties:

$$Fitness = V_{Mine} - k_1 * P_{Level} - k_2 * P_{Unique} - k_3 * P_{Overlap}, \tag{20}$$

where V_{Mine} is the calculated value of the mine configuration, P's are the penalties incurred by each respective constraint violation, and k's are constants, chosen large enough such that even if one penalty occurs, the fitness will indicate that the mine configuration will not be economically viable. This ensures that all economically viable configurations follow all the constraints. The goal of the algorithm is to maximise the fitness. Figure 5 shows the flow chart for the mine configuration optimisation.

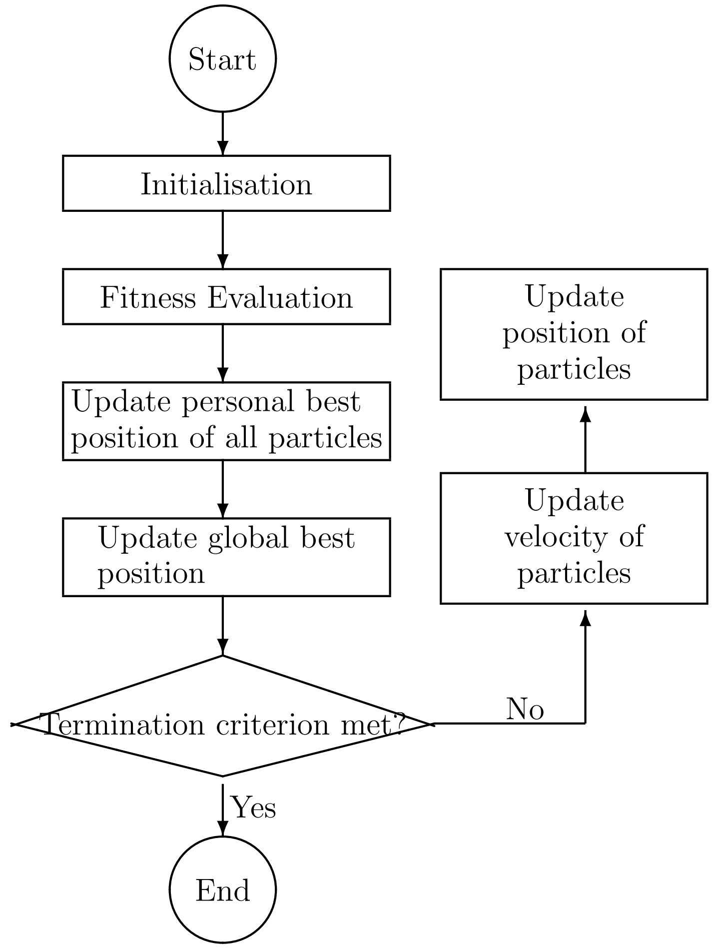


Figure 5: Flowchart of PSO Algorithm.

The experimental method utilising PSO for the purpose of mine configuration optimisation can be described as follows:

1. Initialise swarm size, maximum number of generations and initial velocities and positions of each of the particles, where each particle represents a particular mine configuration.
2. Calculate the fitness of each member according to Equation 20.
3. Update model parameters if necessary; the personal best position of each particle and the global best position at the current time step.
4. Update the velocity and positions of each particle according to Equation 18.
5. The algorithm terminates when the maximum number of generations occurs. The output is an optimal mine configuration.

This method was conducted for a varying number of stopes of a fixed size. Twenty experiments were conducted for each stope number.

3 Results

The optimisation of a mine configuration was attempted using PSO as an optimisation tool. Training data from a conceptual ore body was used to test the optimisation algorithm and the results are discussed.

The algorithm requires a regularized economic block model. The orebody model used in this study represents a theoretical gold deposit. The block model consists of 15572 blocks of a uniform block size. The geological attributes assigned to each block are the gold grade and the rock density. The metal content per block was determined from these two attributes, taking into consideration the block size. An economical value, the Net Smelter Return (NSR), was calculated for each block based on assumptions of the mining costs, processing costs, logistical costs and metal price. The economic block model data is summarised in Table 3.

Table 3: Summary of Economic Block Model Data

Attribute	**Value**
Number of blocks	15572
Block sizes (x,y,z)	5m × 5m × 5m
Rock density variation	2.8 - 3.6 t/m^3
Net Smelter Return Variation	0.6 - 301 USD/tonne

The block model data was then imported into the *Python*® script that was developed for this research. Then the respective optimisation algorithm was run using the fixed block sizes and stopes sizes given in Table 4.

