

DETERMINING THE SOURCE OF MOISTURE VARIATION IN PRODUCED PAPER

M.C. Jeffreys*, N.D. Fowkes[†] and T.G. Myers[‡]

Abstract

Sappi Forest Products experience an unacceptable amount of moisture variation across their paper sheets. The task was to establish where in the production these variations are created, as well as possible solutions for correcting the problem. Spectral analysis of the moisture data was done. This suggested that the nozzles arrangement was probably not the cause of the variation, since the periods of the variations are unrelated to those of the nozzles and the spectrum from after the nozzle layout was changed still had a similar sub-harmonic pattern. However, there was insufficient data to draw any conclusions. The proposed cause of the variation is the onset of ribbing at the exit of the spreader, due to small amplitude Görtler type vortices within the head-box. This ribbing was modelled and suggestions for increasing the decay rate of the ribbing are given.

1 Introduction

Paper consists of bonded discrete wood fibres laid out in a sheet and is produced from a dilute water suspension of fibres by ponding, draining and drying. Variations in thickness, moisture content and density in the sheet of paper produced by this process (in our case a long roll of width 6m) can cause defects in the final product. Moisture variations (streakiness) across the paper sheet are of particular concern. Such variations in moisture content are usual, however excessive variations are unacceptable. On-line

*School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Wits 2050, South Africa. *email: mjeffreys@cam.wits.ac.za*

[†]School of Mathematics and Statistics, University of Western Australia, Crawley WA 6009, Australia. *e-mail: fowkes@maths.uwa.edu.au*

[‡]Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701, South Africa *email: myers@maths.uct.ac.za*

moisture scanners have been used to detect the problem and hopefully will lead to appropriate corrective adjustments of the machinery. The moisture variations of interest can originate at one or several stages along the process; the latter is more likely the case so that it is a matter of identifying and correcting major problem areas. Sappi asked the Study Group to examine the data to determine the sources in the hope that machinery problems could be identified.

The moisture profile across the paper width appears to exhibit a relatively large amplitude (2%) short wave length (about 20cm or less) periodic variation superimposed on a small amplitude long wavelength (perhaps 1m) variation. A spectral analysis of this data was undertaken and revealed a more detailed/complex picture with a broad range of harmonics (and sub-harmonics) present, see later. Sappi suspected the cause for moisture variations lay in the geometric arrangement of nozzles in the head-box and changed this geometric arrangement of nozzles to possibly achieve a more uniform flow within and from the head-box, and thus a better product. There seemed to be no significant improvements as a result of the change. More details will be given later.

In order to correct the moisture variation problem one needs to locate the source of the problem and then one needs to understand its basic physical/chemical cause. Whilst this was not part of the brief the group attempted to identify possible causes. The most likely problem areas lie in the head-box and sheet formation zones. The dilute (0.35% fibre) slurry is fed from the head-box, through a flow spreader (20mm high 7.35m wide), and onto a fine screen belt moving with a speed of about 6-9 m/sec. Water drains from, and is sucked from the suspension, so that at the end of the zone (15m long) one is left with a layer of wet fibres roughly 1mm thick. Any non-uniformity introduced during this process will be reflected in the final product. Subsequently the paste is pressed (by passing through rollers), dried, and the paper sheet rolled up. The final sheet contains 4 - 10% moisture. Immediately before leaving the head-box the slurry needs to be turbulently stirred (and diluted) to prevent flocculation and possibly separation out of fibre types. However, after the slurry discharges onto the belt, uniform conditions are ideal (whether turbulent or laminar); any persistent pattern of flow including eddies will result in a nonuniform paste thickness which will in turn leave an imprint on the final product. Careful matching of the belt and flow speeds is thus required and is achieved by adjusting the pressure head in the head-box. Also side-wall effects should be minimized. The before and after conditions described above are of course incompatible and this is possibly the origin of the difficulties. This means that one

