# PISTON EFFECT DUE TO ROCK COLLAPSE

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**Abstract**

We consider a cave-in of the roof of an underground cavity which is a cause of air blasts in mining operations. Four mathematical models are proposed to describe the air pressure and air speed in the cavity. We solve some of the models and put forward an appropriate composite solution which can be used as the initial boundary value for a suggested model for the air flow in tunnels connected to the cavity.

## 1 Introduction

In mining operations large excavations or cavities, connected to a network of tunnels, often exist. Collapsing roofs cause high pressures and/or wind speeds (called air blasts) to be generated in connecting tunnels. These air blasts are sufficient to overturn vehicles and cause major destruction to infrastructure [4]. An understanding of the process may enable one to modify the design of the tunnel system to mitigate the hazardous effects of an air blast.

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The aims of the study group:

- To model the collapsing rock in the cavity in order to shed light on the following:
  - Understand the escape of air through the gaps between rock blocks that make up the collapsing rock mass.
  - In a piston model , give an idea of the effectiveness of the piston.
- To find a model that explains the air flow in the tunnels.

There is usually interconnection of underground excavations via tunnels, shafts, etc to other underground excavations, with different surface roughnesses, and one or more connections to the surface (the atmosphere). The rock collapse itself may result in a break through to the atmosphere.

In our analysis it is important to know whether or not we expect shock waves in the cavity and/or the adjoining tunnels. The existence of shock waves is connected to the wind speeds attained. If the speed of the air nears the speed of sound in ambient conditions, $c_0 = 331\text{ms}^{-1}$, then one expects shock waves to form.

We consider first a cavity of depth 200 m where the roof collapses under the influence of gravity. The movement of the roof is governed by Newton's second law of motion and thus

$$\frac{d^2z}{dt^2} = g, \tag{1}$$

where $z(t)$ is the position of the roof at time $t$ and $g = 9.8\text{m s}^{-2}$ is the acceleration due to the earth's gravitational field. Integration of equation (1) with initial condition $dz/dt = 0$ at $t = 0$ gives the speed of the roof as a function of time,

$$\frac{dz}{dt} = gt. \tag{2}$$

Since the air is pushed down by the collapsing roof, the speed of the air should be in the order of $gt$. By integration of (2) with $z(0) = 0$ we find the position of the roof as a function of time,

$$z(t) = \frac{1}{2}gt^2. \tag{3}$$

It follows that in a (typical) 200 m deep cavity the falling rock would hit the bottom approximately 6 seconds after it started falling with a speed of 63 $\text{ms}^{-1}$. Therefore we can assume that no shock waves are formed in the cavity.

Next we consider the worst case scenario of a single tunnel connected to the cavity. From a simple consideration of conservation of mass we see that the speed of the air in the tunnel should be of the order of $A/a_0$ times the speed of the air in the cavity. Here $A$ is the (horizontal) cross section area of the cavity and $a_0$ is the cross section area of the tunnel. A typical value for $a_0$ is 16 $\mathrm{m}^2$ whereas $A$ in open stopes can be 400 - 800 $\mathrm{m}^2$. Thus the air could travel 25 to 50 times the speed reached in the cavity.

These numbers suggest subsonic flow within the cavity and supersonic and/or subsonic flow in the adjoining tunnels (decaying shocks). This also suggests that the cavity flow and the tunnel flow problems can be effectively decoupled.

This work focuses on the solution of the cavity problem. Section 2 presents a range of mathematical models that model the pressure levels to be expected in the cavity. The pressure in the cavity defines the pressure at the tunnel entry points. These results may be used to determine the flow into the tunnels. On the basis of the results obtained we suggest an appropriate model to describe the air flow in the connecting tunnels in the last section, the conclusion. (The problem of determining air flow in the tunnels will be addressed in the Proceedings of MISGSA 2006.)

In this report:
$P_0$ is atmospheric pressure (about $10^5$ Pa). We assume that the air pressure in the cavity before collapse is $P_0$.
$P$ is the pressure in the cavity. In general it varies with time $t$ and position $x$ measured downward from the roof of the cavity (before collapse).
$P_t$ is the pressure at the mouth of the tunnel.
$A$ is the cross section area of the cavity which is assumed to be uniform in this report.
$L$ is the height of the cavity before collapse.
$H$ is the height of the rock that collapses (before collapse).
The function $z(t)$ is the position of the collapsing roof of the cavity at time $t$, not to be confused with the spatial variable $x$ which is independent of the variation in the position of the roof.
The volume of the cavity as a function of time is given by $V(t)$ and the initial volume of the cavity is $V_0 = AL$.

# 2 Models of the rock collapse

In this section we present various models that model the collapse of the roof in the cavity and the resultant air pressure and air speed. The falling roof causes compression waves to propagate at speed $c_0$ into the cavity from the moving roof. Such waves reach the floor after a time $L/c_0$ (1-2 seconds for a typical $L = 200$m) and are reflected back towards the roof and thereafter successive reflections lead to a more complex flow pattern. In Section 2.2 we use the one dimensional compressible flow equations to obtain results that are useful for describing the flow field in the early stages. Whilst in theory these results could be used to obtain results for longer time spans it is sensible to filter out the small fluctuations associated with wave interactions and look for a spatially uniform description of the flow field. This approach is followed in Section2.1. This model is presented first because of its simplicity. A useful composite description is thus obtained by combining the models in Sections 2.1 and 2.2. These simple models assume the falling rock mass acts like an impermeable piston. Evidently this is not the case in practice; gas within the cavity initially will "leak" through the falling rock and be forced into the adjoining tunnels. We consider models that take the leakage into account in Sections 2.3 and 2.4.

## 2.1 Model A: An impervious piston model

The simplest model for the collapse is that of a non-porous piston with no friction. The rock falls as a solid mass under gravity. Thus from (3) the position of the rock-face at time $t$ after initiation is $z(t) = gt^2/2$. If the total height of the cavity is $L$ then $z = 0$ when $t = 0$ and $z = L$ after the collapse, i.e. $t = \sqrt{2L/g}$. In this model we assume uniform adiabatic conditions within the cavity. Thus for a constant temperature $T$,

$$P(t)V^{\gamma}(t) = k, \tag{4}$$

with $\gamma = 1.4$, is considered a good approximation. In (4), $k$ is a constant.