Table 4: Mine Configuration

Attribute	**Value**
Block size (x, y, z)	5m x 5m x 5m
Single mine stope dimension $x \times y \times z)$	10m × 10m × 20m
Single mine stope dimension (Blocks) $x \times y \times z$	$2 \times 2 \times 4 = 16$

3.1 Multi-Start Algorithm

The MS algorithm is used as an optimisation tool to obtain a mine configuration. The MS algorithm applied specifically to this mining problem is deterministic as the algorithm constantly makes a greedy choice.

The MS algorithm uses a number of η predefined stope configurations which are used in making stope selections for the mine design. This research compares the mine configuration and its associated mine value to the number of predefined stope configurations used for the search. An overview of the predefined stope configurations is given in Table 5.

Table 5: Predefined stope configurations.

Individual Configurations Overview			
Configuration	Maximum Available Stopes	Maximum Mineable Stopes	Maximum Mineable Stope Value
0	3200	643	754782.0192
1	3100	608	716899.0144
2	3136	663	767009.0959
3	3038	626	727526.7213
4	3100	573	654135.1559
5	3003	541	621057.3940
6	3038	562	648486.8863
7	2943	532	616518.0707

The maximum available stopes in the configuration is the number of stopes that the orebody is divided into. Maximum mineable stopes is the number of stopes that meet the constraints which are the stopes that are feasible to mine, that is, if the stope value is greater than 480 NSV.

The predefined stope configuration overview from Table 5 is visually depicted in Figure 6.

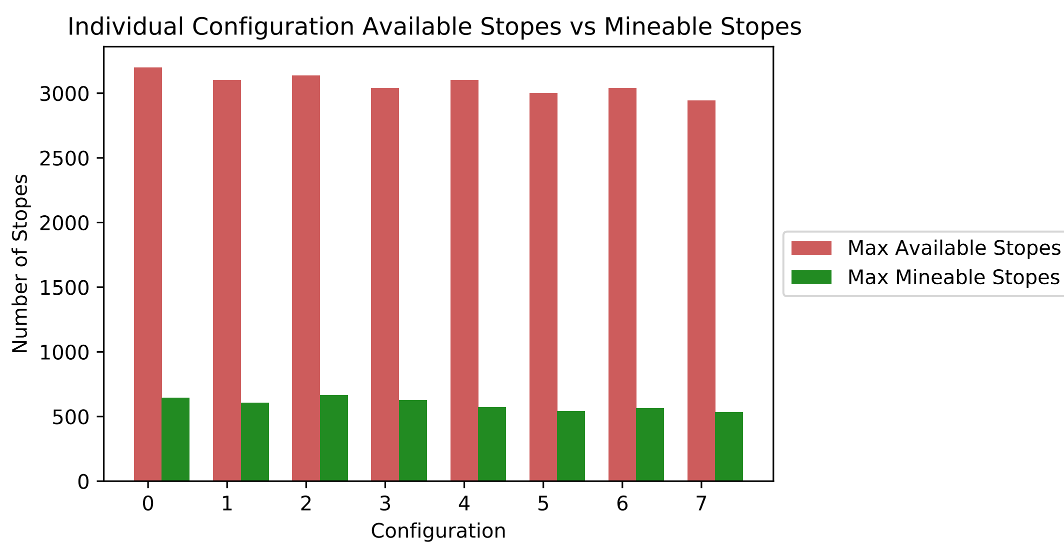


Figure 6: Stopes available compared to number of mineable stopes.

Three particular configuration combinations are selected to form part of the search space to deduce how the predefined stope configurations impact the mine configuration design obtained by the MS algorithm. All the selected predefined stope configurations have a shift in two dimensions. An overview of selected configurations is tabulated in Table 6.

Table 6: Individual Configuration Overview

Configuration Selection Overview					
Search Space	Configurations Selected for Search Space	Maximum mineable stopes	Maximum stopes mined	Maximum mineable stope value	Maximum stope value
A	3 and 5: (x, y) and (x, z) shifts	1167	12	1348584.12	21845.35
B	3 and 6: (x, y) and (y, z) shifts	1188	31	1376013.61	65405.38
C	5 and 6: (x, z) and (y, z) shifts	1103	49	1269544.28	109496.89

An overview of the configurations selected for the search space is visually depicted in Figure 7.