needs to design and tune this system carefully. Turbulent rolls generated within the head-box damp out slowly, sharp edges shed eddies, and end effects associated with flow spreader can initiate instabilities and cause fiber orientation difficulties. The angle of flow entry onto the belt is critical; a small wedge entry angle will tend to lead to the smoother flow desired on the belt but may allow flocculation. All such flow effects will cause surface waves which in turn will result in paste thickness variations. Additionally small differences between the belt and flow speed and capillarity effects can generate instabilities commonly associated with coating problems. To make things worse the fibers may separate out preferentially or may align themselves with the flow and thus change the effective viscosity of the fluid, and also may clog up the wire mesh and thus initiate suction related instabilities. It can be seen that there are a large number of possible sources for the observed non-uniformity and interactions between these various mechanisms are likely. The above possible sources of difficulty all have their own particular signature which may be identified. Effective modelling can be used to identify the appropriate dimensionless groups or combinations of tunable parameters (for example flow rate and belt speed, slurry concentration, suction pressure, settling plate length etc.) that may be changed to effect a remedy. To do this one needs to isolate the important issues and more detailed information about the factory operation would be required to do this. One obvious feature of the data is the presence of longitudinal moisture streaks, which suggests that an instability has been triggered; one would expect a much more random pattern if the variations were a result of purely turbulent oscillations.

Two possible mechanisms for the generation of such longitudinal streaks were suggested at the meeting and certainly the mechanisms are present in isolation and in combination. These mechanisms will be described in section 3. In the absence of specific information concerning the factory process such investigations could be misdirected and misleading. There are likely to be more obvious possible mechanisms in Sappi's specific context. Nevertheless it is hoped the discussions here will assist the company by highlighting likely problem areas.

2 Spectral analysis

The basic aim of spectral techniques is to identify 'preferred' wave lengths or frequencies associated with data. In the present context the aim was to identify wave lengths so that problem machinery components could be

identified. As indicated earlier Sappi changed this geometric arrangement of nozzles to possibly achieve a more uniform flow within and from the head-box, and thus a better product. Moisture profiles, provided by Sappi, from before and after the change were investigated using spectral techniques.

The spectral analysis revealed that significant changes in the moisture profile did in fact occur, although the changes were not beneficial. The harmonic and sub-harmonic structures from the post change data were similar to the earlier ones but at a lower frequency. There was no significant change in the amplitude of these harmonics. The actual wavelengths involved are larger than the spacing between nozzles, so that the link between the spacing and the moisture variations is not obvious. The similar harmonic structures suggest that the problem has not changed and no additional information was provided about possible changes in other properties of the process, such as belt speed or viscosity of the slurry. Possibly the wavelength of a fluid dynamic instability mode is modified by the changed forcing, see later. This suggests that head-box design is important but there may be no obvious simple fix; it is the combined effect of various features of the flow and the geometry that determines the outcome. The moisture spectrum has the breadth normally associated with turbulent phenomena. It may also be the case that the slurry properties play an important role. The data investigations performed at the meeting were incomplete; there being insufficient available information/data to support firm conclusions. The group felt that further data investigations of the type undertaken at the meeting would be both useful for the present investigations and for straight quality control purposes.

3 Fluctuations generated within the head-box

As indicated above it is desirable to have turbulent mixing within the box, but undesirable to have such variations in the flow leaving the box. It might be hoped that the turbulent fluctuations within the box would be suppressed as the flow is squeezed through the spreader. The shape of the spreader will affect the generation of eddies and in particular the change in sectional area with distance along the spreader and the vertical wedge angle (particularly at the exit end) will be important. The flow will be accelerated as it passes down the spreader but then, depending on the belt speed, decelerated as the flow leaves the spreader. Sustained eddies tend to be generated in deceleration conditions and at sharp edges or at obstructions. Random turbulent eddies will be generated if the flow speed is large enough.