The volume at time $t = 0$ is $V(0) = V_0 = LA$ and at any time $t$, ignoring the leakage from the cavity into adjoining tunnels, $V(t) = A(L - z)$. As previously mentioned we assume $P(0) = P_0$. Thus, from (4),

$$P(t)V^{\gamma}(t) = P_0V_0^{\gamma}.$$

It follows that

$$P(t) = P_0\left(\frac{L}{L - \frac{1}{2}gt^2}\right)^{\gamma} \quad \text{for} \quad 0 < t < \sqrt{\frac{2L}{g}}. \tag{5}$$

Consider a tunnel $\delta L$ from the top of the cavity. The rock reaches the tunnel at time $t_e$ when $\delta L = \frac{1}{2}gt_e^2$ and thus $t_e = \sqrt{\frac{2\delta L}{g}}$. The pressure at the mouth of the tunnel is therefore given by

$$P_t(t) = P_0 \left(\frac{L}{L - \frac{1}{2}gt^2}\right)^\gamma \quad \text{for} \quad 0 < t < \sqrt{\frac{2\delta L}{g}}.$$

Once the rock face reaches the tunnel, the tunnel could be blocked off or, after a while, be exposed to open air. The study group did not consider a model for the pressure $P_t(t)$ at the mouth of the tunnel after $t_e = \sqrt{\frac{2\delta L}{g}}$. What is important is that we have established an upper bound for $P_t(t)$, namely $P_u = P_0/(1-\delta)^{1.4}$ when $t = t_e$. Note that if the tunnel is near the roof of the cavity (i.e. $\delta \ll 1$) the rise in pressure is small. Also note that, because this model assumes no loss of gas either into the tunnels or through the falling rock (an impervious piston) the pressure rise, according to (5), becomes infinite in the cavity when the rock nears the bottom of the cavity. Figure 2.1 shows the pressure $P(t)$ in the cavity as a function of time in a cavity of height 200m as determined by (5) for the simple piston model .

## 2.2 Model B: An impervious piston with isentropic compressible flow

In this section we refine the simple piston model by the consideration of isentropic compressible flow [5].

The one-dimensional gas dynamics equations are

$$\rho_t + (\rho u)_x = 0, \tag{6}$$

$$u_t + uu_x + \frac{1}{\rho}P_x = 0, \tag{7}$$

$$P = k\rho^\gamma. \tag{8}$$

where $\rho(x,t)$ is the air density, $u(x,t)$ is the air speed, $P(x,t)$ is the air pressure at depth $x$ and time $t$. The constant $\gamma = 1.4$ for air. We assume that the air is stationary at time $t = 0$. The appropriate initial conditions are given by

$$P(x,0) = P_0 = k\rho_0^\gamma \text{ where } \rho(x,0) = \rho_0, \quad u(x,0) = 0, \quad u(z(t),t) = \frac{dz}{dt}. \tag{9}$$

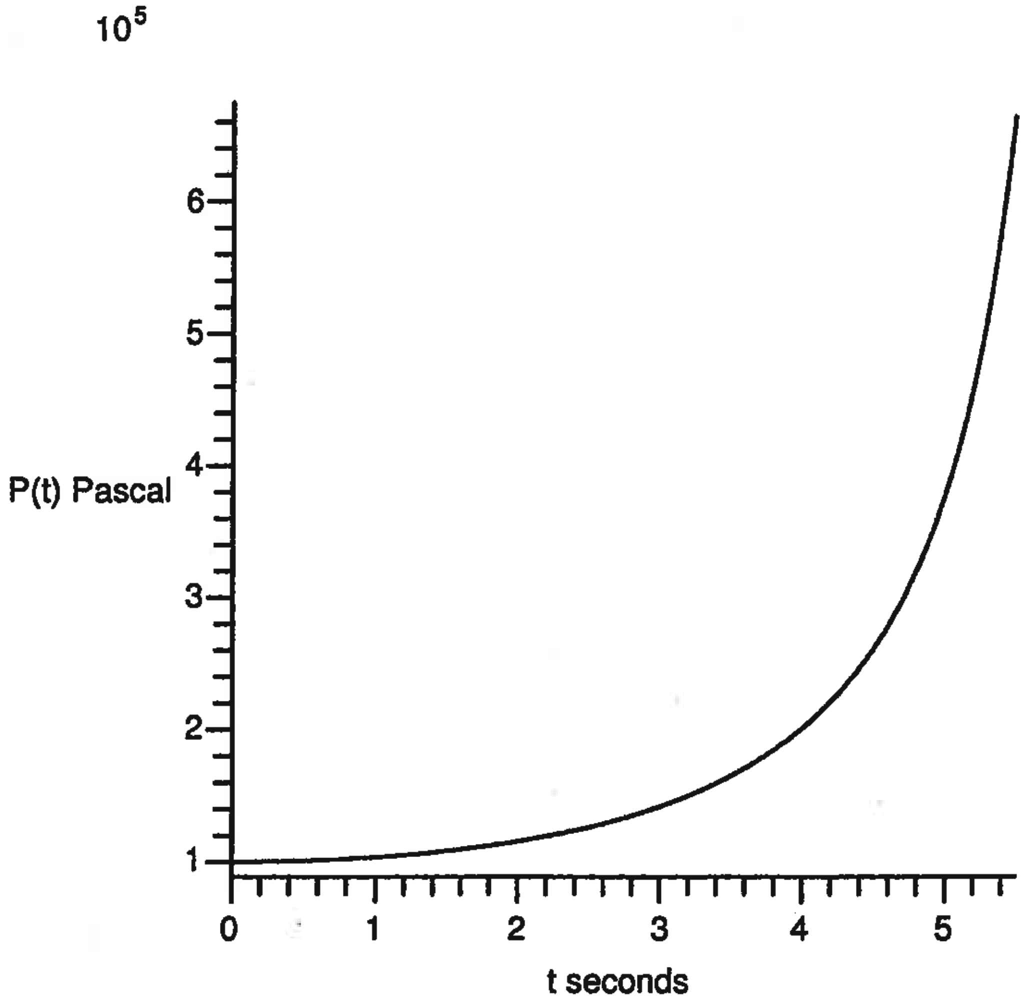


Figure 1: Model A: Pressure in the cavity as a function of time for a cavity with depth $L = 200$ m.

Let $c^2 = k\gamma\rho^{\gamma-1}$. Then

$$2cc_x = k\gamma(\gamma-1)\rho^{\gamma-2}\rho_x,$$
$$2cc_t = k\gamma(\gamma-1)\rho^{\gamma-2}\rho_t,$$
$$P_x = c^2\rho_x.$$

We multiply (6) and (7) by $k\gamma\rho^{\gamma-1}$ and rewrite in terms of $c$:

$$c_t + uc_x + \frac{(\gamma-1)}{2}cu_x = 0, \tag{10}$$

$$u_t + uu_x + \frac{(\gamma-1)}{2}cc_x = 0. \tag{11}$$

Thus multiplication of (11) by $2/(\gamma-1)$ and addition and subtraction of (10) yield

$$\left(\frac{\partial}{\partial t} + (u+c)\frac{\partial}{\partial x}\right)\left(u + \frac{2c}{\gamma-1}\right) = 0 \tag{12}$$

and

$$\left(\frac{\partial}{\partial t} + (u-c)\frac{\partial}{\partial x}\right)\left(u - \frac{2c}{\gamma-1}\right) = 0\,, \tag{13}$$

respectively.

