POSITRON EMISSION PARTICLE TRACKING

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Abstract

Positron Emission Particle Tracking (PEPT) refers to a procedure for tracing the movement of particles that uses the gamma rays emitted from radioactive materials imbedded in the particles. This procedure has been used to trace the movement of rocks in the tumbling mills used by the mineral industry. The emitted rays strike detectors surrounding the tumble mill and triangulation can be used to determine the particle's position. However a large proportion (often a majority) of the received strikes are false and thus incorrectly locate the particle. The group was asked to examine and evaluate effective procedures for filtering out such false information. A number of procedures (including those currently in use) were examined and evaluated using experimental and artificially generated data. The results suggest that the procedure currently in use is effective, but marginal improvements may be realized if the initial culling procedure is improved and if statistical methods are used to identify false events.

1 Introduction

Gamma rays readily pass unhindered through 11 cm of steel, so that it is possible to detect and trace the movement of a radioactive particles in opaque environments. PERT uses this procedure to trace the movement of particles in fluid and granular material flows. The radioactive materials are imbedded in particles that are typical

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of the flow system being investigated; in granular flow systems rock fragments are used. The radioactive source emits back to back gamma rays which are detected when they strike detectors mounted around the tumble mill. Under true pairing conditions the back to back rays form a straight line passing through the particle position and striking the detectors and (using successive emissions) a triangulation procedure can be used to locate the moving source. However incorrect pairing results if either or both the emitted back to back rays are scattered, or if random pairing occurs, see Figure 1. An example of a short time slice of detected events is shown in Figure 2. The proportion of spurious events varies enormously depending on the activity of the particle, its size and state of motion, as well as the environment in which it is imbedded. Also scattering is more likely if rays travel large distances before hitting a detector, and of course under such circumstances the solid angle subtended at the detectors by the particle will be small so that (even without s-cattering) geometric errors will arise. The geometric arrangement of the detectors can in fact greatly effect accuracy. As a result of all such sources of error typically 75% to 90% of events need to be rejected in order to guarantee a certain maximum deviation from the true particle trajectory and at present this proportion needs to be calibrated on a case to case basis. The effect of filtering out false lines is shown in Figure 3. A better filtering and computational positioning technique may avoid calibration, reduce processing time and improve accuracy.

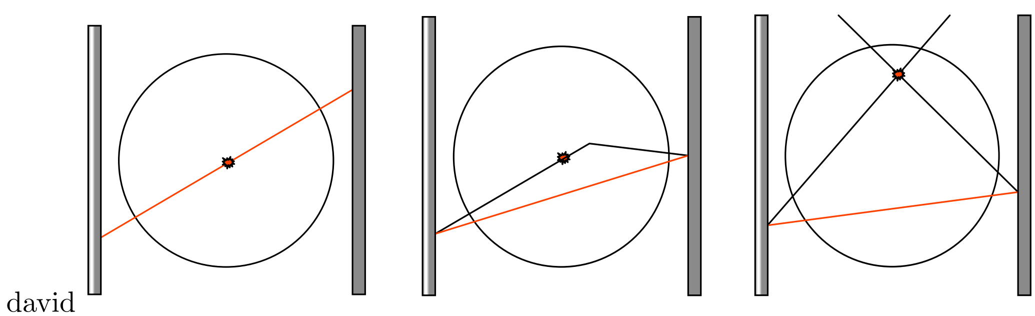


Figure 1: True pairing, Scattered pairing, and Random pairing

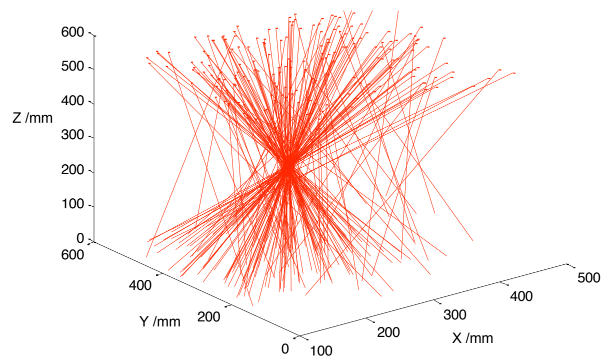


Figure 2: A time slice of detected events.