The Reynolds number of the flow is $\frac{UL}{\nu} \approx 10^5$ (based on a flow speed of 6m/sec and $L = 20\text{mm}$), and this is well above the required level $Re \approx 100$ to generate eddies even without subtleties associated with flow geometry, so that one would expect the flow leaving the spreader to be fully developed turbulence. One might anticipate the axes of eddies generated to be either random or at right angles to the flow direction, however secondary flows can be generated in the spanwise direction, even under strong turbulent conditions and these flows would give rise to longitudinal streaks if the associated velocities are small compared with the longitudinal velocity, and typically this is the case for secondary flows.

Many factors can cause the generation of steady or unsteady streamwise (longitudinal) vortices or streaks in either laminar or turbulent flow. These include small steady or unsteady perturbations of any orientation, imperfections at the exit slot, cross flow instabilities generated by side wall effects etc. Also low frequency components of 3D disturbances in the main stream flow can be entrained into the boundary layer owing to the non parallel flow effects producing alternate thickening and thinning of the layer in a spanwise direction. Surface curvature effects in the direction of flow (or their equivalent) can also generate longitudinal vortices referred to as Görtler vortices. Essentially such vortices are generated by the centrifugal mechanism originally described by G. I. Taylor for the flow of a viscous fluid between rotating cylinders, but the same mechanism generates Görtler vortices in the case of flow in a boundary layer on a concave wall or even within a channel [9]. Since this early work there has been a great deal of research done on the generation, evolution and breakup of Görtler vortices, but the understanding is still incomplete, basically because the equations are three-dimensional and nonlinear, and modal interactions¹ complicate the picture. The resulting flows are spanwise dependent but essentially unidirectional, that is, the crosswise velocity component is much smaller than the streamwise component. Because the cross flow is slow the flow appears to be made up of rolls with axes along the direction of flow generating longitudinal streaks. Such streaks break up if the external flow is large enough [2].

The growth rate versus wavelength stability curve for Görtler vortices is near constant with a maximum growth rate occurring for small wavelengths in the infinite plate case². Given the flatness of this curve one would expect the preferred wavelength or wavelengths to be determined by the forcing; that is by the turbulent flow within the main head-box.

¹Tollmien-Schlichting modes destabilize the Görtler vortices.

²Finite geometry effects seem to determine the wavelength with maximum growth rate.

One might expect the effect of the spreader on the frequency spectrum and orientation of the eddies to be dramatic, with longitudinal vortices to be damped out much more rapidly than Görtler vortices, so that one would expect the flow exiting the slot to consist of a turbulent profile across the gap with Görtler type rolls superimposed.

Perhaps the only easy control mechanism for reducing the amplitude of these vortices within the head-box and spreader is the production speed and the tapering and length of the spreader. There is, however, a suggestion that crossflow can render the Görtler mechanism inoperable, [12].

4 Cross-flow instabilities: onset of ribbing

In general terms the problem of laying out the slurry on a substrate has been studied under the label of premetered coating flows which are flows for which the thickness of the coating layer is determined by the flow rate. In particular the instabilities arising in such situations are of interest here. Such flows are of major industrial interest and have received much attention. For a good review of the research done in the area (steady state only) the reader is referred to Ruschak [13] and Weinstein and Ruschak [14]. A quote from the Weinstein and Ruschak review is of interest in context. 'Genuinely predictive modelling of complex coating processes is not yet possible and coating practice remains largely empirical. Nonetheless coating science is sufficiently advanced that physical insights and mathematical models greatly benefit design and practice.'

Ribbing is a term used to describe the longitudinal streaks or 'ribs' that can appear on sheets produced during coating processes and are due to the non-uniform flow of the liquid onto the substrate or to the triggering of an instability in the coating flow. Articles of particular interest in context are those of Coyle, Macosko and Scriven [7], Castillo and Patera [6] and Carvalho and Scriven [5]. The situation they describe is associated with the roll coating, but the physics (and field equations) are similar, although the geometry is somewhat different. Liquid is passed through two rollers, one with a web on top of it, and the liquid adheres to the web. If the roll speed is too fast, or the liquid viscosity is too large, the meniscus in contact with the rollers becomes wavy in the cross flow direction, so that the films emanating from the rollers become longitudinally ribbed. In this case relatively small fluctuations in pressure in the liquid as it exits the rollers are translated into meniscus fluctuations along the roller, which in turn lead to ribbing. Such wavy variations in the thickness of the layer in the spanwise direction can