The functions

$$R_\pm = u \pm \frac{2c}{\gamma-1}$$

are the Riemann invariants of the system and are constant on the two sets of characteristic curves, $X_\pm(t)$, where

$$\frac{dX_\pm}{dt} = u \pm c.$$

The characteristics $X_+$ are downward characteristics (moving with speed $c$) and $X_-$ are upward characteristics (moving with speed $-c$). We consider characteristics that emanate from a point $x_0 > 0$ at time $t = 0$. Thus from the initial conditions (9) we have the Riemann invariants

$$u \pm 2c/(\gamma-1) = \pm 2c_0/(\gamma-1) \quad \text{where} \quad c_0^2 = k\gamma\rho_0^{\gamma-1}\,.$$

It follows that $u = 0$ and $c = c_0$ in the region where ever both of these sets of characteristics (that started at $t = 0,\ x_0$) occur. Here $dx/dt = \pm c_0$ or $x = \pm c_0 t + x_0$. Thus $u = 0, c = c_0$ for $x > c_0 t$. This is the region of silence.

Now we consider $x < c_0 t$. In this region we still have upward characteristics that started at $t = 0$, thus

$$u - \frac{2c}{\gamma - 1} = -\frac{2c_0}{\gamma - 1}. \tag{14}$$

We also have downward characteristics. These characteristics start at the moving rock face. As in Model A, we assume that the rock falls under the influence of gravity only. Thus the downward characteristics emanate from the rock face at time $t = \tau$ at position $x(\tau) = z(\tau) = g\tau^2/2$. From the initial condition (9) the fluid velocity at the rock face is $u(z(t), t) = gt$. The characteristic curve is described by

$$dx/dt = u + c \tag{15}$$

on which $u + 2c/(\gamma - 1)$ is a constant. From this and (14) it follows that $u$ is the constant $u = g\tau$ on the downward characteristic curve that starts at $t = \tau$. It follows that $u + c$ is a constant on (15) and therefore we can integrate (15) to find the characteristic curves

$$x = \frac{1}{2} g\tau^2 + (u + c)(t - \tau).$$

We thus have the three equations

$$u - \frac{2c}{\gamma - 1} = -\frac{2c_0}{\gamma - 1}, \quad x = \frac{1}{2} g\tau^2 + (u + c)(t - \tau), \quad u = g\tau ,$$

which are solved for $u, c$ and $\tau$ explicitly:

$$\begin{aligned} u(x,t) &= \frac{(\gamma+1)}{2\gamma} gt - \frac{c_0}{\gamma} \\ &\quad + \left[ \left( \frac{c_0}{\gamma} - \frac{(\gamma+1)}{2\gamma} gt \right)^2 - \frac{2g}{\gamma} (x - c_0 t) \right]^{\frac{1}{2}}, \\ c &= c_0 + \frac{(\gamma - 1)}{2} u, \\ \tau &= \frac{u}{g}, \end{aligned}$$

where $\frac{1}{2} gt^2 < x < c_0 t, \quad 0 < t < \sqrt{\frac{2L}{g}}$.

The line $x = \frac{1}{2}gt^2$ represents the position of the rock fall and $x = c_0 t$ is the line that separates the region of "silence", where the air is not disturbed, from the air that is disturbed. A shock could form at $t = \frac{2c_0}{g(\gamma+1)}$.

A point to reflect on is that at $t < L/c_0$ the "leading characteristic", $x = c_0 t$, reaches the bottom of the cavern. Subsequently the disturbance is reflected back up into the air which destroys the region of silence. The consequence of this was not addressed by the study group and is left as an open problem.

The pressure in the cavity is found from $c^2 = k\gamma\rho^{\gamma-1}$ and $P = k\rho^\gamma$; thus

$$P(x,t) = \gamma^{\frac{\gamma}{1-\gamma}} k^{\frac{1}{1-\gamma}} c^{\frac{2\gamma}{\gamma-1}}.$$

If $\gamma = 1.4$ then $P \propto c^7$.

The pressure at the mouth of the tunnel is found if we set $x = \delta L$ where $\delta L$ is the distance from the roof of the cavern, before collapse, to the tunnel. Thus

$$P_t(t) = P(\delta L, t).$$

Figure 2 presents the pressure $P_t$ for tunnels 20m, 60m, 80m and 100m from the roof of a 200m high cavern. The disturbance reaches the floor of the cavity when $t = L/c_0$ and the reflection of the disturbance takes a further time $(L - \delta L)/c_0$ to reach a tunnel at $x = \delta L$. This wave model is therefore the appropriate model for $t < (2-\delta)L/c_0$. Figure 3 shows the accurate part of the solution presented in Figure 2. One can construct a composite solution by considering Model A for $t > (2-\delta)L/c_0$. Figure 4 gives a graphical representation of such a composite solution.

The results from Model A is inaccurate for long time scales because of the fact that air leakage out of the cavern was not considered. We can thus better the composite model if we construct a model which takes the air leakage into account. We endeavor to find such a model in the next two subsections.

## 2.3 Model C: A porous piston

In this section we consider a porous rock piston under the influence of gravity as well as a drag force and a force due to the pressure difference over the piston. We assume the pressure above the piston to be $P_0$, that is, the roof collapses to open air.