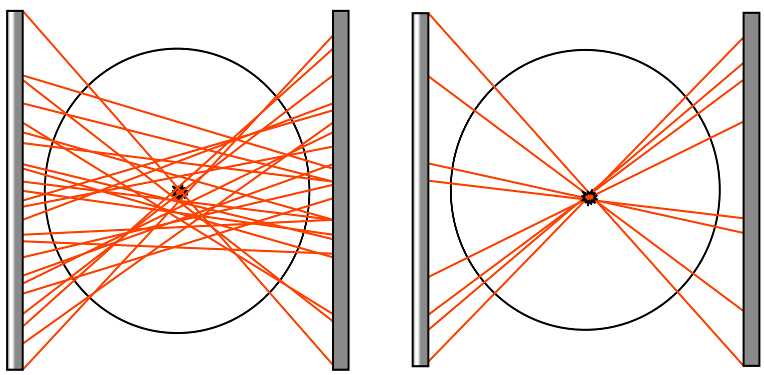


Figure 3: *Left:* True and false pairings. *Right:* Resulting figure after the removal of false pairings.

1.1 Tumbling mills

In the minerals industry (gold, platinum, copper, etc) mined boulders are initially crushed and then tumbling mills are used to reduce the rocks to a powder suitable for mineral extraction. The tumbling mills are of typical diameter 4.5 m and of length 6 m and rotate at speeds of 20-60 revs/min (approximately 2-6 radians/sec). The rotation speed is normally chosen to be 75% of the 'critical' rotational speed $\sqrt{2g/D}$, which is the speed corresponding to the rocks just lifting off the drum surface above the axis of rotation. Typically rocks occupy 30% of the drum volume and lifters are used to facilitate rock falling and tumbling. Heavy steel balls help crush the material. This is a energy-intensive and inefficient process which could be greatly improved by better drum and lifter design and tumbling control, however the design

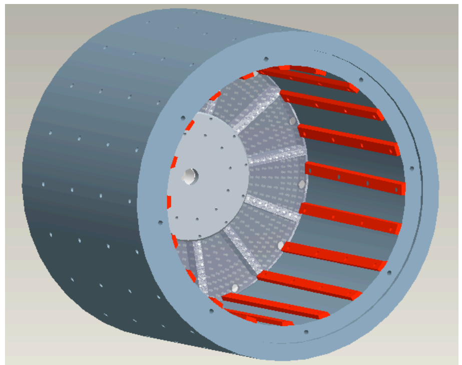


Figure 4: A drawing of the University of Cape Town experimental tumble mill.

requirements are strongly dependent on the ore being crushed and are mill specific. Present state design and operation is largely ad hoc, however computational models (particle models) have been developed that can track the movement of a relatively large number of relatively small pebbles so that quantitive scientific design may be realizable. These models run into computational and modelling difficulties as particle size decreases (and numbers correspondingly increase) and as the flow becomes more fluid-like. Such computational models rely on an adequate description of the particle to particle interactions and in fact results are very sensitive to the interaction model adopted. Evidently such models need to be tested and will need to be calibrated for on site use. PEPT enables one to see what is going on within the drum and so may provide immediate information concerning the behaviour of rocks in a tumble mill and hopefully will also provide a necessary check on the developing computational models.

The University of Cape Town's PEPT group has an experimental tumbling drum of diameter $D = 0.3\,\mathrm{m}$ and length $0.27\,\mathrm{m}$ with 20 lifters which tumbles both rocks and marbles, see Figure 4. The tumble drum is typically 30% filled with rocks and rotated at a speed of about 1 revolution per second. Detectors are arranged around the tumble drum. Data collected from this apparatus was used at the study group to evaluate various filtering procedures.