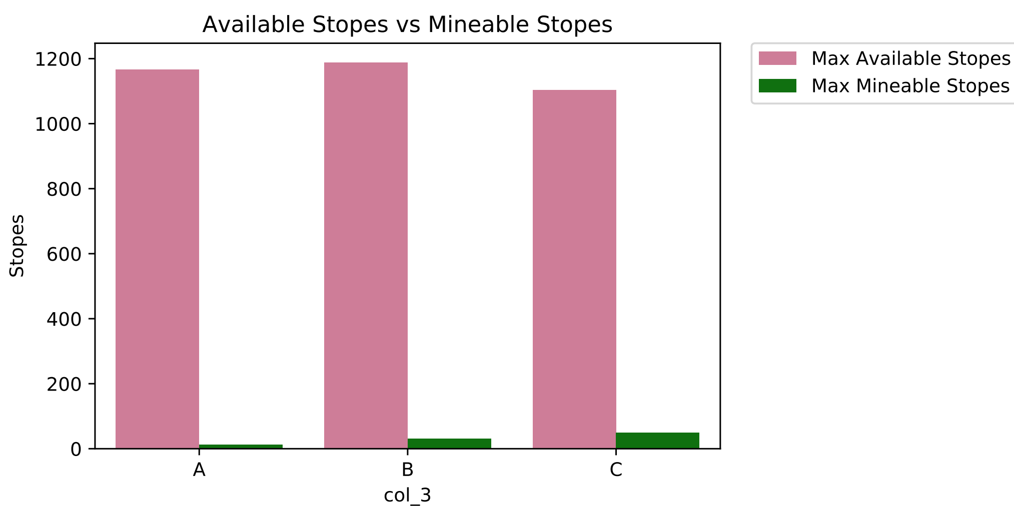


Figure 7: Stopes available compared to number of mineable stopes forming the search space.

The the value of the available stopes to the mineable stopes of the 3 specified search spaces is given in Figure 8. From these results it can be deduced that the predefined stope configurations impact the mine design. The design of the applied MS algorithm is of a deterministic nature, hence is restrictive in exploring the search space. The highest valued search space is C, which obtains a mine design valued at 109496.89 NSV for 49 stopes. This should be considered in conjunction to the single configuration search spaces given in Table 5. It is highlighted that using a single configuration of predefined stopes in a search space may be more viable as the greedy technique reduces the number of stopes that form a part of the mine design as a results of violating the level constraints.

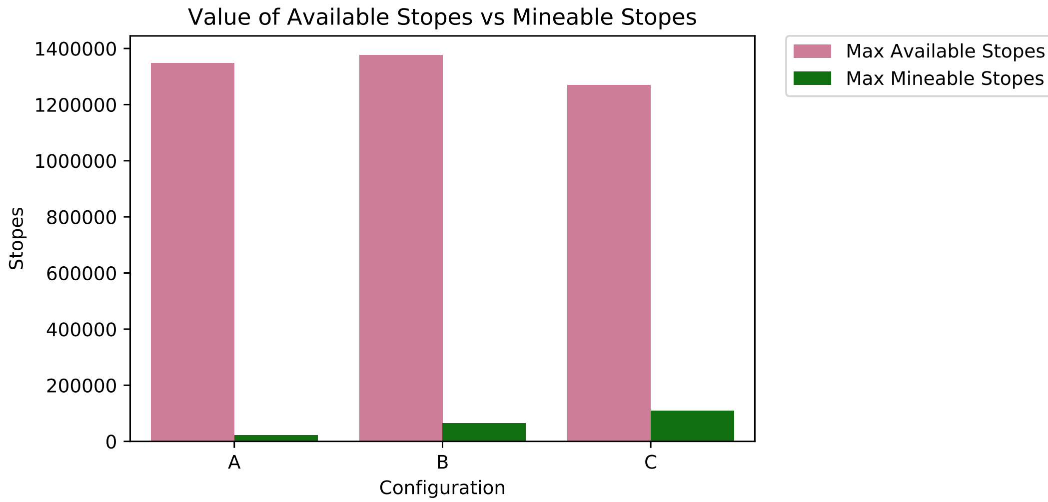


Figure 8: Value of stopes available compared to number of mineable stopes forming the search space.

3.2 Particle Swarm Optimisation Algorithm

The optimisation of a mine configuration was attempted, using PSO as an optimisation tool. Training data from a conceptual ore body was used to test the optimisation algorithm, and the results are discussed.

The algorithm requires a regularised economic block model. The orebody model used in this study represents a theoretical gold deposit. The geological attributes assigned to each block are the gold grade and the rock density. The metal content per block was determined from these two attributes, taking into consideration the block size. An economical value, the Net Smelter Return (NSR), was calculated for each block based on assumptions of the mining costs, processing costs, logistical costs and metal price. The economic block model data is summarised in Table 3.

