THE EFFECT OF ALLOWING MINIBUS TAXIS TO USE BUS LANES ON RAPID TRANSPORT ROUTES

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Abstract

In order to facilitate the flow of traffic in Gauteng province, South Africa, during peak hours the transport authority is investigating the effect of allowing minibus taxis to use the lanes presently reserved for buses either throughout the day or just during peak hours. One would expect the reduction of flow in the normal lane to result in increased car speeds in this lane and also increased speeds for the minibus taxis in the bus lane, however bus speeds may be reduced and therefore timetables not adhered to. The results obtained show that for initial traffic densities exceeding a critical value the flux through the system will be increased if switching is allowed. Inevitably disruptions will be caused by lane changes which will propagate through the system and also into the oncoming stream. Such dynamics issues are also briefly discussed.

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Figure 1: Competing transport modes in Gauteng: minibus taxis, cars and cabs, and buses

1 Introduction

Gauteng is the smallest (1.5% of the land area) but most populous (12.3 million) province in South Africa and is highly urbanised, containing Johannesburg, Pretoria and large industrial areas. About 65% of Gauteng population use minibus taxis as their mode of transport. The minibus taxi (or share taxi) is a mode of transport which falls between a taxicab (in future called a cab) and a bus; each minibus taxi (in future called a taxi) takes about 8 – 15 people. These taxis are for hire and usually take passengers on fixed or semi-fixed routes without timetables, but instead departing when all seats are filled. They may stop anywhere to pick up or drop off passengers. There are over 100,000 taxis in Gauteng Province. The competing transport modes are primarily taxis, cars and cabs, and buses, see Figure **??**. Buses have seating for 50 to 100 passengers.

Gauteng recently introduced the bus rapid transport (BRT or Rea Vaya) to provide a modern public transport service to support urban development. The BRT ensures that passengers have safe, fast and affordable urban mobility with dedicated right-of-way infrastructure. The present arrangement on the BRT system is shown in Figure **??**. Bus stops are 10 -15 minutes apart with stations located between the two bus lanes and with passenger access such that the normal and bus lanes are effectively separated. This arrangement ensures that buses are unhindered so that the timetable is reliable even in peak traffic flow conditions. This system also tries to reduce passenger travel times in that the centre of roadway keeps buses away from the busy curb-side. However, the down side of this system is that these new bus lanes are often empty during peak hours and that the bus lanes were built where the lanes used to be available to other vehicles.

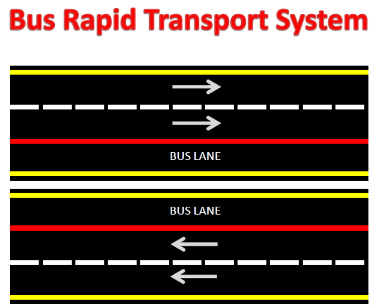

Figure 2: The arrangement of normal and bus lanes in the BRT. Barriers separate the normal and bus lanes.

ADVANTAGES	DISADVANTAGES
The buses get from station to station more quickly	Bus lanes are often empty
Bus lanes are seldom or never congested	Regular traffic is often congested in places where the bus lane used to be available to other vehicles
The system is easily scaled to a much larger number of buses and passengers if needed	Buses block the bus lane at stations

Figure 3: Advantages and disadvantages of maintaining separation of the bus and normal lanes.

The Province is investigating the effect of allowing taxis to share the bus lanes on certain routes and perhaps only during peak hours, and has asked the Mathematics-in-Industry-Study-Group (MISG) to examine the issues. The thought is that by allowing taxis access to the bus lane one might increase the total flow rate without compromising the bus timetable. The BRT setup envisaged is one in which taxis will only be permitted to change lanes at prescribed locations also coinciding with bus stations, see Figure **??**. A list of the advantages and disadvantages of the present arrangement is shown in Figure **??**.

If access of taxis to the bus lane is allowed then the traffic authorities may either choose to enforce the taxis to use the bus lane, or allow taxis to make a choice between taking the normal lane or the bus lane. Moving the most people in the shortest time in the roads is the primary objective. One might hope that allowing taxi drivers to freely choose between the two options might increase flow, but this could well result in instabilities in the normal lanes spreading to the bus lanes. Things to consider when developing a model can include the need to maximize the number of people transported per unit time, taxis stopping to pick up passengers, as well as bus stops, robots and intersections.

