

LIQUID FLOW IN A SUGAR CENTRIFUGAL

C.P. Please^{*}, N.D. Fowkes[†], D.P. Mason[‡], C.M. Khaliq[§],
A. Hutchinson[¶] and M. C. Rademeyer^{||}

Industry representatives

Richard Loubser and Steve Davis

Abstract

Masseccuite is a mixture of sugar crystals and molasses produced during the manufacture of sugar. A centrifuge, which is a rotating cylindrical basket, is used to separate the sugar crystals from the molasses. During the latter part of the process water and steam are introduced into the centrifuge to further facilitate drainage. The model developed indicates that a fifty percent increase in molasses drainage can result from the addition of steam, whereas water does not significantly affect drainage.

1 Introduction

The process of extracting molasses from sugar crystals is not optimal. In the early stages of the extraction process, a significant percentage of molasses is not drained from the centrifuge and more time is required to complete the process. Currently, the addition of water or steam is used to aid in the efficiency of this process. It has

^{*}Oxford Centre for Collaborative Applied Mathematics, University of Oxford, Oxform OX1 3LB, United Kingdom. *email: please@maths.ox.ac.uk*

[†]Mathematics Department, University of Western Australia, Crawley, WA 6009, Australia. *email: neville.fowkes@uwa.edu.au*

[‡]School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg, Private Bag 3, Wits 2050, South Africa. *email: David.Mason@wits.ac.za*

[§]Department of Mathematical Sciences, North West University, Private Bag X2046, Mmabatho 2735, South Africa. *email: Masoos.Khaliq@nwu.ac.za*

[¶]School of Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg, Private Bag 3, Wits 2050, South Africa. *email: hutchinson.ash@gmail.com*

^{||}School of Mathematics, University of the Witwatersrand, Johannesburg, Private Bag 3, Wits 2050, South Africa. *email: maryke.rademeyer@students.wits.ac.za*

been proposed that hydrogen peroxide may be added to act as a bleach to reduce the colour of the sugar. In this paper, we use a simple mathematical model in order to investigate the mechanism of this process. We show that the extraction process can be modified to improve the drainage of molasses from the sugar crystals and thus decrease the time required for its completion. According to this model, the addition of steam leads to improving the drainage of molasses by fifty percent.

During the first stage of the sugar manufacturing process juice is extracted from shredded sugar cane in a diffuser. The concentration of sugar is then increased to the point of supersaturation by evaporation. The resulting substance termed massecuite consists of roughly 50 % sugar crystals suspended in the molasses which is a non-Newtonian fluid. The combination has an average viscosity 0.8 Pa.sec. The massecuite is deposited into a batch centrifuge, consisting of a cylindrical basket which is spun at high speed so that the liquid surrounding the crystals, now referred to as molasses, is extracted through the walls of the basket as shown in Figure 1. The basket has a diameter of $2R = 1.2$ m and is lined by a fine mesh which allows for the passage of liquid but ensures that the crystals remain trapped inside.

While the massecuite is being deposited into the basket, the rotational velocity of the centrifuge is about 200 rev/min. After some time the rotational velocity is increased to 960 rev/min and by this stage a layer of massecuite of about 100 mm has built up on the interior vertical sides of the basket. This is known as the spin up stage of the extraction process. After molasses ‘removal’ the centrifuge is slowed down and the sugar crystals are removed by a plough-like mechanism. The addition of water and the washing off of the molasses regulates the colour of the crystals. Further bleaching may be achieved by means of an oxidising agent such as hydrogen peroxide.

This investigation was prompted by difficulties experienced in the sugar industry during this process. Under sub-optimal loading conditions for either the composition of the massecuite, its feeding rate, or the spin up process, the mixture will not uniformly coat the basket and this can lead to centrifuge instabilities either immediately or later during the spin cycle when spin speeds are large. Nonuniform drainage can also result and blockages may occur. The spin up stage is briefly discussed in Section 2. During the molasses stripping process, additional difficulties can also arise during the water and steam addition stage of the process in that nonuniform washing can result in quality degradation and the dissolution of crystals. Based on experience and simple models practical measures have been adopted to deal with the above difficulties, however it is hoped that a better understanding of the processes might lead to better operational procedures. A useful reference to similar problems (but in a geophysical context) is Greenspan [1]. The parameter range is somewhat different in the present context and there are other two phase/lubrication issues that play a role. In Sections 3 to 6 a simple model for the molasses stripping process is

examined. Conclusions and suggestions are presented in Section 7.