From Newton's second law of mechanics

$$M\frac{d^2z}{dt^2} = Mg - F_{drag} - (P - P_0)A$$

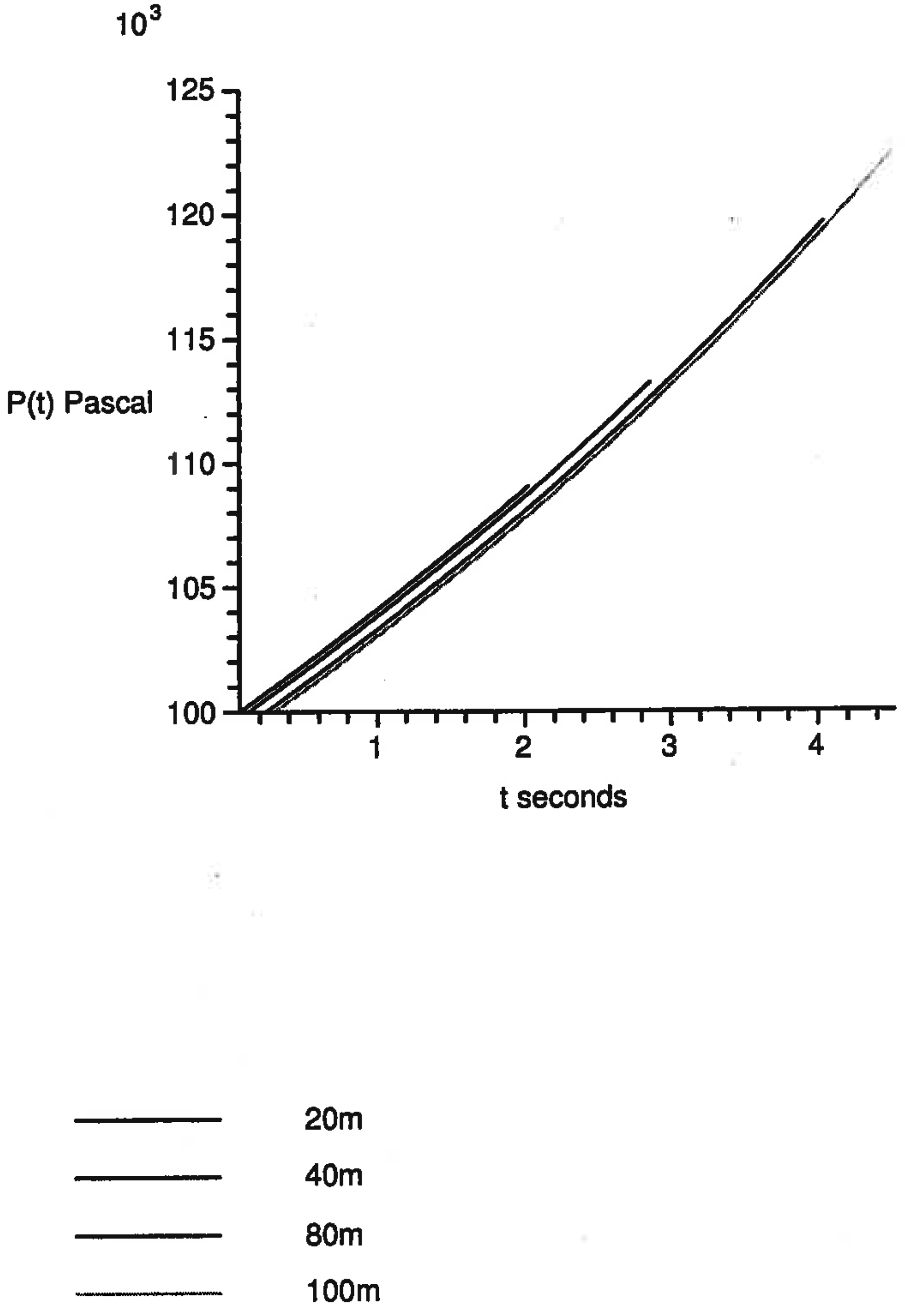


Figure 2: Model B: Pressure $P(t)$ at the mouth of tunnels (from left to right) 20, 40, 80 and 100 meter from the roof of a 200 m cavern.

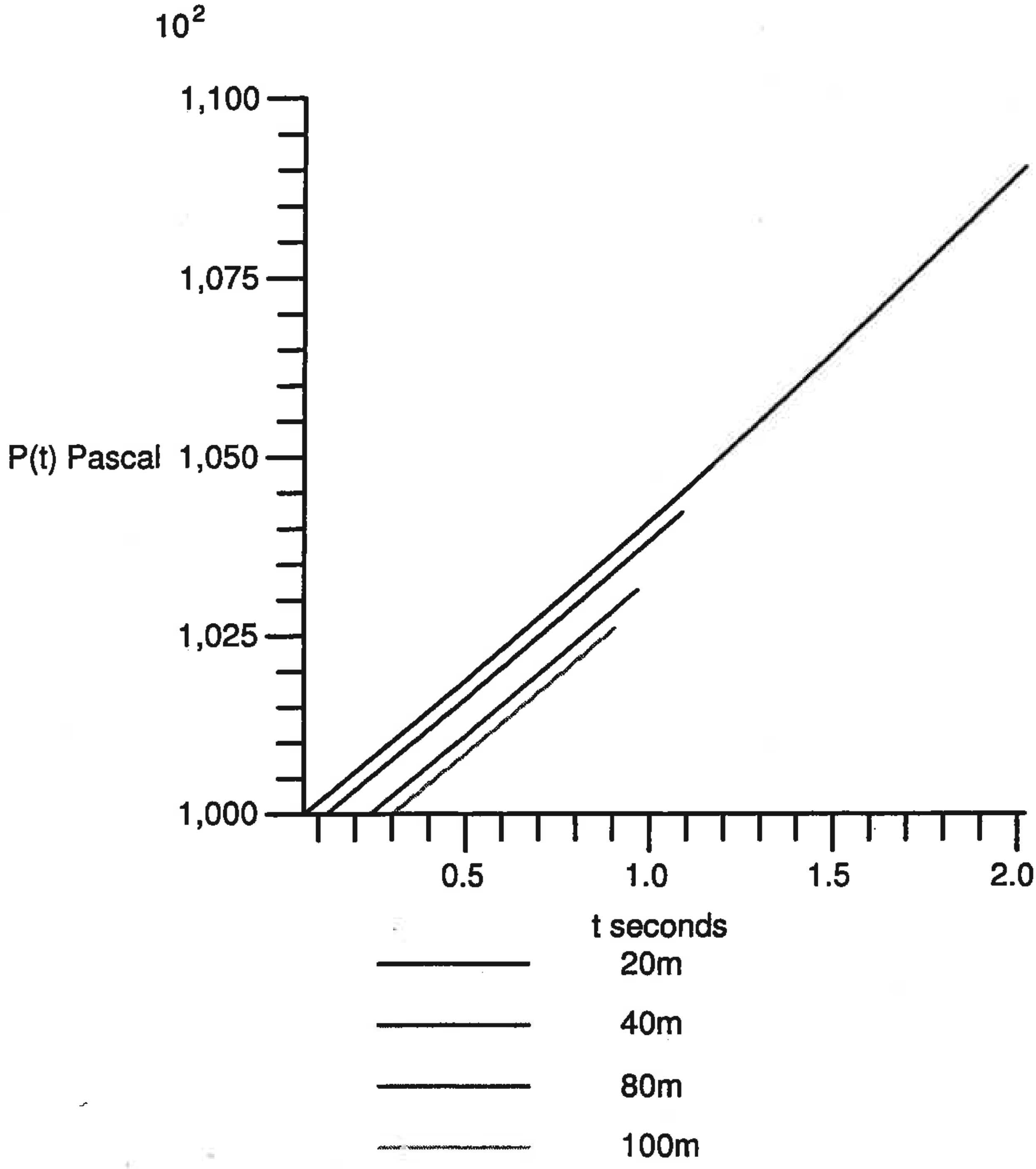


Figure 3: Model B: Accurate pressure $P(t)$ at the mouth of tunnels (from left to right) 20, 40, 80 and 100 meter from the roof of a 200 m cavern.

