OPTIMIZATION MODEL FOR CAMPUS PARKING SPACE ALLOCATION

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Abstract

In South Africa and around the world, cars are used daily to go to one's workplace and back. During the day, cars must be parked and managing the use of parking spaces is a complex matter. Several methods of attribution are investigated and discussed. A review of possible approaches will then be applied to a simple example.

1 Introduction

On every site, parking must be organised for people working there and/or visitors. From the moment an organisation reaches a threshold size, it becomes difficult to have enough parking spaces for all people potentially in need of them. For example, at the University of the Witwatersrand, there are around 25000 students and 2000 staff for a number of parking spaces considerably less. At this stage, a parking policy must be put in place: some people may be allocated a permanent space while others will have to share and will potentially not find a parking space. Defining

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a parking policy cannot be done randomly. If the policy is not defined correctly, the parking will not be used to its best potential and this could create a lot of problems for management. This problem was submitted to the Study Group. The group investigated different ways to model policies with mathematics. A review of possible approaches will be carried out in Section 2. A simple example will then be presented in Section 3.

2 Theoretical approach

The group first considered general rules for parking space management and their translation into mathematical terms. The typical configuration studied here is shown on Figure 1. N people work on the site considered in three different buildings. When drivers arrive on site, they may choose between parking areas $P1$, $P2$ and $P3$. Depending on the building where they work, people will favour different parking areas:

- Building 1 people will obviously prefer parking area P1, parking area P2 will be acceptable for them and parking area P3 will be their least favoured choice.
- Building 2 people will favour parking area P1 and P2 and try to avoid parking area P3.
- Building 3 people will prefer parking area P2, tolerate parking area P1 and avoid parking area P3.

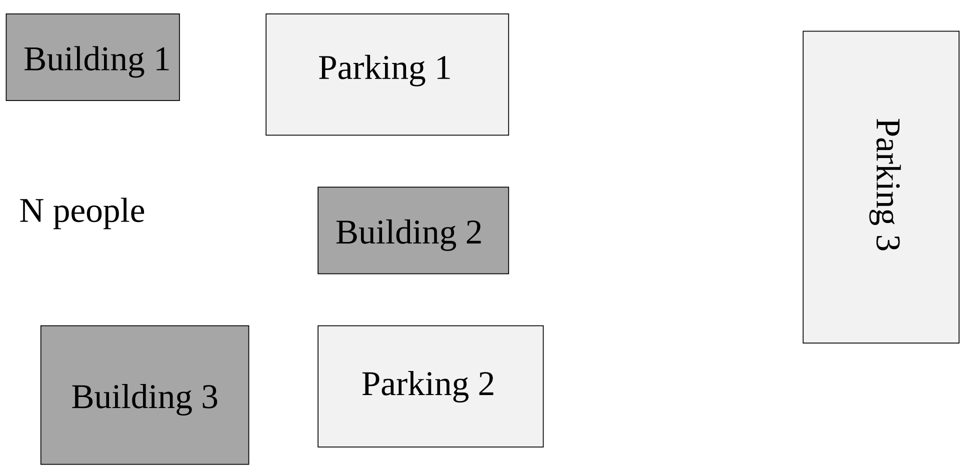

Figure 1: Typical configuration

There is no guarantee however that there are enough parking spaces for all N people. Managing the parking areas efficiently could mean the following:

- **Maximising the parking use**. The objective of this optimisation method is to make sure that each parking space is used as much as possible during the day. Parking spaces must be accessible to as many people as possible: when a space becomes free, it will be filled by a new car very quickly. In this configuration, parking will always look very full but it might be difficult to find a free space rapidly.

- **Maximize income**. In this approach the parking management office is trying to charge users as much as possible to maximise its revenues. Parking usage could be charged as follows:

  - A parking space could be attributed to one person in particular and would be booked for this person for a given period, typically 6 months.
  - A flat yearly charge could be imposed for any parking space and no parking space could be reserved.
  - A yearly charge could be imposed at a different level for each of the parking spaces.
  - Parking meters could be introduced.
  - Parking spaces could be auctioned for a given period, typically 6 months.

  Many other pricing options could be considered. This strategy could lead to a situation where all parking areas are nearly empty. This could potentially lead to a lot of friction within the staff as people without a parking space would feel let down by the system while people with a parking space could feel they are paying too much for the service.

- **Minimise unhappiness**. The satisfaction of people will vary depending on their parking situation:

  - They may or may not be allowed to use parking spaces on site.
  - The space they find or are allocated might be close or far away from the building. Typically in Figure 1, the satisfaction of a person working in building 1 will be very different from a parking space in zone P1 and P3.