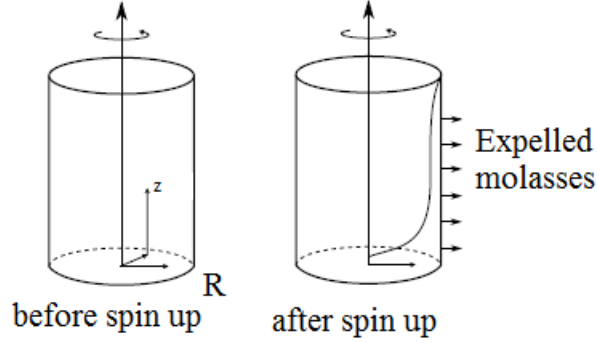


Figure 1: Cylindrical polar coordinate system (r, θ, z) for a thin film of molasses draining from sugar crystals. The coordinate system moves with the centrifuge rotating with angular velocity Ω .

2 Extraction process

During the initial stages of spin up the massecuite that is fed into the bottom of the centrifuge first spreads out in the radial direction and then creeps up the insides of the basket. Alternatively the massecuite may be sprayed directly onto the basket. During this spin up stage of the process the mixture behaves like a homogeneous viscous liquid [1]. After the initial spin up, the centrifuge reaches a constant angular velocity of which it then maintains. The liquid component of the mixture, in other words the molasses, filters through the crystal matrix formed against the walls of the basket. For sufficiently low liquid concentration levels, the flow of molasses through the wall of the centrifuge essentially ceases and the remaining liquid is flushed out from the crystals via the addition of water or steam.

Our main concern is the extraction of molasses from the sugar crystals once the flow of molasses has basically stopped and the centrifuge has maintained a constant angular velocity for a sufficient length of time. Water or steam is required to facilitate further molasses removal in this stripping stage of the extraction process.

3 Model description

When considering the problem of removing molasses from the crystals, the theory of thin fluid films applies. The model for the drainage of molasses from the sugar crystals is illustrated in Figure 2. It is assumed that the crystals form a flat wall with initially a uniform layer of molasses. It is also assumed that the molasses can flow freely through the outer wall of the centrifuge. The outer wall is made of a fine mesh which permits the passage of liquid but not the sugar crystals. The initial thickness of the molasses layer is of order 2×10^{-4} m. In Figure 2 the horizontal length scale is exaggerated for clarity. The model fixes the sugar crystals in the form of a wall of crystals 10 cm high. The molasses forms a thin fluid film on the crystal wall which drains from the surface. The flow is driven by the centrifugal force of magnitude $\Omega^2 R$ per unit mass where Ω is the angular velocity of the centrifuge and R is its radius. The drainage of molasses is a coating flow problem and thin fluid film theory may be applied. Although the molasses is a non-Newtonian fluid it will be modelled as a highly viscous Newtonian fluid.

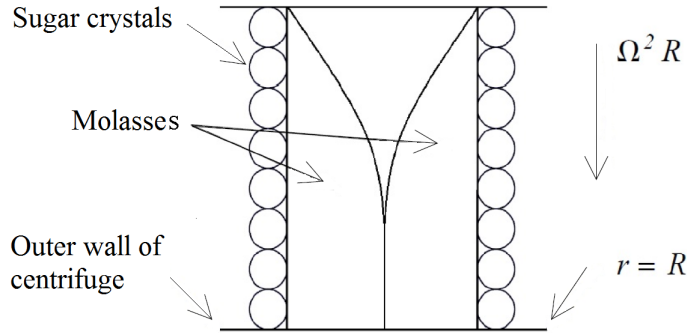


Figure 2: Drainage of molasses from crystals

The rectangular coordinate system is shown in Figure 3. The flow is two-dimensional and there is no dependence on x . The equation of the free surface of the molasses is $y = h(z, t)$. The components of the fluid velocity are