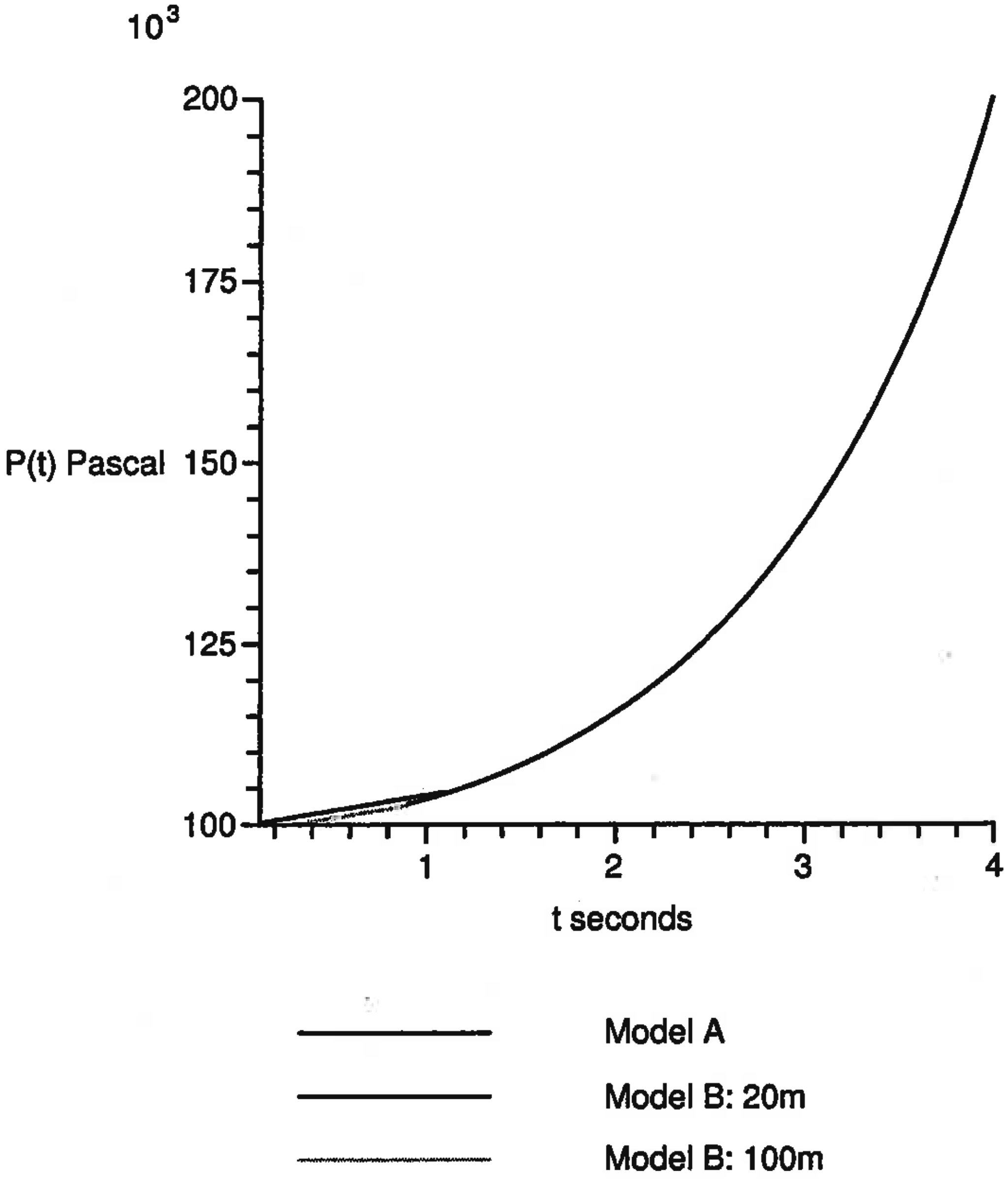


Figure 4: A composite solution from Model A for $t > (2-\delta)L/c_0$ and Model B for $t < (2-\delta)L/c_0$ for tunnels 20 and 100 meter from the roof of a 200 m cavern.

where $M$ is the mass of the rock that collapses, $Mg$ is the force due to gravity, $F_{drag}$ is a friction force and $(P - P_0)A$ is an upward force due to the pressure difference. The distance from the roof before collapse to the position of the roof at time $t$ is $z(t)$ and $A$ is again the area of the roof. It should be mentioned that one expects the drag effect as well as the cushion effect of the pressure difference to be small compared to the effect of gravity but we have no idea at this stage how small. If there is no drag or cushion effect then the rocks can be assumed to fall freely under gravity as in Model A.

We now try to model the falling rock as a porous plug across which there is a pressure drop and through which there is a flow of air which depletes the mass of air trapped between the falling rock layer and the floor of the cavern. The pressure drop and flow rate will be strongly dependent on the 'average gap thickness between rocks' and thus the ratio (volume of rock)/(volume occupied by rock) and the thickness $H$ of the zone. Generally $H$ will increase as rock peals off from the roof and rocks accelerate, and then decrease as the rock piles up on the floor. This scenario is taken into consideration in Model D. However, here we consider a fixed thickness $H$. The flow through such a large gap porous medium (with $H$ as a constant) has been described by use of the Ergun equation, a nonlinear extension of the Darcy's law, see [2, 3],

$$\frac{P - P_0}{H} = \beta v + \alpha v|v|. \tag{16}$$

In (16) $v$ is the average velocity if the obstructing rock was not there,called the fluid superficial velocity, the constants $\alpha$ and $\beta$ are defined by

$$\alpha = \frac{1.75(1-\epsilon)\rho}{\epsilon^3 D} \tag{17}$$

and

$$\beta = \frac{150\nu(1-\epsilon)^2}{\epsilon^3 D^2}\,, \tag{18}$$

where the fluid density and viscosity are $\rho$ and $\nu$, $D$ is the rock diameter and $\epsilon$ the void fraction (volume of voids/total volume). Note the similarity to the Bernoulli equation with a stagnation point.

Again, as with Model $A$, we assume for simplicity that the temperature is constant. The mass of air in the cavity is then $\rho(L-z)A$. The rate of decrease of this mass is given by the mass-flow rate through the porous rock, this rate being $\rho vA$. Thus, since the cross sectionsl area, $A$, of the cavity is assumed constant,

$$\frac{d}{dt}\Big(\rho(L-z)\Big) = -\rho v\,.$$

For adiabatic behaviour, $P/\rho^\gamma$ is a constant and therefore

$$\frac{dP}{dt} = \frac{\gamma P}{L-z}\left(\frac{dz}{dt} - v\right). \tag{19}$$

The appropriate initial value problem for $z, P$ and $v$ is therefore

$$\begin{aligned} M\frac{d^2z}{dt^2} &= Mg - F_{drag} - (P - P_0)A, \\ \frac{P - P_0}{H} &= \alpha v|v| + \beta v, \\ \frac{dP}{dt} &= \frac{\gamma P}{L-z}\left(\frac{dz}{dt} - v\right), \end{aligned}$$

with

$$z(0) = 0, \quad \frac{dz}{dt}(0) = 0, \quad P(0) = P_0.$$

The pressure at the mouth of the tunnel is $P_t(t) = P(t)$ for $0 < t < t_\delta$ where $t_\delta$ is the time when $z = \delta L$.