The block model data was then imported into the *Python*® script that was developed for this research. Then the PSO algorithm optimisation was run using a fixed stope size of $10m \times 10m \times 20m$ along the x, y and z axis, respectively.

Figure 9 shows the final results of all experiments. The maximum mine values are plotted as a function of the number of stopes. The maximum value found by the PSO algorithm is about 22000 with 12 stopes.

Figure 10 shows how the mean and maximum fitnesses in the population change with iteration number from one of the experimental runs using 13 stopes. Figure 10 illustrates the convergence process of the algorithm. Convergence does not mean that the population

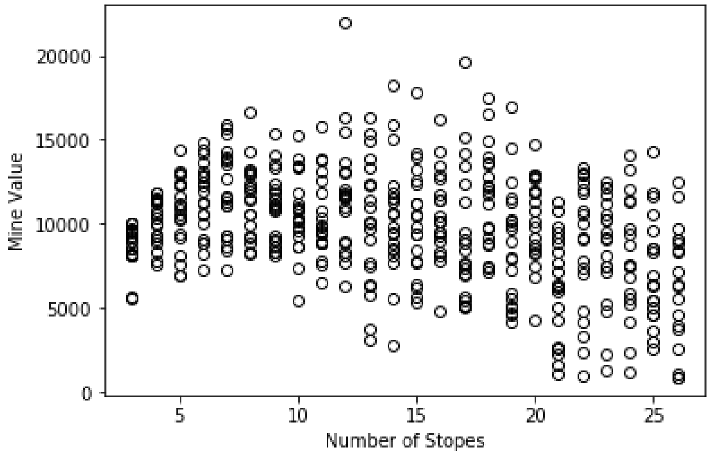
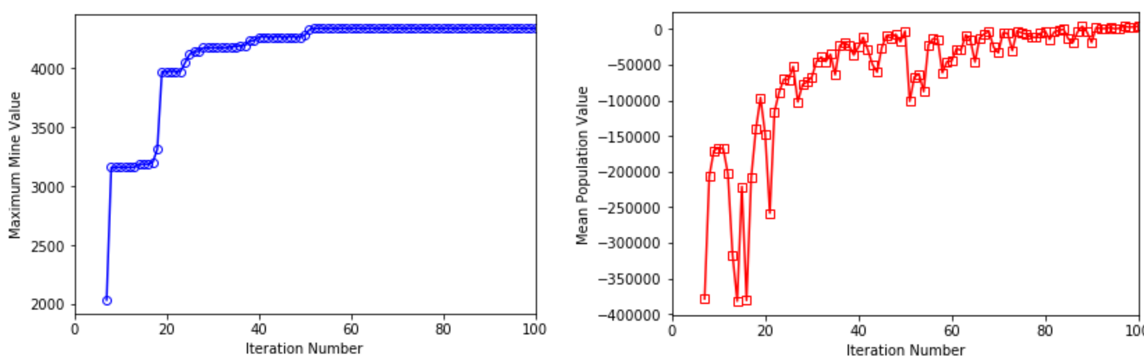


Figure 9: Maximum Mine Values for All Experiments.

has reached an optimum (local or global). Rather it means that the population has reached an equilibrium state, that is, the particles converged to a point, which may not be an optimum point [5].



(a) Maximum Mine Value (NSR)

(b) Mean Mine Value (NSR)

Figure 10: Results from an experimental run with 13 stopes.

From Figure 10(a), the algorithm clearly converges to a maximum value within 100 iterations. In this particular case, the maximum value is relatively small, at approximately 4400 and therefore this value is a local maximum, not the global maximum. The algorithm becoming trapped in local maxima is the reason multiple experiments are run.

From Figure 10(b), the effect of the heavy penalties on the constraints violations can be observed. The mean mine value starts off at approximately -3800000, indicating a large number of constraints violations. The mean mine value then increases with the number of iterations to about 4000 as the violations are resolved.

4 Conclusions

The following conclusions can be drawn from the results:

- The mine layout was successfully optimised by both the specified methods.
- Convergence of the PSO was clearly shown.
- The PSO is feasible with this development and warrants further research.
- The predefined mine configurations impact the mine design obtained by the MS algorithm.
- The MS is a possible solution technique which should be developed by applying it to the SBOP using stochastic methods.

5 Recommendations

The following recommendations are made for further research in the field:

- Hyper-parameters of the PSO were chosen at random; actual parameter selection techniques should be considered.
- MS algorithm designed to be deterministic; stochastic schemes should be considered.
- All current work is defined for fixed stope sizes; in reality, stope size is a variable that changes within an underground mine. It is therefore a useful additional parameter to include in the problem formulation.

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