$$v_x = 0, \quad v_y = u(y, z, t), \quad v_z = w(y, z, t). \quad (1)$$

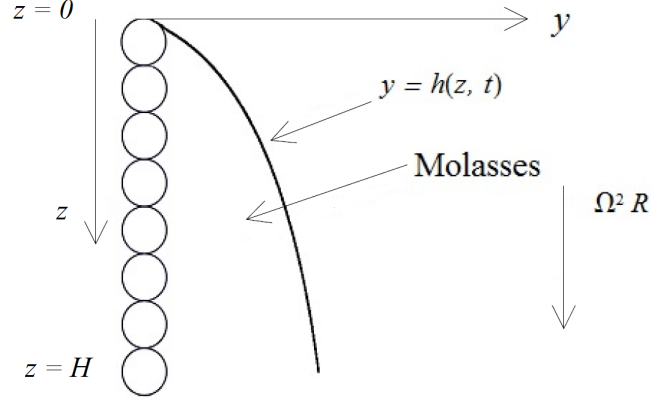


Figure 3: Coordinate system for a thin film of molasses draining from sugar crystals.

For the drainage problem considered here the fluid pressure is constant. The fluid sticks to the wall and there is no shear force on the surface of the molasses, $y = h(z, t)$. The boundary conditions are

$$w(0, z, t) = 0, \quad \frac{\partial w}{\partial y}(y, z, t) = 0. \quad (2)$$

In the lubrication approximation of thin fluid film theory, the inertia term is neglected. The viscous force and the centrifugal force therefore balance and the thin film equations reduce to

$$\mu \frac{\partial^2 w}{\partial y^2} = -\rho \Omega^2 R. \quad (3)$$

Integrating (3) twice with respect to y and imposing the boundary conditions in (2), we obtain

$$w(y, z, t) = \frac{\rho \Omega^2 R}{2\mu} y(2h - y). \quad (4)$$

Now, by considering the conservation of mass equation and the free surface boundary condition it can be shown that

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial z}(h \langle w \rangle) = 0, \quad (5)$$

where

$$\langle w \rangle = \frac{1}{h} \int_0^h w(y, z, t) dy. \quad (6)$$

Using (4) it follows that

$$\langle w \rangle = \frac{\rho\Omega^2 R}{3\mu} h^2 \quad (7)$$

and we have that

$$\frac{\partial h}{\partial t} + \frac{\rho\Omega^2 R}{\mu} h^2 \frac{\partial h}{\partial z} = 0. \quad (8)$$

This equation corresponds to the exact equation within thin film theory describing two-dimensional drainage of a thin layer of viscous incompressible fluid down a vertical surface due to gravity if $\Omega^2 R$ is replaced by g [2].

In order to non-dimensionalise (8) we define:

$$h = h_0 \bar{h}, \quad t = T \bar{t}, \quad z = H \bar{z}, \quad (9)$$

where h_0 is the initial width of the thin film, and the characteristic time T is to be determined. When (9) is substituted into (8) we obtain

$$\frac{\partial \bar{h}}{\partial \bar{t}} + \frac{\rho \Omega^2 R h_0^2 T}{\mu H} \bar{h}^2 \frac{\partial \bar{h}}{\partial \bar{z}} = 0 \quad (10)$$

and the initial conditions become,

$$t = 0, \quad z = 0: \quad 0 \leq \bar{h} \leq 1, \quad (11)$$

$$t = 0, \quad 0 < z \leq 1: \quad \bar{h} = 1. \quad (12)$$

The characteristic time, T , which is an estimate of the time taken for the molasses to drain from the sugar crystals, is therefore

$$T = \frac{\mu H}{\rho \Omega^2 R h_0^2}. \quad (13)$$

Using (13), (10) becomes

$$\frac{\partial \bar{h}}{\partial \bar{t}} + \bar{h}^2 \frac{\partial \bar{h}}{\partial \bar{z}} = 0. \quad (14)$$

We will suppress the overhead bars in the subsequent analysis it being understood that dimensionless variables are being used.