either grow or be damped out with distance from the slot by surface tension and viscous effects. The issue is whether these ribs are sufficiently flattened or grow insufficiently to be problematic before the liquid solidifies. Coyle, Macosko and Scriven [7] discuss the linear stability problem and Castillo and Patera [6] extend their results to take into account non-linear effects; in particular they determine rib height as a function of parameters of the problem. Ribbing can be reduced by 'softening' the flow at the exit of the rollers using a rubber coated roller or using a flexible plate [5]. The Sappi situation is similar, but in our case the liquid exits from a slot, and pressure variations along the slot are likely to be of the Görtler type with a broad spectrum of disturbance wavelengths being presented, and of course in our case suction/drainage freezes patterns into the sheet laid down.

Of particular interest is the preferred wavelength of the resulting wavy pattern and the damping or rib growth length scale. In the roller situation the two most important dimensionless parameters that determine the flow are the capillary number $Ca = \mu V / \sigma$, where μ is the viscosity, V the roller speed and σ the surface tension, and the geometric factor H_0/R where H_0 is the spacing between rollers and R the radius of the rollers. The capillary number measures the ratio of the viscous and surface tension forces, and ribs formed as a result of viscous fingering at the roller exit will be damped out if Ca is small enough (roughly $Ca < 0.1$, see p449 Coyle, Macosko and Scriven [7]). The resulting transverse waves damp out over a distance of the order of the film thickness. If on the other hand the capillary number $Ca = \frac{\mu V}{\sigma} > 0.1$, ribs will grow with distance from the exit, reaching an amplitude that depends on the operating parameters. The distance between these stable ribs is roughly $10H_0$. In the Sappi case $Ca = 0.06$, if one assumes the flow is laminar in the important region of the flow (debatable) and this capillary number would be critical for values of the ratio H_0/R less than about 0.001 (see p 449, Coyle, Macosko and Scriven [7]). Now the equivalent geometric factor is α , the vertical wedge angle at the end of the spreader, so that one might expect ribs to be generated for small wedge angles. Also the geometry is somewhat different than for the roller and the other relevant dimensionless groups (the Stokes number and the Reynolds number) are different to those for which calculations are presented so the most one can say is that ribbing is a possibility; the correct equations would need to be investigated. Observations may help. As the capillary number is increased ribs first appear at the ends of the roller (slot) and then quickly spread out across to roller (slot) and grow rapidly as the critical capillary number is reached.

5 On the fate of ribs

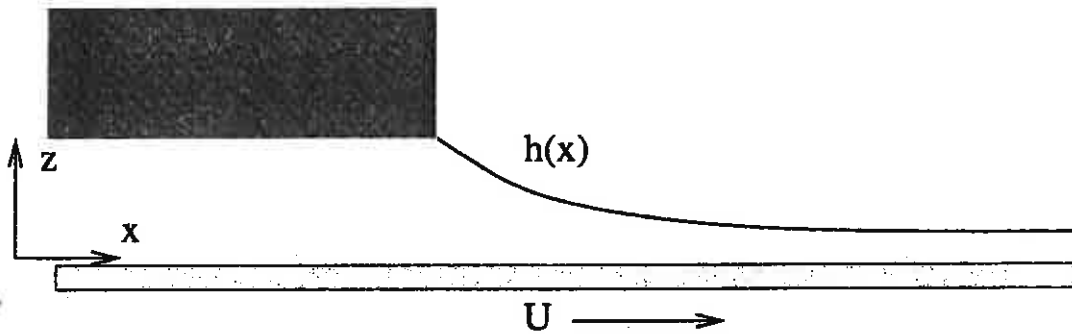


Figure 1: Problem configuration

To understand the fate of the ribs that emerge from the outlet we must first investigate the fluid flow. We start by considering the flow shown in Figure 1. The fluid moves due to the motion of the substrate, which travels with a prescribed speed U . On emerging from the channel the fluid has height h_i , it subsequently decreases and is described by the curve $h(x)$. To simplify the analysis we assume the viscosity is constant, although in reality it will increase as the fluid drains away through the substrate.