1.2 Positron emission particle tracking, detection

Roughly 100,000 (back to back) gamma rays per second are emitted from the source and the emission time and ray direction are random. The rays, which may or may not be scattered, strike the detectors surrounding the drum. The detectors can locate the strike within 1 mm and the response time of the detector system is very short, so that distinct events are normally seen by the system. A strike results in a detector voltage which is registered providing it exceeds a threshold level. Two strikes are paired if the timing is such that they appear to be associated with the same event, and unpaired strikes are discarded. However many of these retained pairings correspond to scattered rays, or separate events that are too close to be distinguished (random pairing). The filtering system (a software program) needs to be able to filter out the majority of such false pairings in order to correctly position the particle. Typically 75-90% of the pairings need to be discarded in order to determine the particle's location within acceptable accuracy. This can be problematic in that (under high scattering conditions) the remaining data may not be sufficient to detail the movement of the particle to sufficient accuracy. A more efficient scheme for identifying true pairings would overcome this difficulty. It should be noted that, whatever mathematical scheme is used to filter out false lines, the numbers of lines to be examined is large so that efficient culling is essential. It is also important to note that false pairings result in false lines that are normally miss the particle by 'a long shot'; a result supported by available data, see later. What is required is a computationally efficient (possibly crude) true pairing test, followed up by a major culling, then more detailed calculations.

In section 2 we will describe an efficient initial cull scheme that should be useful and we then go on to examine various schemes for processing the data including the scheme presently in use (with variants), and a new scheme. In the remaining section we summarize our conclusions and make recommendations.

2 Position Tracking: Filtering Methods

The number of events in each slice of the data set is chosen to limit the distance the tracer travels before repositioning is undertaken. This depends on both the expected maximum speed of the tracer and on its radioactivity. Throughout the work to follow $\mathbf{r_0}$ will denote the estimated location of the particle at the beginning of this time interval δt chosen for position estimation and N will denote the number of (un-discarded) pairings over this time interval. In the experimental drum case typical values for N are $2000 - 10,000$ and $\delta t \approx 0.1$sec. As indicated earlier in the rock tumbling context maximum particle velocities of the order of 10 m/sec are to be expected in a typical mill and 2 m/sec for the experimental mill, so that over the time interval $\delta t = 0.1$sec the travel distance will be less than 1 cm in the experimental setup and about 5 cm in the mill.

2.1 An initial cull

In the tumble mill context impulse forces will be acting, so that no assumptions can be made about the direction of movement of a rock particle over the time interval δt (for example direction continuity). However, the particle cannot move a distance greater than $R = V_m \delta t$ over the time interval, so that (knowing $\mathbf{r_0}$) one can determine a region within which the particle must be found. This observation can be used to cull false lines. Thus if γ is the direction corresponding to a (true or false) line then, in order to correspond to a true pairing, the associated strikes must fall within the two shadow zones on the detector surface formed by light rays parallel to γ striking a sphere of radius R centered on the previous estimated location $\mathbf{r}_0$. Now the locations and sizes of the shadow zones will depend on the particle position, the ray direction, and the orientation of the detector surface within the shadow zone, all of which can be calculated precisely or estimated experimentally, but with considerable computational effort. More simply one can reject a pairing if either of the strikes are more distant than R from the intersection points on the detector corresponding to a ray through $\mathbf{r}_0$ in the direction γ. An initial cull based on these simple observations should significantly reduce the followup computations. It should be noted that there are a range of techniques (Kalman filtering being the best known) that can be employed to better estimate a particle position and movement based on particle velocity continuity. Such techniques might be useful in fluid flow PEPT applications but in the rock tumbling context, impulse forces dominate and need to be detected.