Consider now an estimate of the characteristic time T . For the centrifuge an estimate of the parameters in (11) at 60 °C is:

$$\begin{array}{lll} h_0 = 2 \times 10^{-4} \text{m} & \rho = 2 \times 10^3 \text{kg/m}^3 & H = 0.1 \text{m} \\ R = 0.5 \text{m} & \mu = 100 \text{Pa s} & \Omega = \frac{100\pi}{3} \text{radians/sec} \end{array}$$

With these values of the parameters, $T = 25$ sec. This estimate for T is within the expected range of values for the characteristic time for filtration.

4 Mathematical solution

Consider the model of drainage of molasses from sugar crystals as shown in Figure 3. The thickness $h(z, t)$ of the molasses between $z = 0$ and $z = 1$ is described by the first order quasi-linear partial differential equation

$$\frac{\partial h}{\partial t} + h^2 \frac{\partial h}{\partial z} = 0, \quad (15)$$

subject to the initial conditions

$$t = 0, z = 0 : \quad 0 \leq h \leq 1, \quad (16)$$

$$t = 0, 0 < z \leq 1 : \quad h = 1. \quad (17)$$

The conditions, (16) and (17), which are illustrated in Figure 4, assume that there is initially a uniform layer of molasses on the sugar crystals. The molasses escapes from the centrifuge for $z > 1$.

The differential equations of the characteristic curves of (15) are

$$\frac{dt}{1} = \frac{dz}{h^2} = \frac{dh}{0} \quad (18)$$

and the characteristic curves are thus

$$h = c_1, \quad (19)$$

$$z - c_1^2 t = c_2, \quad (20)$$

where c_1 and c_2 are constants. The general solution of (15) is

$$h = F(z - h^2 t), \quad (21)$$

where F is an arbitrary function.

Consider first $0 < t \leq 1$. From the initial condition (17),

$$F(z) = 1, \quad 0 < z \leq 1, \quad (22)$$

and therefore

$$h(z, t) = 1, \quad t < z \leq 1. \quad (23)$$

To determine the solution for $0 \leq z \leq t$, consider the initial condition (16). From (20), $c_2 = 0$ and since $0 \leq h \leq 1$,

$$h(z, t) = \left[\frac{z}{t} \right]^{\frac{1}{2}}, \quad 0 \leq z \leq t. \quad (24)$$

For $t > 1$,

$$h(z, t) = \left[\frac{z}{t} \right]^{\frac{1}{2}}, \quad 0 \leq z \leq 1. \quad (25)$$

The spatial gradient is infinite at $z = 0$:

$$\frac{\partial h}{\partial z}(z, t) = \frac{1}{2(zt)^{\frac{1}{2}}} \rightarrow \infty \text{ as } z \rightarrow 0, \quad t > 0. \quad (26)$$

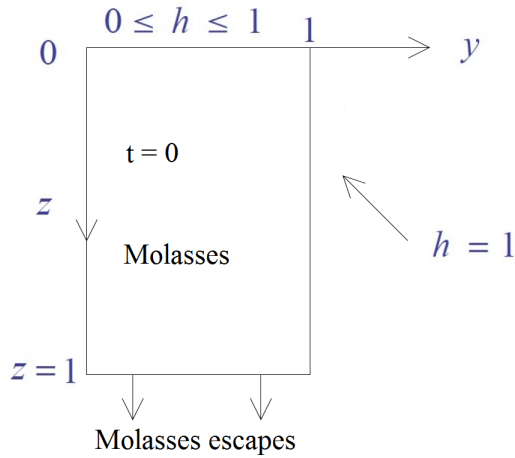
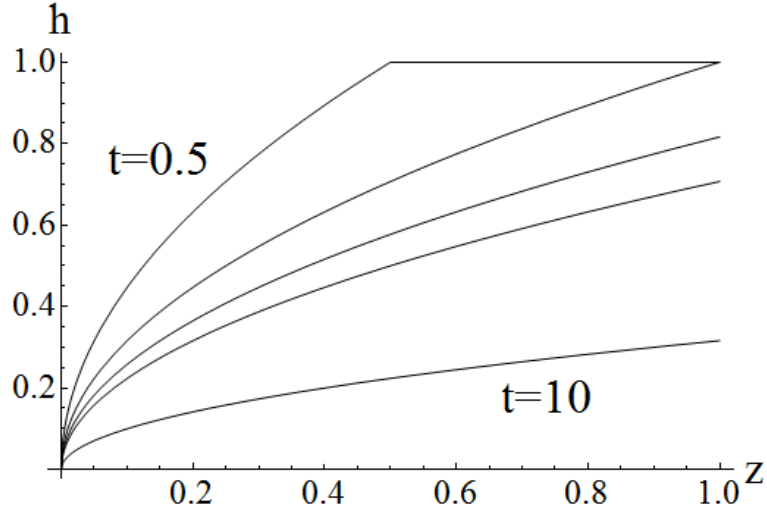