  In this approach, the unhappiness of each person will be measured. This parameter may be evaluated using many methods. It could be, for example, a positive number, with very happy people attributed the value 0 and unhappy people getting larger values. The global unhappiness of people on site can then be evaluated and the goal of the parking manager is to reduce the value of this parameter.

The different approaches are not necessarily contradictory and could be mixed. Other objectives could also be considered. However all strategies have very similar modelling requirements.

- Parking areas should be split into categories. In Figure 1, zone $P3$ is the least desirable for people working in all buildings: a parking space should be easier to get there than in any other area. The ranking of parking areas will vary with each building as shown in Table 1. The ranking of parking areas could be established using various criteria such as the average distance between the parking and the building, improved security, covered parking or any other additional service offered.

	Building $B1$	Building $B2$	Building $B3$
First Choice	$P1$	$P1$	$P2$
Second Choice	$P2$	$P2$ (joined first)	$P1$
Third Choice	$P3$	$P3$	$P3$

Table 1: Ranking of parking areas for each building

- Parking users must be split into categories. Here again, there are multiple options to model the ranking of potential users. Several elements could be used to classify people such as the position in the hierarchy, the number of years the car owner has been working on site, special services provided. For example, in a company, the CEO must have his/her own bay, so should other essential staff as opposed to those who require it less frequently. In the context of universities, potential categories would be academic staff, general staff and students for example.

All the elements above will help define a parking policy. As already observed, the number of people requiring a parking space is going to be much larger than the number of parking spaces available. Defining a parking policy means choosing who will have access to the parking areas and what proportions of people will actually find a space. The policy for each category may be defined using the ratio between the number of people willing to use the parking and the number of spaces actually available:

- x_1 spaces are reserved for the N_1 people of category 1.
- x_2 spaces are reserved for the N_2 people of category 2.
- More generally x_i spaces are reserved for the N_i people of category i.

Defining the policy means defining the categories and the ratio α_i between the spaces available and the number of people in each category:

$$\alpha_i = \frac{x_i}{N_i} \ .$$

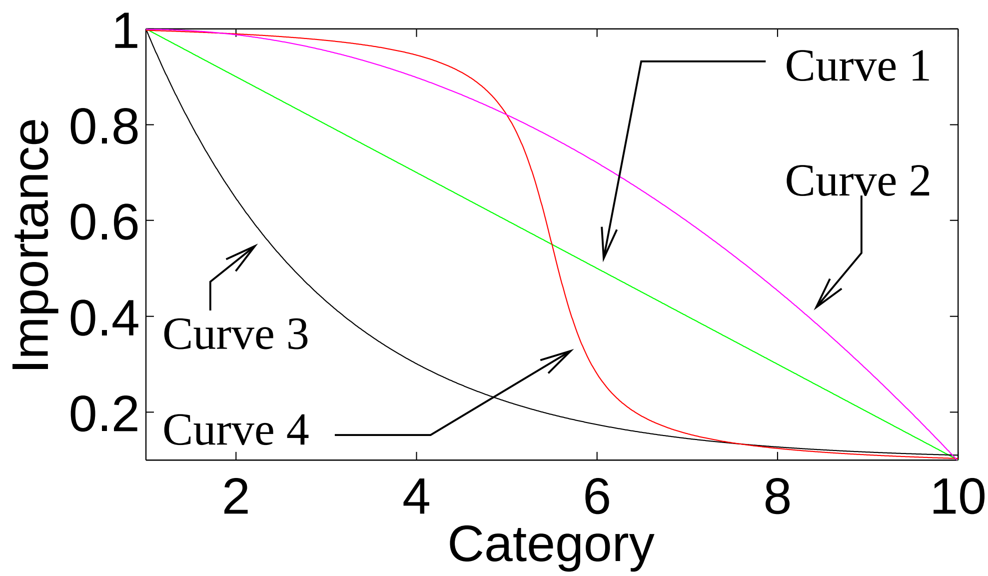

Figure 2: Possible importance functions

Figure 2 shows different possible policies for ten categories of people, represented by importance functions. In all cases, for the first category, the ratio $\alpha = 1$ means that each person in the category is guaranteed to find a parking space when they require one. For category 10, the minimum ratio $\alpha = 0.1$ shows that for each parking space, there are on average ten people wanting to park their car. This minimum value is an element of the parking policy: depending on the global ratio between the total number of people and the global number of parking spaces, this value may be lower or higher. The four curves in Figure 2 describe four different parking management policies.