This initial value problem was not solved during the study group. (To do so we would need information about $D, H$ and $\epsilon$. It is left as an open problem.)

## 2.4 Model D: Rock-rain

Here we consider the case where the rock breaks up and does not fall down all at once. The air is assumed to escape through the falling rock to open air where $P = P_0$ and $v$ is again the fluid superficial velocity (average velocity in the absence of the rock) of the escaping air. The case where the air can not escape to open air is left as an open problem. Let the initial thickness of the rock that falls down be $H$. Then the initial volume of the rock that falls down is $\tilde{V}_0 = AH$. The volume that the falling rock occupies at time $t$ is denoted by $\tilde{V}(t)$. Note that this is not in general the same as the volume of the cavern $V(t)$. A graphical representation is given in Figure 5.

The first column (a) in Figure 5 shows the rock at the top of the cavern, occupying a volume $\tilde{V}(0) = \tilde{V}_0 = AH$. In column (b) some of the rock has started falling down. Thus $\tilde{V}(t) > \tilde{V}_0$.

Column (c) shows a time after the first rocks have settled on the bottom, but some of the rock is still falling, and in the last column, all the rock has settled at the bottom.

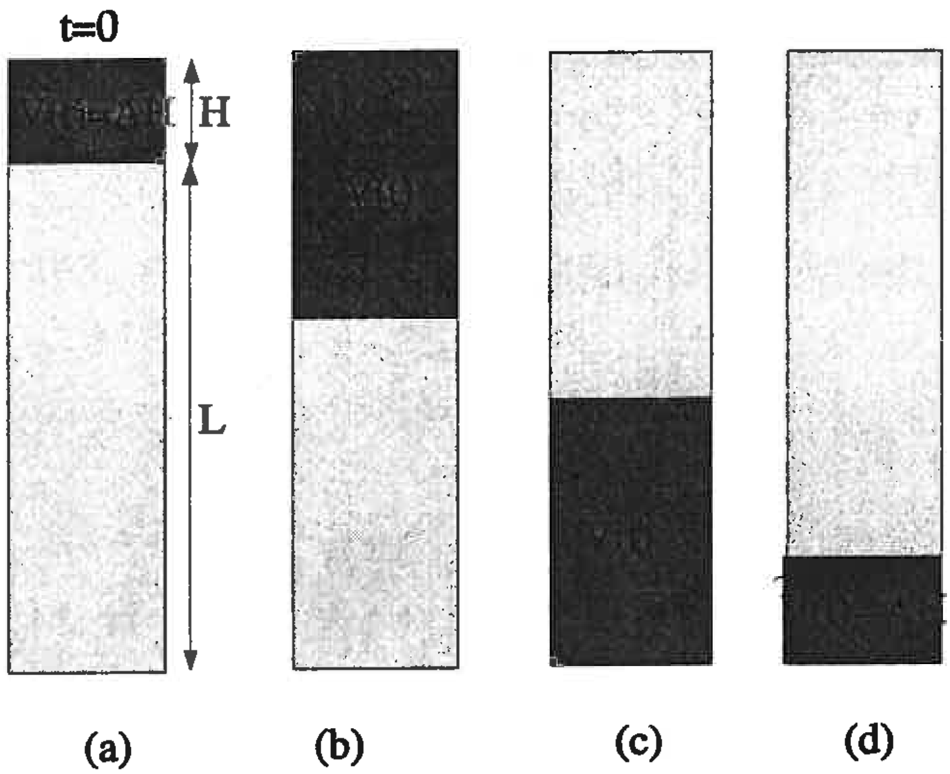


Figure 5: Model D: The rock initially occupies volume $\tilde{V}_0 = AH$. As time progresses the rock spreads out and falls down until, at the end, it forms a pile of rock with volume slightly larger than $\tilde{V}_0$ (typically 1.2 to 1.6 times $\tilde{V}_0$).

We define $\tilde{V}(t)$ as a parabola which goes through the following three points: The initial point $\tilde{V}(0) = \tilde{V}_0$. The maximum point is chosen such the first rocks hit the bottom when the last rocks start to fall, viz:

$$\tilde{V}\left(\sqrt{\frac{2L}{g}}\right) = AL,$$

which is the whole cavity. The final point is

$$\tilde{V}\left(2\sqrt{\frac{2L}{g}}\right) \simeq \tilde{V}_0.$$

Since we have no data on how the rock falls in a specific rock fall, this crude approximation will do as well as any other. Some studies can be done on varying the form of $\tilde{V}(t)$.

Thus

$$\tilde{V}(t) = g\frac{(\tilde{V}_0 - AL)}{2L}t^2 - 2\left(\tilde{V}_0 - AL\right)\sqrt{\frac{g}{2L}}t + V_0.$$

To simplify the equations, let $H = \delta L$ and thus $\tilde{V}_0 = \delta AL$ where $\delta < 1$. Then (see Figure 5)

$$\frac{\tilde{V}(t)}{AL} = g\frac{(\delta - 1)}{2L}t^2 - (\delta - 1)\sqrt{\frac{2g}{L}}t + \delta.$$

We consider the Ergun equation (16) as described in Model C. This formula may be rewritten as

$$P - P_0 = (\alpha u|u| + \beta u)\delta L.$$

where $\alpha$ and $\beta$ as given in (17) and (18). Typical values of the density and viscosity of air are $\rho = 1.23$ kg m$^{-3}$ and $\nu = 1.73 \times 10^{-5}$N s m$^{-2}$. Since $\epsilon$ is the ratio of void space to total occupied space, $\epsilon = 1 - \tilde{V}_0/\tilde{V}(t)$ which varies with time. However, $\epsilon$ is a constant in the Ergun equation and substitution of this function gives a very strange result in which the pressure is much larger than expected, so we will stay with a constant. Lastly $D$ is some characteristic size of the medium through which it is flowing, that is, a typical diameter of a chunk of stone times the sphericity of the stones. Stones may typically vary in size from relatively small pieces (less than half a meter in diameter) to huge (larger than 6m in diameter) rocks. Some research is therefore needed to find appropriate values for $D$ and $\epsilon$.