Figure 4: The initial conditions (16) and (17) for the partial differential equation (15).

Graphs of $h(z, t)$ plotted against z ($0 \leq z \leq 1$) for a range of values of t are presented in Figure 5. The graphs illustrate the evolution of the thickness of the layer of molasses on the crystals. For $t > 1$ the thickness of the layer of the molasses on the crystals is less than unity and channels start to form in the molasses which is illustrated in Figure 5. Since this first occurs at $t = 1$, it shows that the time T defined by (13) is a suitable characteristic time for drainage of the molasses.

5 Analysis of results

The objective is to drain as much molasses as possible from the sugar crystals in the least amount of time. Let $V(t)$ be the volume of the molasses remaining on the crystal wall per unit width after draining for time t . The volume $V(t)$ is a useful

Figure 5: Graph of $h(z, t)$ against z for $t = 0.5, 1, 1.5, 2$ and 10 .

measure of the efficiency of the draining process. The cases $0 < t \leq 1$ and $t > 1$ need to be treated separately. For $0 < t \leq 1$,

$$V(t) = \int_0^t h(z, t) dz + \int_t^1 h(z, t) dz, \quad (27)$$

and using (23) and (24) we obtain

$$V(t) = \int_0^t \frac{\sqrt{z}}{\sqrt{t}} dz + \int_t^1 1 dz \quad (28)$$

and thus

$$V(t) = 1 - \frac{1}{3}t. \quad (29)$$

Also,

$$\frac{dV}{dt} = -\frac{1}{3}. \quad (30)$$

For $t \geq 1$ and using (25),

$$V(t) = \int_0^1 \frac{\sqrt{z}}{\sqrt{t}} dz, \quad (31)$$

which simplifies to

$$V = \frac{2}{3\sqrt{t}} \quad (32)$$

and we have

$$\frac{dV}{dt} = -\frac{1}{3t^{\frac{3}{2}}}. \quad (33)$$

From (30) and (33) we see that the molasses drains rapidly and at a constant rate for $0 < t \leq 1$ but for $t > 1$ the rate of drainage decreases. A graph of the volume remaining, V , plotted against t is shown in Figure 6. After draining for time $t = 15$, approximately 20 % of molasses is not drained and thus this percentage of molasses is not extracted.

Consider now the thickness of the molasses layer on the crystals. At $z = 1$, $h(1, t)$ provides an indication of the spacing between adjacent fluid segments at the wall of the centrifuge as shown in Figure 2. It gives a measure of the growth of the channeling in the molasses. From (23) and (25)

$$0 < t \leq 1 : \quad h(1, t) = 1, \quad (34)$$

$$t \geq 1 : \quad h(1, t) = \frac{1}{\sqrt{t}}. \quad (35)$$

The graph of $h(1, t)$ plotted against time t is shown in Figure 7. We see that a

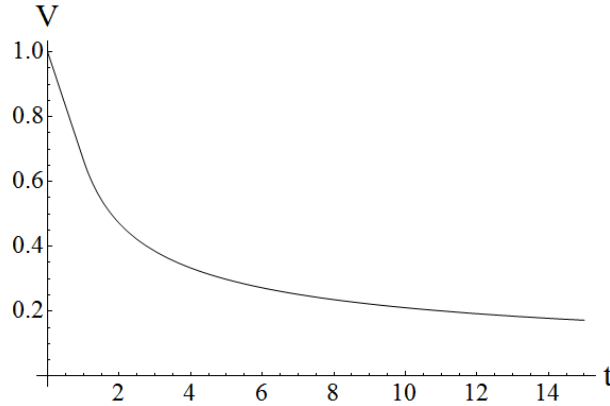


Figure 6: Graph of the volume remaining, $V(t)$, plotted against drainage time t .

channel is formed in the molasses at time $t = 1$. The rate of increase of the width of the channel decreases as t increases. Channelling of the wash water is sometimes observed where water flows through part of the molasses layer and does not wash the remainder. We also see from Figure 7 that after time $t = 15$, about 25 % of the molasses layer remains at the wall of the centrifuge.