However, the general situation is complicated because:

1. The flow on all lanes is traffic robot controlled. Bus priority is often provided at signalized intersections to reduce delays by extending the green phase or reducing the red phase in the required direction compared to the normal sequence. By modifying the timing to suit the circumstances one may optimize the flow.

2. The presence of petrol stations and shopping centres will affect the flow. The best option is likely to be specific to the particular local situation and the time of day.

3. Under close to maximum flux circumstances the flow of traffic is strongly affected by disruptions (accidents etc) so that dynamic considerations may be of primary importance.

4. The long term (weeks) effect of modifying the present arrangement of lanes is likely to be very different to the immediate (days) response in that commuters may change their mode of travel. For example car drivers may choose to become taxi customers because of the reduced travel times, and bus passengers may also choose the taxi option because of the decreased reliability of the bus service.

Lighthill and Whitham [?] (1955) developed a continuum model for the flow of traffic along a single lane under crowded conditions. Following on from this work a variety of gas-kinetic continuum models have been developed to describe multi-lane traffic flow, see Helbing and Greiner [?], Helbing [?], Shvetsov and Helbing [?], and Helbing [?]. These Boltzmann equation models take into account local velocity distributions and overtaking and lane changing manoeuvres, and are more relevant for the flow of traffic under less crowded conditions where such velocity variations and manoeuvres are possible. Our concern here is with flow under crowded conditions so that the simple Lighthill and Whitham model will be used.

In Section 2 we review the important results of single lane flow. Then in Section **??** we go on to determine the effect of allowing taxis to switch lanes on the total flow under steady conditions. Switching will however cause dynamic changes in both traffic streams. Such effects can effect the total through flow locally and may also cause major disruptions in the traffic streams which can be of concern for passenger safety. These issues will also be addressed in Section **??**. Conclusions and recommendations for further work are presented in Section **??**.

2 The Single Lane Model

We first review the important results for single lane unidirectional traffic flow according to the Lighthill and Whitham model. A more complete, but simple, account of this material can be found in Fowkes and Mahony [**?**] and Haberman [**?**]. Also related work on road blocks was done at the MISG 2009, see Ockendon et al [**?**].

2.1 Steady state

For traffic flow in a single lane the continuum model introduces the variables:

$$\begin{aligned} N &: \text{the number of vehicles per km } [1/\text{km}], \\ V &: \text{the velocity of vehicles } [\text{km/hr}], \\ F = NV &: \text{the flux of vehicles past a location } [1/\text{hr}]. \end{aligned} \tag{1}$$

In the simplest model, the driver (or traffic) behaviour is described by

$$V = V(N), \quad \text{or equivalently} \quad F = F(N), \tag{2}$$

and often the linear relationship

$$V(N) = V_{max}(1 - N/N_{max}) \tag{3}$$

is adopted, where (N_{max}, V_{max}) are the maximum density and speed on the lane. The associated flux is given by (the parabolic model)

$$F = V_{max}N(1 - N/N_{max}), \tag{4}$$

see Figure **??**.

Note that there exists a maximum possible flux F_{max} along the lane, given by $F_{max} = N_{max}V_{max}/4$ when $N = N_{max}/2$.

Note also that for flux levels F^0 less than this maximum there are two possible solutions: a small density/high speed solution (N_A^0, V_A^0), and a large density/low speed solution (N_B^0, V_B^0). Passengers and traffic engineers would much prefer the high speed solution, however, without external intervention, they do not have the choice. Under normal circumstances if the flow starts from rest (with vehicles initially behind a red traffic light that then turns green) then