To compensate for a constant $\epsilon$ we take into account that $P - P_0$ should be inversely proportional to $\tilde{V}(t)$ (the more the rock is spread out, the more

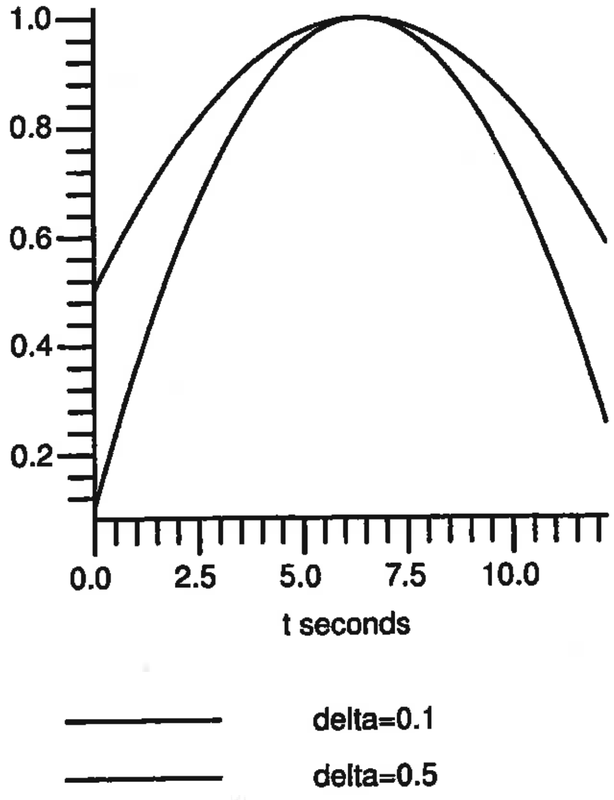


Figure 6: Model D: The occupied volume of the rock $\tilde{V}(t)/$ (the volume of the chamber $AL$) for a 20 m ($\delta = 0.1$) and a 100 m ($\delta = 0.5$) thick roof caving in.

air can escape, thus lowering the pressure difference). We further assume that the rock falls through the air under the influence of gravity and that the speed of the rock through the cavern (and the air) is roughly the speed of the air through the rock (an assumption which of course is very inaccurate if the pressure below the rock is much higher than the pressure above the rock). Thus $u \simeq gt$. Then

$$P \simeq \left( \frac{1.75(1-\epsilon)\rho}{\epsilon^3 D} g^2 t^2 + \frac{150\nu(1-\epsilon)^2}{\epsilon^3 D^2} gt \right) \frac{\delta A L}{\tilde{V}(t)} + P_0. \tag{20}$$

The pressure at the mouth of the tunnel is therefore given by

$$P_t = \left( \frac{1.75(1-\epsilon)\rho}{\epsilon^3 D} g^2 t^2 + \frac{150\nu(1-\epsilon)^2}{\epsilon^3 D^2} gt \right) \times \frac{2\delta L}{\left(g(\delta-1)t^2 - 2(\delta-1)\sqrt{2gL}t + 2\delta L\right)} + P_0$$

for the time it takes for the rock to reach a tunnel $\delta L$ from the roof, that is $0 < t = \sqrt{\frac{2\delta L}{g}}$, and $P = P_0$ when the rock has passed the opening, that is,

$$t > \sqrt{\frac{2L}{g}} + \sqrt{\frac{2\delta L}{g}}.$$

This is a result of the assumption that the air escapes to open air (that is, the roof caves in or at least has holes to the open air. The case where the air does not escape to open air is not studied here.

Some research is also needed to model the pressure in the time that the rock takes to pass the tunnel namely

$$\sqrt{\frac{2\delta L}{g}} < t < \sqrt{\frac{2L}{g}} + \sqrt{\frac{2\delta L}{g}}.$$

Figure 7 is a graph of the pressure $P$ in a cavity $L = 200$m deep as defined in (20). Here $\epsilon = 0.4, 0.5, \delta = 0.5, 0.1, D = 1, 0.6$ are specified in the legend.

# 3 Conclusions

All of the models in Section 2 give an estimate of the pressure at the tunnel entrance from the start of the collapse to the point where the collapsing rock

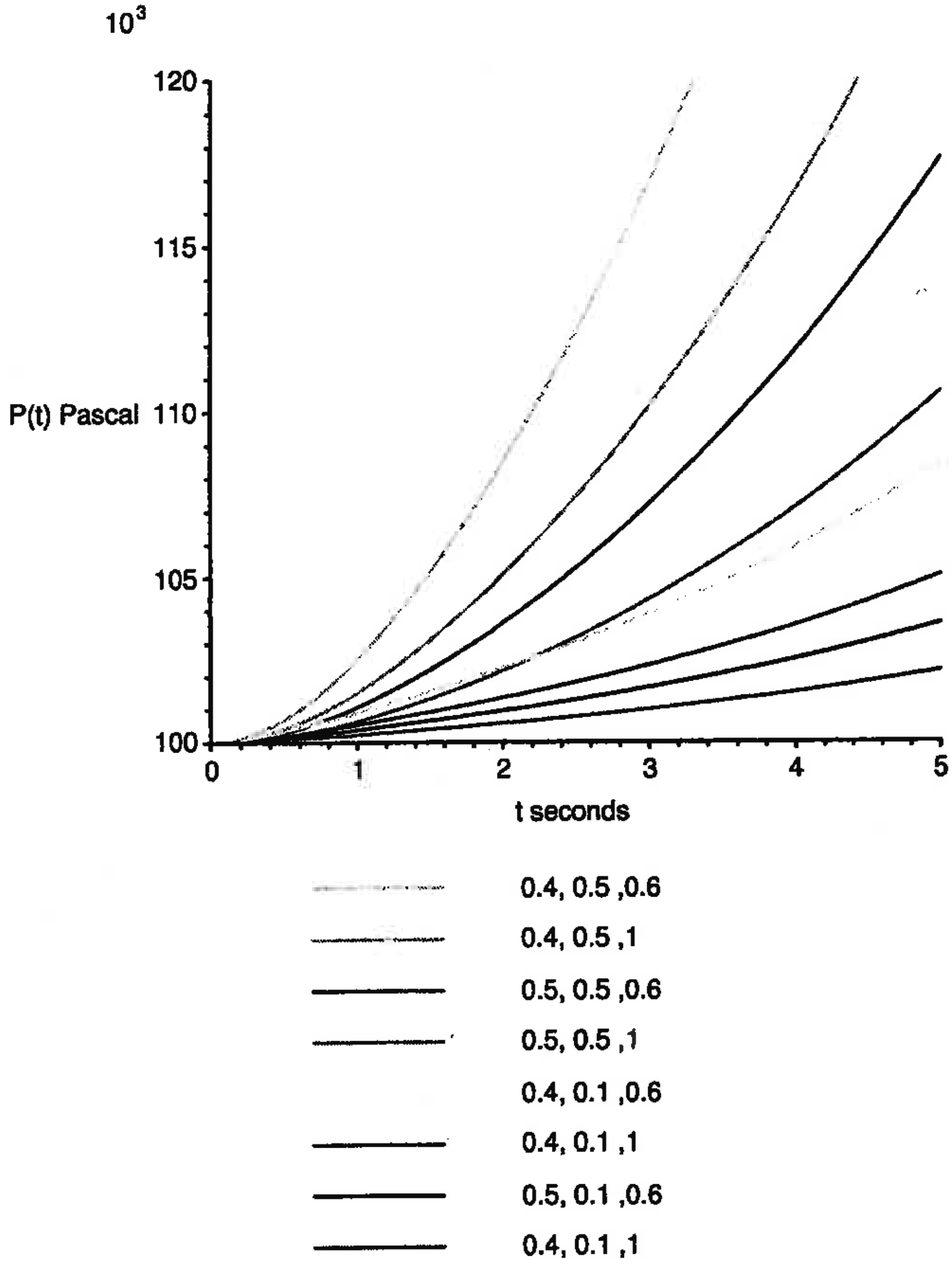


Figure 7: Model D: Pressure $P(t)$ in a cavern, $L$ = 200 m, for specified values of void ratio $\epsilon$, initial rock volume $\delta$ and rock size $D$ (in that order).