Given that the aspect ratio of the flow is small, the motion may be described by the standard lubrication approximation to the Navier-Stokes equations

$$-p_x + \mu u_{zz} = 0, \quad -p_z - \rho g = 0. \quad (1)$$

The equations basically state that in the z direction the pressure variation is due solely to gravity. In the x -direction any variation in velocity across the film is due to pressure variation in the x -direction. Far from the outlet we expect all the fluid to move with the same velocity as the substrate. Since the ambient pressure is constant the velocity variation must come from the surface tension of the fluid. This is reflected in the boundary condition on $z = h(x)$,

$$p = p_a - \sigma h_{xx}, \quad (2)$$

where σ is the surface tension and p_a is the atmospheric pressure. The stress due to surface tension is proportional to the curvature, h_{xx} . The standard zero shear stress condition also applies here, $u_z = 0$ on $z = h$. On the substrate the velocity $u = U$.

Integrating equation (1 b) gives

$$p = p_a - \sigma h_{xx} - \rho g(z - h) . \quad (3)$$

Substituting this into equation (1a) and integrating gives

$$\mu u = (-\sigma h_{xxx} + \rho g h_x) \frac{z}{2} (z - 2h) + \mu U \quad (4)$$

Equation (4) involves the unknown film height h . To determine h we note that the fluid is incompressible. This is expressed mathematically by

$$u_x + w_z = 0 . \quad (5)$$

We integrate equation (5) across the film, that is, with respect to z . We first need to consider the boundary conditions at $z = 0$ and $z = h$.

At $z = 0$ we apply a condition that reflects the fact that fluid drains into the substrate $w = -\alpha h$, where α depends on the porosity of the mesh. This model is based on the simple assumption that the infiltration is proportional to the weight of the fluid above the mesh. Obviously we could make it more realistic by making the inflow proportional to the pressure, which includes both weight and surface tension, however to keep the initial model simple we neglect the surface tension component to the pressure. Wide mesh spacing makes $\alpha \gg 1$, fine spacing gives $\alpha \ll 1$. To satisfy the lubrication assumptions we require the vertical velocity to be significantly less than the horizontal velocity. This places a restriction on the size of α . For the present problem the height goes from 10mm to 1mm in around 15m; this requires $\alpha \sim 0.77s^{-1}$. If the substrate moves with velocity $U = 5m/s$ and the vertical velocity scale $W \sim \alpha h \sim 10^{-2}$ then $W \ll U$ and the lubrication assumptions remain valid.

On the free surface we apply the standard kinematic condition [1]

$$\frac{d}{dt}(z - h) = 0 ,$$

which implies that

$$w(h) = h_t + u(h)h_x .$$

This condition merely states that fluid particles do not pass through the free surface.

The integration of (5) then gives

$$h_t + u(h)h_x + \alpha h = - \int_0^h u_x dz = - \left(\frac{\partial}{\partial x} \int_0^h u dz - u(h)h_x \right) , \quad (6)$$

where we used Leibniz's theorem to move the derivative outside the integral. The resultant equation is a mass balance describing the film height

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = -\alpha h, \quad (7)$$

where the fluid flux Q is

$$Q = \frac{h^3}{3\mu} (\sigma h_{xxx} - \rho g h_x) + Uh.$$

To deal with the problem of ribbing we must first identify the base state to this flow. Since the flow is steady the base state must be described by

$$\frac{\partial}{\partial x} \left(\frac{h^3}{3\mu} (\sigma h_{xxx} - \rho g h_x) + Uh \right) = -\alpha h.$$