2.2 Iterative centroid methods

The original triangulation technique, developed at the University of Birmingham, is an iterative one, see Parker et al 1993. A preliminary centroid is calculated by finding the point which minimises the sum of the perpendicular distances from the N lines to that point. The furthest-lying lines of response from the preliminary centroid are then discarded, and the process repeated until only some fraction, f, of the original N lines remains. The choice of this fraction, f, depends on the attenuation environment of the experiment. In theory, by performing a radiation transport simulation of the experiment, it is possible to determine the expected fraction of events which are not scattered and to use this fraction in the triangulation routine. In practice, however, it is unrealistic to perform such a simulation for each experiment and so this fraction f, is usually chosen so that the maximum distance of un-discarded lines to the calculated centroid is below some threshold. A major drawback of this method is that no qualitative assessment of the true and false lines is performed beyond the distance to the calculated centroid, and so it is possible that extremely spurious events may be skewing the calculated centroid. One might hope that the initial culling scheme described above will in part address this difficulty. A second major drawback of the method (and its variants) is that the process is iterative; the method requires the recalculation of the centroid at each step and so is computationally expensive, especially if N is large and many iterations are required. In this context one might well ask if the set reduction is worthwhile in the sense that the improvement in positioning is worth the computational effort, see later. We examined possible filtering schemes based on the distance distribution, the hope being that convergence rate might be greatly improved by a clever choice. Statistical theory might be brought to bear on these problems, however we only addressed these issues experimentally.

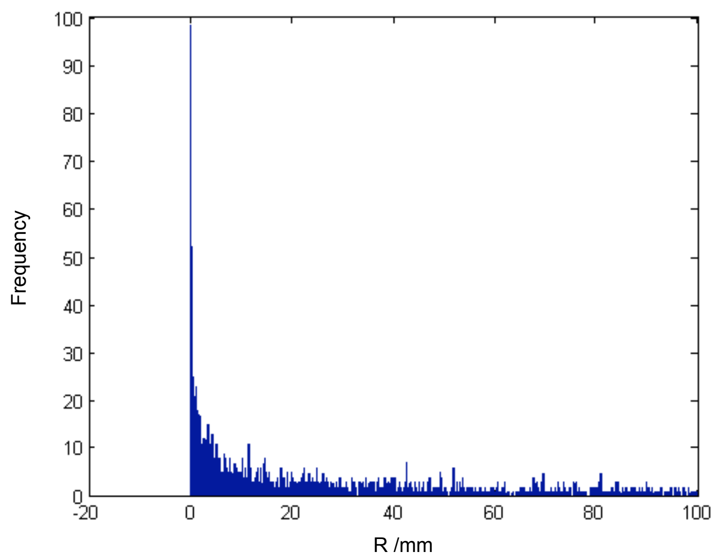


Figure 5: The distance to the calculated centroid distribution function.

A typical distribution obtained using the experimental data is displayed in Figure 5. Note that, whilst most of the data falls within 20 mm of the centroid, the distribution has a very long tail. The explanation is straightforward and is evident in Figure 1; scattered and random rays generally lead to particle position estimates that miss the mark by a long shot (the 3D the situation is much worse than in the 2D situation depicted)! The implication of this is that these outliers, corresponding to false events, greatly bias the position estimate. For example, in Figure 5 one would expect the estimate to be greatly improved if all lines with $d > 20$mm were removed. Furthermore one would expect the numerical convergence to be greatly improved by successively applying a scheme in which all lines with distance outside the presently determined centroid by (say) a standard deviation are removed. Using experimental and artificially generated data, numerical experiments were performed using a variety of rejection criteria based on the locally generated distance distribution. Such schemes have the advantage that they could be dynamically (and automatically) applied to the generated data, removing the need to tune the filtering scheme. Since the number of scattering events will greatly depend on the current location of the particle in the drum (usually more in the rock pile close in the bottom of the drum) a dynamic scheme such as this is really required.