Because of the slow rate of removal of the molasses from the crystal for $t > 1$ methods of enhanced molasses removal were analysed.

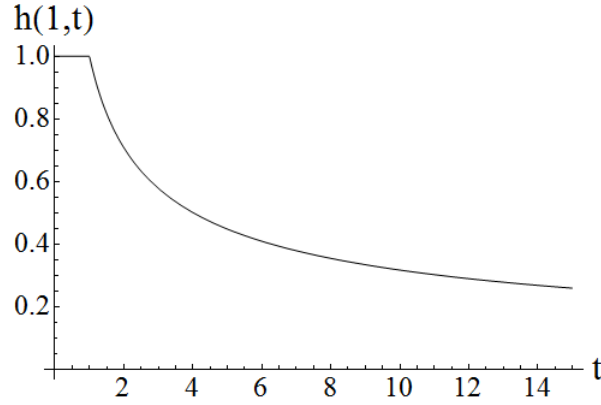


Figure 7: Graph of $h(1, t)$ plotted against time t for $0 \leq t \leq 15$.

6 Enhanced molasses removal

The methods used to enhance the removal of the molasses all reduce the viscosity, μ , of the molasses. The characteristic drainage time, T , defined by (13) is proportional to μ and will be reduced when μ is reduced.

A method which is used to enhance molasses removal is water spraying. The water diffuses into the molasses which is diluted and its viscosity is reduced. The characteristic time, T_D , for water diffusion into the molasses is

$$T_D = \frac{h_0^2}{D_W}, \tag{36}$$

where h_0 is the initial thickness of the molasses layer on the crystals and D_W is the diffusion constant for diffusion of water into the molasses. Now

$$h_0 = 2 \times 10^{-4} \text{m}, \quad D_W = 5 \times 10^{-10} \text{m}^2 \text{s}^{-1}, \tag{37}$$

and therefore,

$$T_D = 80 \text{ sec.} \tag{38}$$

Since the characteristic time for the molasses to drain is $T = 25$ sec, water takes longer to diffuse into the molasses than the time taken for the molasses to drain.

Hydrogen peroxide is also sprayed on the molasses to bleach the molasses to a lighter colour [3, 4]. Spraying is done lightly for several minutes. The hydrogen peroxide will remove the molasses in the same way as water. The characteristic diffusion time for hydrogen peroxide to diffuse into the molasses is assumed to be

the same as for water.

A third way to enhance molasses removal is by spraying with steam. The molasses is heated by the steam which reduces the viscosity. The characteristic time for heat to diffuse into the molasses is

$$T_H = \frac{h_0^2}{D_H}, \quad (39)$$

where D_H is the diffusion constant for the diffusion of heat into the molasses. Now,

$$D_H = \frac{k}{\rho c_p}, \quad (40)$$

where k is the coefficient of thermal conductivity, ρ is the density of the molasses and c_p is the specific heat at constant pressure of the molasses. Now,

$$k = 0.4 \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}, \quad \rho = 2 \times 10^3 \text{ kg m}^{-3}, \quad c_p = 3 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} \quad (41)$$

and therefore

$$T_H = 0.6 \text{ sec.} \quad (42)$$

Since $T_H \ll T$, spraying with steam is a practical way to enhance molasses drainage and is a much better way than diluting with water or hydrogen peroxide. The temperature dependence of the viscosity of molasses is given by [5]

$$\mu = \frac{\mu_0}{\left[\frac{K - 273.15}{K^2} \right]^{4.45}}, \quad (43)$$

where μ_0 is a constant and K is the absolute temperature. The ratio of the characteristic time for the drainage at 100 °C to the characteristic time at 60 °C is

$$\frac{T(100^\circ\text{C})}{T(60^\circ\text{C})} = \frac{\mu(100^\circ\text{C})}{\mu(60^\circ\text{C})} = 0.283. \quad (44)$$