- Curve 1 is a linear function. The ratio, space to user, decreases progressively as the category number increases. This policy does not favour any intermediate category, it only reflects the importance of the category.
- Curve 2 favours low number categories but the ratio, space to user, still decreases progressively as the category number increases.
- In the policy described by curve 3, the importance of the category is still reflected but the highest number categories are nearly all treated in the same way.
- Curve 4 combines the properties of curve 2 and curve 3: all low number categories and all high number categories are treated in a nearly equivalent fashion.

Low number categories are clearly favoured with this policy while high number categories are all treated similarly to category 10.

These models will now be used to describe how a policy may be implemented in a simple example.

3 Example

Each situation will require a specific model using some of the models presented in the previous section. In the following, some results are presented for the test configuration shown in Figure 3

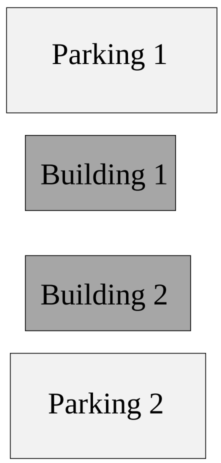

Figure 3: Test configuration

In this example, there are only two categories, people who are allocated a parking space and people who are not and there are as many people allowed to park as there are parking spaces. The only aspect of the policy still to be determined is how people will evolve from one category to the other. This will be decided using unhappiness. As already observed, this parameter may be measured using many methods. In this example, the unhappiness will be measured for each individual using the formula

$$u_i = w_i X_i \tag{1}$$

where

- The subscript i represents a particular individual.
- w_i is the weight of each individual. This parameter may be evaluated using different criteria. For example

- The place in the hierarchy, r_i. This parameter may be estimated using an importance function which could be different from the importance function used to determine the people- parking space ratio.
- The amount of time spent on the site, t_i, expressed in a unit to be decided. This could be for example years or semesters.
- A combination of the two.

In the following, the weight will be evaluated as

$$w_i = r_i t_i \ . \tag{2}$$

- X_i is the dissatisfaction factor

$$X_i = \begin{cases} 0 & \text{if satisfied} \\ \alpha \quad (0 < \alpha < 1) & \text{if parking found but not next to the building} \\ 1 & \text{if no parking found} \end{cases}$$

Other parameters could be included such as the average distance between the building and the parking areas. This aspect could be included easily in the dissatisfaction parameter. In all models, the goal of the parking manger is to reduce the global unhappiness on the site

$$U = \sum u_i$$

On a regular basis, parking management must reassign the parking spaces. During that period, a few people might have given up their parking space and new people will have requested one. In all cases the procedure will be the same:

1. Rank all people requesting a parking space
2. Rank all people who have a parking space
3. Determine who should be allocated a parking space. Note the people who were already allocated a parking space can either be likely to loose their space or be guaranteed to keep it. This is another element of the policy.
4. Determine to which parking area people will be allocated.

In the following, ten people have applied for a parking space but only six spaces became free, three in each parking area. The data for applicants and people likely to lose their parking spaces may be found in Tables 2 and 3, where *unhappiness 1* represents the unhappiness of people with no parking space and *unhappiness 2* is the unhappiness of people allocated a parking space in the wrong parking area. People likely to loose their parking space or change parking area are the persons who have

Person	1	2	3	4	5	6	7	8	9	10
Importance r_i	1	1	1	0.8	0.8	0.8	0.6	0.6	0.6	0.6
Time t_i	2	6	1	10	2	8	12	4	13	7
Work building	$B1$	$B2$	$B1$	$B1$	$B1$	$B2$	$B2$	$B1$	$B1$	$B2$
Unhappiness 1	2	6	1	8	1.6	6.4	7.2	2.4	7.8	4.2
Unhappiness 2	1	3	0.5	4	0.8	3.2	3.6	1.2	3.9	2.1

Table 2: Applicant characteristics

Person	11	12	13	14
Importance r_i	0.8	0.6	1	0.8
Time t_i	9	12	7	1
Work building	$B1$	$B2$	$B1$	$B1$
Parking allocated	$P1$	$P2$	$P2$	$P2$
Unhappiness 1	7.2	7.2	7	0.8
Unhappiness 2	3.6	3.6	3.5	0.4

Table 3: People likely to loose their parking space

been allocated a parking space in the past and whose potential unhappiness reaches the level of the highest person with no parking permit, and all the people who have not been allocated the parking area they wished.

The numbers used in the tables below were calculated manually for the examples considered here. For larger groups, a computer based code should be developed.