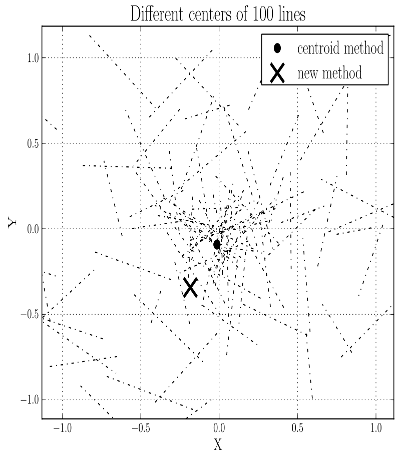
2.3 Other methods

One would expect the line density to be greatest near the particle position. This insight can be exploited in the so-called line density methods, see Bickell (2009). In these methods the volume of interest is discretised into either 2D or 3D voxels, and the number of times that the generated lines pass through each voxel is used to determine the position of highest line density. This simple binning routine has been refined in various ways, see Giovannoni (2009). Whilst reliable the technique is more computationally expensive than the centroid method but may be useful for setting or resetting a datum. For example it is possible that a more 'sophisticated' procedure could lead to an error drift, so that a check or resetting is appropriate.

The minimum distance method consists of determining the midpoint of the perpendicular line between each pair of successive lines of response for a slice of N events as discussed above, and the median of these midpoints can be used as an estimate of the tracer position and movement; more details below. One might hope to avoid the need for iteration using this method. Work on this procedure was reported by Odo in a workshop in April 2010, after the MISG. In practice, this method was used to reduce the number of iterations needed rather than to eliminate the need to iterate entirely. Some investigations related to this method were carried out at the MISG and are reported below.

2.4 The perpendicular lines method

Given two non-parallel lines in $\Re^3$, we can determine the unique line segment which connects, and is perpendicular to, both of them. In context the lines correspond to back to back rays emitted over a short time interval by the tracer, and we would expect the midpoint of the line segment to be a good approximation to the particle's location, providing of course the strikes are not false. Now if we have a temporally ordered set of N lines in $\Re^3$, we can find the $N-1$ points as described above and these $N-1$ points will provide an estimate for the location of the particle, providing



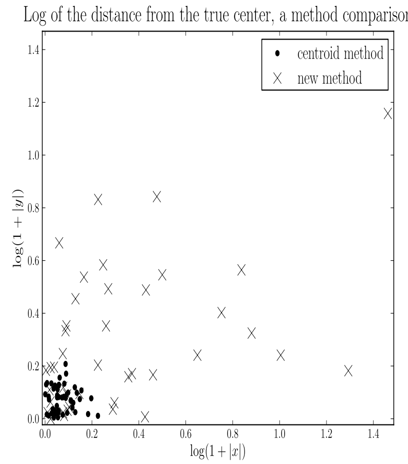


Figure 6: *Left:* Simulations using 50 lines generated from a normal distribution. Note that the perpendicular lines method is inferior to the centroid method. *Right:* The results of 50 such simulations.

it moves little in the emission time. By using the $N-1$ lines one might hope to eliminate the bias caused by false strikes. By stringing together such estimates one can trace the particle's movement. In order to assess the usefulness of this technique as a predictor of particle position we carried out the following simple analogous numerical experiment.

A particle located at $(0,0,0)$ emits back to back gamma rays in a random direction[1]. For the purpose of our present investigation, we assume the detection system is flawed in the sense that the strikes appear to emanate from points near to, but not necessarily at, the origin. Our aim is to correctly locate the particle. For definiteness we assume that the strikes appear to emanate from points whose distance is selected from a normal random distribution centered on the origin and with a length scale determined by the detection system. We will scale the observations so that the standard deviation is unity; the distribution of strikes is thus given by Normal (0,1). In other words the N locations of emission form a cloud centered at the origin with the density of the cloud decreasing as one moves away from the origin with the decrease the same in any direction. The successive midpoints of the lines are used to estimate the particle's position.