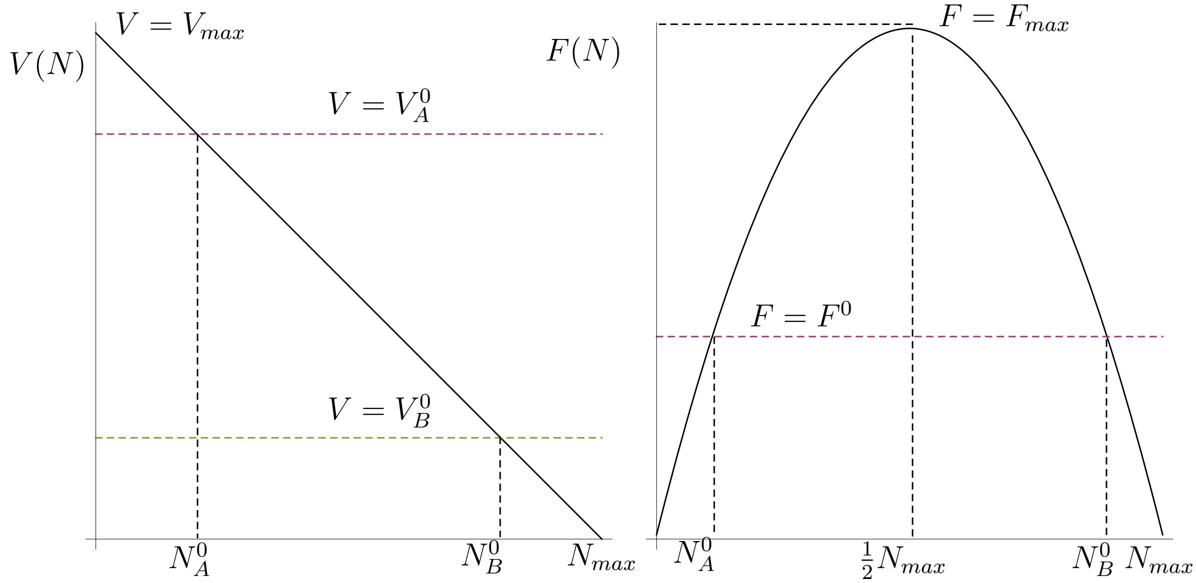

Figure 4: Single Lane Traffic Flow: *Left:* $V(N)$ *Right:* $F(N)$. Note there are two solutions for prescribed $F^0 < F_{max}$

the high speed (low density) solution is initially chosen. However if at some later stage flux levels approach F_{max} then the flow can switch between the two solutions or can become stuck in the the low speed solution. Normally vehicles inter the BRT system after passing through traffic robots so the high speed solution will be initially chosen, but under peak flow conditions difficulties can arise.

2.2 Dynamics

Under dynamic conditions, and in the absence of inflow or outflow, car conservation requires that

$$\frac{\partial N}{\partial t} + \frac{\partial F}{\partial x} = 0, \qquad (5)$$

and if we assume that drivers adjust immediately to their local density environment, then the steady-state flux relation (**??**) applies and we obtain the 'traffic equation'

$$\frac{\partial N}{\partial t} + F'(N)\frac{\partial N}{\partial x} = 0, \qquad (6)$$

for $N(x,t)$. This equation normally requires that the initial condition density $N(x,0)$ be specified, and that a road entry condition on the flux $F(0,t)$ be prescribed.

Solutions to this hyperbolic equation can be obtained by noting that $N(x,t)$ remains constant along curves $\dot{X}(t) = F'(N)$ in the (x,t) plane referred to as characteristics. Since N remains fixed along such a characteristic $F'(N)$ also remains fixed so that the characteristic curves are in fact straight lines in the (x,t) plane. Explicitly if $x = \xi$ denotes the intersection of a specific characteristic on the $t = 0$ axis, so $X(0) = \xi$, then

$$N(X(t),t) = N(\xi,0), \text{ along } X(t) = \xi + F'(N(\xi,0))t. \tag{7}$$

By plotting out all such characteristics one can determine the solution for all

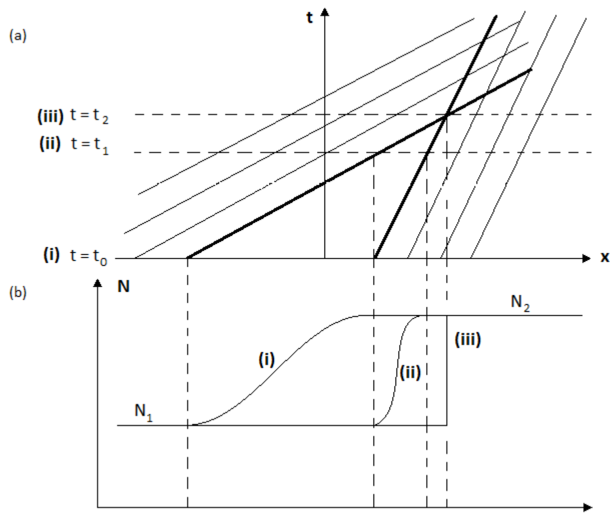

Figure 5: (a) The characteristic curves in the (x,t) plane (increasing density case) and (b) Density profiles at times $t = t_0, t_1, t_2$. Note the steepening of the density profile with increasing time and the eventual overlapping of characteristics and discontinuity after $t = t_2$.

cases in which cars are travelling into lower density road conditions.