reaches the tunnel. The behaviour of the pressure after this event has not been considered. It clearly depends on whether the cavity collapses to open air, or not, among other factors. The pressure drop caused by the air that escapes through the tunnel is not taken into account. The cross section area of the tunnel is considered to be small. Figure 8 gives a comparison of the pressure $P_t$ for some of the models discussed in Section 2.

Each of the models have weaknesses and strengths. The strength of Model A is its simplicity. However the model predicts a large pressure rise for tunnels near the bottom of the cavity. The accuracy of Model A when a tunnel close to the floor of the cavity is considered, is debatable since it is highly unlikely that the roof of the cavern will move down as a solid mass with no air escaping. Model B does not take air leakage into account either, but it is a better model than Model A initially. To obtain a result for longer time scales is difficult, so a composite picture as obtained in Figure 4 is suggested. Although we endeavoured to find a more accurate model that takes into account that air escapes from the cavity during the rock collapse, results obtained from Models C and D are useless until appropriate values for the constants involved can be found. We expect however that the resultant rise in air pressure in the cavity should be lower than those predicted in Models A and B and therefore a composite picture from Models A and B, can be seen as a worst case scenario. It is also the most useful (and accurate) model at this stage.

The study group did not complete the study of the dynamics in the tunnel and made the following comments. The results obtained in Section 2 show that the rise in pressure at the entrance of the tunnel is probably not very large. Therefore we may assume the pressure at the mouth of the tunnel to be of the form $P = P_0(1 - \epsilon\tilde{p}(t)), 0 < \epsilon \ll 1$. A key modelling assumption is that the tunnel is long enough for turbulent dissipation to be important. The study group suggests the Fanno model for turbulent compressible flow to study the dynamics in a tunnel which is described in [8]. If the Fanno model gives good results, a Fanno network with branching and loops should be studied.

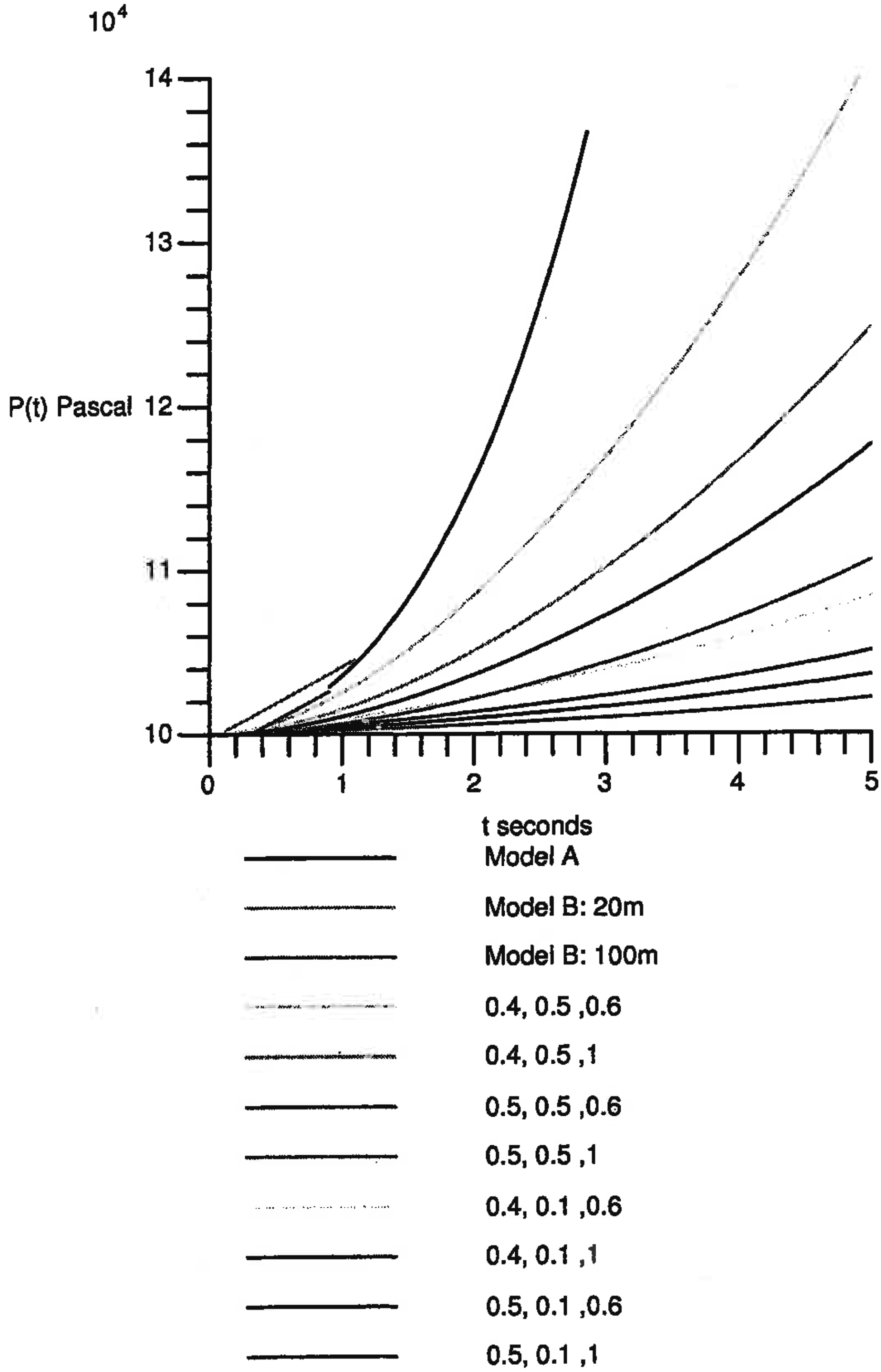


Figure 8: Comparison of Models A, B and D: Pressure $P(t)$ at the mouth of a tunnel 20 m from the roof of a 200 m deep cavern. For Model D, the void ratio $\epsilon$, initial rock volume $\delta$ and rock size D are given in that order.

## Acknowledgements

Thanks to N. Fowkes and T.G. Myers for valuable comments and recommendations

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