Figure 6 Left displays the results of a simulation using 50 points generated as above. The projections of the associated lines on the (x,y) plane are displayed and

[1]That is such that the probability of a strike on a patch of the surface of a unit sphere is independent of its location.

the associated mean is displayed as a cross. Also displayed is the centroid. The results of 50 such simulations are displayed in Figure 6 Right. Evidently the new method does not produce as good an estimate as the standard approach. In an intuitive sense this is understandable. The effect of an incorrect strike on the position of the (false) line, and thus the estimated position, is dramatic because of the geometry as has been displayed in Figure 1. Furthermore the incorrect strike will result in two incorrectly positioned lines and associated locations. On the other hand the centroid technique simply estimates 'the centre of mass' and is *much* less sensitive to an incorrect strike. Now in the PEPT situation the distribution of strikes has a long tail, as indicated in Figure 2 so that the perpendicular distance method is likely to produce worse results than suggested in the above simulations. Further simulations were carried out using actual experimental data (and thus a more realistic distribution), and also artificially generated data. These results confirmed the above observations; the centroid technique is much superior.

2.5 Set reduction

We now return to the 'set reduction' question: 'Is it worthwhile repeatedly recalculating the centroid and removing the furthest line, or should we stick to the centroid calculated using all the lines?' Several experimental tests were performed to check this out and the answer, as illustrated in Figure 7 Left and Right, is that the set reduction does result in an improvement, but it is not significant at least for this particular data set.

3 Conclusions

This was a very small but hard working group. The group was asked to suggest possible ways for improving the software used for particle tracking using PEPT technology. The PEPT procedure is able to accurately record strikes, but many of the strikes are false, so the challenge was to design software that will efficiently recognize and filter out the associated false lines. The presently used software calculates the centroid and uses an iterative scheme to eliminate false lines. A useful observation was that false events give rise to lines that generally miss the particle by a long shot, so that identifying such false lines based on the geometry should be reasonably straightforward in theory. A suggestion was made for distinguishing such false lines based on the information about the maximum possible speed of the particle. As indicated in the text many very efficient techniques have been developed in smooth flow situations that rely on a continuously varying particle velocity but such techniques are not useful in the rock tumbling situations of primary interest here. One new technique was investigated but it was found to be not not nearly

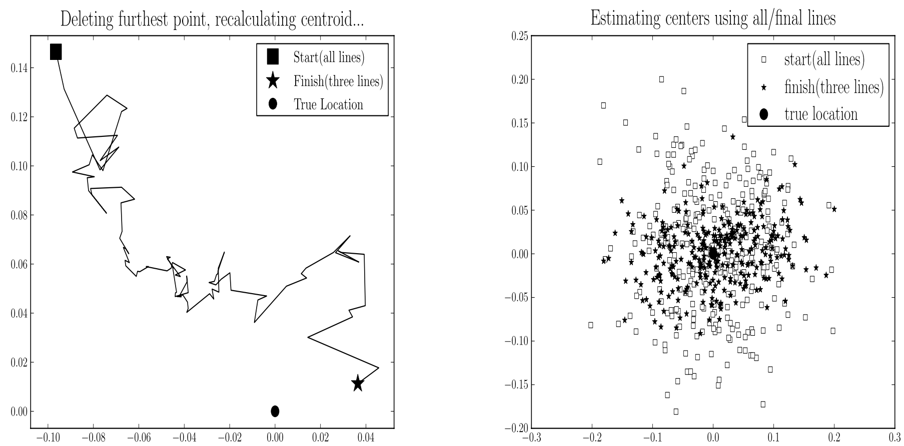


Figure 7: *Left:* Illustration of the 'centroid path', each successive centroid is calculated after removing the line furthest from the previous centroid. This process is repeated until only 3 lines remain. *Right:* Three hundred simulations of the above reduction process. We observe that more often than not the final centrod, calculated using the three lines which at no stage were the furthest from any centroid, is better than the starting centroid.

as good as the currently used technique. A variety of schemes were examined for improving the convergence rate of the iterative centroid scheme currently in use. These schemes were based on the centroid to line distance distribution and made use of the above 'poor shot' observation. The results suggest that refinements of the centroid technique can result in marginal improvements in convergence. One advantage of the data based refinements suggested is that dynamic and automatic tuning of the filtering scheme can be achieved using such schemes.

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