But for $t > 1$, the volume V of molasses remaining is, from (57),

$$V = \frac{2}{3} \left[\frac{T}{t} \right]^{\frac{1}{2}}, \quad (45)$$

where t is the unscaled time and therefore

$$\frac{V(100^\circ\text{C})}{V(60^\circ\text{C})} = \left[\frac{T(100^\circ\text{C})}{T(60^\circ\text{C})} \right]^{\frac{1}{2}} = 0.53. \quad (46)$$

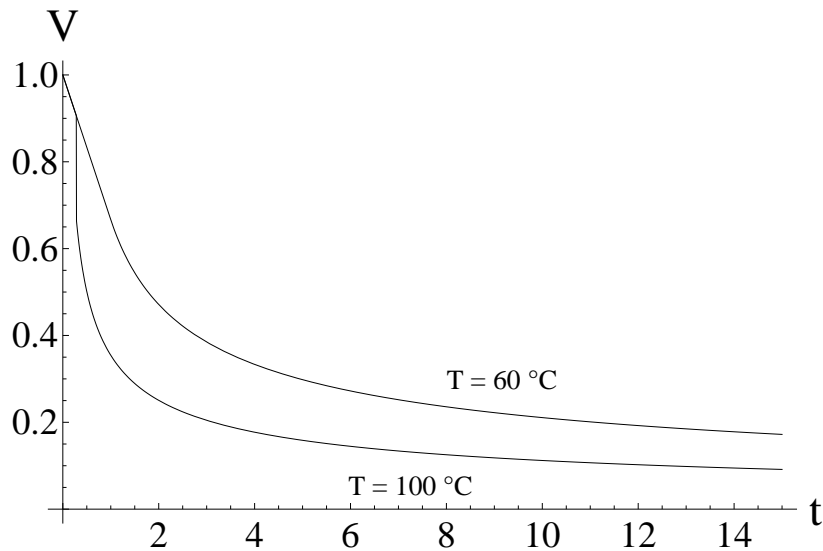


Figure 8: Graph of the volume, V , of molasses remaining plotted against time t at $T = 60^\circ\text{C}$ and $T = 100^\circ\text{C}$.

In Figure 8 the volume of the molasses remaining at 60°C and 100°C are plotted against the scaled time t for 60°C for $t > 1$. We see that spraying by steam reduces by about half the volume of the molasses remaining.

We conclude that heating the molasses by spraying with steam is a better way to enhance the molasses removal than by diluting the molasses by spraying it with water or hydrogen peroxide. Spraying with hydrogen peroxide of course has the other use in the manufacturing process of bleaching the molasses to a lighter colour.

7 Conclusions

The characteristic time for the drainage of molasses, defined by (13), was calculated to be 25 sec and is within the correct order of magnitude. It is useful because it shows how the parameters μ , Ω and R can be varied to control the removal of the molasses from the crystals. The characteristic time does not depend on the permeability of the massequite, the value of which would be difficult to estimate accurately. The permeability would occur in drainage models formulated in terms of Darcy's law for flow through a porous medium.

The spraying of hydrogen peroxide is used to bleach the molasses. The diffusion time for hydrogen peroxide into the molasses is about 80 sec. The molasses layer

therefore needs to be kept for several minutes for the hydrogen peroxide to diffuse into the molasses. The hydrogen peroxide will also remove the molasses from the crystals in the same way as water.

We found that channels can occur in the bed of crystals at the outer wall of the centrifuge. This is due to the draining of the molasses from the crystals. Channelling is sometimes observed when wash water flows through part of the bed and does not wash the remaining part.

Washing by water was found not to be a good way to enhance the removal of molasses because the characteristic time for diffusion of water into the molasses is about 80 sec. Also the spraying with water dissolves some of the sugar crystals leading to losses in production. By far a better way to remove the remaining molasses is to heat the molasses by spraying with steam. The characteristic time for the diffusion of heat into the molasses is about 0.6 sec. Spraying with steam can reduce by about half the volume of the molasses remaining on the crystals.

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