This is a standard type of equation, known as a draining equation [11]. It may be solved numerically using a simple marching scheme. In the far field $h \rightarrow \text{constant}$ (which in fact is zero), so for $x \gg 1$

$$\frac{\partial}{\partial x} (Uh) = -\alpha h, \quad (8)$$

with solution $h = Ae^{-\alpha x/U}$ where A is a constant. This solution can provide all of the necessary boundary conditions in the far field. We choose an arbitrary $A \ll 1$ and $x \gg 1$ and calculate h, h_x, h_{xx} and h_{xxx} . The solution may then be marched backwards. The only boundary condition at the inlet is that $h = h_i$. Hence we continue marching backwards until the height reaches h_i . Since the governing equation is autonomous (x does not appear explicitly), we can then shift this solution appropriately until $h = h_i$ at $x = 0$. We denote this solution $h_0(x)$.

Now, to deal with ribbing we need to consider the problem in three-dimensions. We set the y -direction as pointing into the page in Figure 1. The three-dimensional equivalent of (7) is [11]

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{Q} = -\alpha h, \quad (9)$$

where

$$\mathbf{Q} = (Q_1, Q_2) = \left(\frac{h^3}{3\mu} \left(\sigma \frac{\partial}{\partial x} \nabla^2 h - \rho g \frac{\partial h}{\partial x} \right) + Uh, \frac{h^3}{3\mu} \left(\sigma \frac{\partial}{\partial y} \nabla^2 h - \rho g \frac{\partial h}{\partial y} \right) \right),$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Ribbing corresponds to a wave in the y direction. The standard way to model this wave, whilst examining its decay, is to carry out a linear stability analysis [8]. This method consists of defining the base flow, described by $h_0(x)$ and adding a small wave which decays or grows exponentially. It may be shown formally that the exponential behaviour is a result of choosing a small perturbation and linearising. However, to save time we will take this behaviour as given. For the present problem this means we look for a film shape of the form

$$h = h_0(x) + \epsilon h_1(x) \exp(iky - st), \quad (10)$$

where k is the wave number (the wavelength $\omega = 2\pi/k$) and h_0 is the base state determined earlier. The purpose of the exercise is to determine s . If the wave (ribs) decay then $s > 0$, if the waves grow then $s < 0$.

If we substitute for h from (10) into (9) we find that the leading order problem for h_0 is simply equation (8). At $\mathcal{O}(\epsilon)$ we obtain an equation for h_1 . However, due to the high order derivatives this equation is difficult to solve and therefore the s dependence is not clear. To simplify the problem and so identify the dominant behaviour of the system we can reduce the model further as follows.

Our real interest lies in the decay of the ribbing. The curvature in the x -direction dies down within the order of the capillary length-scale

$$L = \sqrt[3]{\frac{\sigma H^3}{3\mu U}}$$

Due to an absence of practical data we will use typical values for water, namely $\sigma = 7 \times 10^{-2} \text{N/m}^2$, $\mu = 1 \times 10^{-3} \text{Ns/m}^2$. A typical velocity is $U = 5 \text{m/s}$ and $H = 0.02 \text{m}$. Thus the capillary length-scale is $L \sim 3.3 \text{cm}$. Beyond this region the curvature is small in the x -direction but, due to the ribbing, may not be small in the y -direction. We may therefore determine the appropriate behaviour by considering a film shape given by

$$h = h_0(x) + \epsilon e^{iky-st}$$

where $h_0(x) = Ae^{-(\alpha/U)x}$ and note that, provided x is sufficiently large h_0 varies only slowly with x . An even simpler model, with h_0 constant,

has been studied by Bixler and Scriven [3] in an analysis of ribbing. In the present situation we retain some x dependence, since the leading order problem involves the draining rate. At first order in ϵ (neglecting derivatives in h_0) we arrive at

$$s = \frac{h_0^3}{3\mu} (\sigma k^4 + \rho g k^2) + \alpha > 0. \quad (11)$$

The consequence of this result is that for any instability wavelength, the instability will die down given sufficient time (and provided $\epsilon \ll 1$, so that the linear theory may be applied). Physically we may interpret the result as stating that all the relevant forces act to stabilise the film. It is well known that surface tension acts to reduce wave size on thin films. Since the draining term is proportional to the height, peaks will drain quicker than troughs, similarly the draining acts to reduce peaks quicker than troughs. Given that all the forces act to stabilise the film, the real issue is how quickly do the ribs decay and what, if anything, may be done to accelerate the decay.