3.1 Case 1: nobody can lose their parking space

In this situation, the people who have been allocated a parking space in the past are guaranteed to be allocated a parking space in the next round but maybe not in the parking area they want. People applying for a parking space will therefore be ranked according to the unhappiness they could generate and the first 6 will be allocated a parking space. Results are described in Table 4. The person numbers 4, 9, 7, 6, 2, and 10 will be allocated a parking space: they would cause the largest unhappiness if they were not selected.

In the second stage of the process, parking areas should be allocated. In this situation, the newly selected people must be combined with people already in possession of a parking permit but not in the parking area they would like. In the end, there are 4 spaces allocated in parking area 1 and 6 in parking area 2. At the end of the allocation/re-allocation process, all new people allocated a parking space obtained the parking area they wished. The only person not fully satisfied is

Person	4	9	7	6	2	10	8	1	5	3
Importance r_i	0.8	0.6	0.6	0.8	1	0.6	0.6	1	0.8	1
Time t_i	10	13	12	8	6	7	4	2	2	1
Work building	$B1$	$B1$	$B2$	$B2$	$B2$	$B2$	$B1$	$B1$	$B1$	$B1$
Unhappiness 1	8	7.8	7.2	6.4	6	4.2	2.4	2	1.6	1
Unhappiness 2	4	3.9	3.6	3.2	3	2.1	1.2	1	0.8	0.5

Table 4: Ranked applicants

Person	4	9	11	12	7	13	6	2	10	14
Importance r_i	0.8	0.6	0.8	0.6	0.6	1	0.8	1	0.6	0.8
Time t_i	10	13	9	12	12	7	8	6	7	1
Work building	$B1$	$B1$	$B1$	$B2$	$B2$	$B1$	$B2$	$B2$	$B2$	$B1$
Unhappiness 1	8	7.8	7.2	7.2	7.2	7	6.4	6	4.2	0.8
Unhappiness 2	4	3.9	3.6	3.6	3.6	3.5	3.2	3	2.1	0.4
Parking	$P1$	$P1$	$P1$	$P2$	$P2$	$P1$	$P2$	$P2$	$P2$	$P2$

Table 5: Parking allocation case 1

the person with the lowest unhappiness who will still have to remain in the parking area furthest from its building but the unhappiness of this person is considerably smaller than the unhappiness of the first person denied a parking space. The total unhappiness in the system may then be evaluated as $U = 7.4$.

3.2 Case 2: people can lose their parking space

In this situation, people who were granted a parking permit in the past may lose it. The algorithm is therefore changed. All applicants and all people likely to lose their parking space or change parking area should be ranked together. This is performed in Table 6. Four people are likely to loose/change parking, and once again, in the end, 10 parking spaces should be allocated, 4 in parking area 1 and 6 in parking area 2.

In this configuration, 7 of the 10 applicants were allocated a parking permit and only one person in the wrong parking area. One person saw their parking allocation cancelled. The total unhappiness in the system is then $U = 6.6$. This policy seems to be fairer to the parking users: a regular re-evaluation of parking permits will reduce unhappiness.

Person	4	9	11	12	7	13	6	2	10	8	1	5	3	14
Importance r_i	0.8	0.6	0.8	0.6	0.6	1	0.8	1	0.6	0.6	1	0.8	1	0.6
Time t_i	10	13	9	12	12	7	8	6	7	4	2	2	1	6
Work building	$B1$	$B1$	$B1$	$B2$	$B2$	$B1$	$B2$	$B2$	$B2$	$B1$	$B1$	$B1$	$B1$	$B1$
Unhappiness 1	8	7.8	7.2	7.2	7.2	7	6.4	6	4.2	2.4	2	1.6	1	0.8
Unhappiness 2	4	3.9	3.6	3.6	3.6	3.5	3.2	3	2.1	1.2	1	0.8	0.5	0.4
Parking	$P1$	$P1$	$P1$	$P2$	$P2$	$P1$	$P2$	$P2$	$P2$	$P2$	No	No	No	No

Table 6: Ranked applicants case 2

4 Conclusion and recommendations

This report underlines the complexity of parking allocation policies. Organisations have a variety of ways to tackle this problem. Policies which can involve numerous parameters should be translated in mathematical terms to optimise their implementation. Importance functions can be used to determine the ratio between the number of people of a certain category and the number of parking spaces that should be made available to them. These models can also be used to evaluate the unhappiness of the users. This parameter should be reduced as much as possible. To achieve this, the policy should stipulate that parking allocation may be cancelled at the end of each time period. Clearly, modelling optimal use of parking areas will depend on the case studied, whatever the objectives.

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