The decay rate is given by s . It is controlled by the fluid properties, the instability wavelength and the draining rate. Taking parameter values for water, $\alpha = 0.767$ and an instability wavelength $\omega = 0.3\text{m}$, hence $k = 2\pi/\omega \approx 21$, we find

$$s \sim 1.3 \times 10^9 h_0^3 + 0.767.$$

Say, for example, we wish the wave to decay to a tenth of its original value then we require $e^{-st} = 0.1$. If we take an average value for the height $h_0 = 0.005\text{m}$, then $t = 0.013\text{s}$. In this time the fluid travels only 6cm.

This analysis appears to indicate that any instability will decay well before the end of the line. However, we have taken values for water. In reality the fluid may have properties similar to water near the head box, but by the end of the belt they will have increased significantly. If we take a viscosity value for castor oil, $\mu \approx 1$ then $t \sim 7$ and the fluid travels 35m.

In conclusion, the only factors affecting the decay rate or the distance over which decay occurs that can be changed in practice are the surface tension and the belt speed. Our calculations indicate s varies linearly with σ and that an increase in σ will result in the more rapid decay of the ribbing. Given a lack of data the calculations above do not allow us to draw firm conclusions. If the viscosity of the fluid is close to that of water then surface tension will remove the ribs long before the end of the line, if it is closer to oil then the ribs may remain. However, in fact, the viscosity changes along the belt and therefore probably varies between these values or it may go to

higher values. In which case the above analysis merely indicates the possibility that changing the surface tension could help alleviate the problem. These issues could certainly be resolved by some relatively simple experiments, involving the use of different surfactants, or amounts of surfactant and by varying the belt speed.

6 Other comments

After the flow leaves the slot it passes over a plate and then onto the moving belt where suction removes water. Evidently the plate should be of sufficient length to allow the flow 'to settle'. The length scale for the removal of water is about 1 – 2m, so that it is unlikely that the dynamics in the belt zone are important, but it is possible that clogging of the wire by fibers could generate nonuniform settling.

7 Conclusions

The spectral analysis was inconclusive. No direct relation was found between the nozzle placement and the moisture variation. The change in the head-box dynamics, resulting from the new nozzle arrangement, may have had some effect on the lowering of the harmonic frequencies of the variation but the underlying problem has remained the same. The lower frequency could also be the result of other parameter changes in the process but this information was not available for the analysis.

It would appear that small amplitude Görtler type vortices could be generated within the head-box and spreader. Any resultant pressure variations at the exit of the spreader would be translated into spanwise thickness variations in the liquid stream. Surface tension and viscous effects would either damp out these ribs or enable their further growth depending on the capillary number and wavelength, with the preferred wavelength being about ten times the slurry thickness. Given the above the following suggestions of adjustments are made:

- A reduction in the production speed is likely to reduce streaks
- Matching the belt speed with that of the stream speed at exit is likely to prevent eddy generation
- Elastic flaps may reduce the streaks; essentially such elastic rubber flaps even out the slot exit pressure variations.

- Strings etc across the top of the slot may effect the meniscus
- Increasing the length of the plate at exit is likely to give the flow time to settle, or in fact give ribs time to develop
- Increasing the length of the spreader is likely to dampen the vortices
- Increase the surface tension of the fluid.

Acknowledgements

Thanks are due to the Industrial Representatives from Sappi, Karel Boon, Corne DeJager, Ryan Silberman and Kevin Hingst, for introducing the problem and for their help during and after MISGSA 2005. We also acknowledge the contributions of the group members, Chris Breward, Daniel Makinde, David Mason, Joel Moitsheki, Jon Chapman, Mubanga Lombe, Pekka Pihlajasaari, Peter Mhone, and Serge Neossi.

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