If cars are heading into more dense conditions then the density profile will steepen and the characteristics will eventually overlap so the procedure fails to produce a unique result, see Figure **??**. Under such circumstances there will be a density and velocity discontinuity across a travelling shock wave. The traffic equation (**??**), (which requires continuity of N_x), needs to be abandoned in favour of a simple flux continuity condition across the moving shock which uniquely determines the shock speed as

$$\dot{X}_S = \frac{F^r - F^l}{N^r - N^l}, \tag{8}$$

where (N^l, F^l) and (N^r, F^r) denote conditions on the left and right sides of the moving shock; a result referred to as the Rankine-Hugoniot condition. On either side of the shock the normal characteristic solution applies, so that the solution process is complete.

3 Lane Switching Models

We return to the lane switching problem and examine the simple scenario depicted in Figure ??. Taxis travelling in the normal lane switch to the bus lane at a switching point which coincides with the bus passenger stop. Under such circumstances the total density of vehicles (cars, buses and taxis) across the two lanes at the stop remains unaltered, but the taxis will now all be travelling in the bus lane. After the switching point all vehicles are required to remain in their designated lanes. The speed of cars in the normal lane will normally increase because of the smaller density while the speed of vehicles in the bus lane will decrease. The question of interest is: will the net flux of vehicles increase as a result of the switch and what will happen to the passengers travel times? A second issue involves the dynamics: will the local disruption caused by switching be serious?

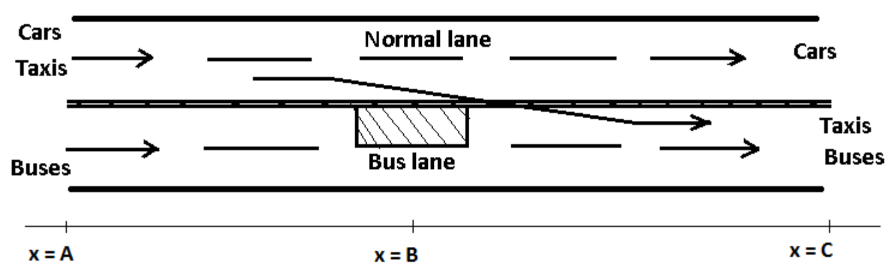

Figure 6: A simple lane changing scenario: taxis travelling in the normal lane switch to the bus lane at a switching point, normally a bus stop.

3.1 Steady state

We assume that vehicles do not stop at the switching point and (for the present) assume that there is no disruption as a result of the lane switching. In practice it will take time and distance for drivers to adjust to the changed circumstances but one would expect the new steady state to be realised in a short distance. The dynamics will be addressed later.

In the work to follow we will use superscripts to refer to lanes: 'b' for bus and 'n' for normal. Subscripts (where necessary) will be used to distinguish vehicle types: 'c' for cars, 't' for taxis, 'b' for buses. Thus (N_b^b, N_t^b) are the densities of buses and taxis in the bus lane respectively. After switching both buses and taxis share the bus lane, and the total number of vehicles in the bus lane, denoted by N^b, is given by $N^b = N_b^b + N_t^b$.

In order to simplify the calculations we will assume that the density, velocity and flux in both the normal and bus lanes is dependent only on the total vehicle numbers in the specified lane. Explicitly we model the flow in the bus and normal lanes using

$$F^n = F^n(N^n), \quad V^n = V^n(N^n), \quad \text{and} \quad \mathrm{F^b = F^b(N^b)}, \quad \mathrm{V^b = V^b(N^b)} \tag{9}$$

where N^n is the total number of vehicles (cars and taxis) travelling in the normal lane, and (V^n, F^n) are the associated (shared) vehicle speeds and (total) vehicle fluxes in this lane. The single lane flux function $F(N)$ is again appropriate however the bus and car lanes will normally differ in capacity and in particular the maximum vehicle flux levels will be different in the two lanes.

Taxis may or may not occupy the normal lane (sharing with cars), or the bus lane (sharing with buses) so we have:

$$N^n = N_t^n + N_c^n, \quad N^b = N_b^b + N_t^b, \tag{10}$$

$$F^n = F_t^n + F_c^n, \quad F^b = F_b^b + F_t^b, \tag{11}$$

and using (**??**) the vehicle velocities in the two lanes are given by

$$V^n = \frac{F^n(N_t^n + N_c^n)}{(N_t^n + N_c^n)}, \quad V^b = \frac{F^b(N_t^b + N_b^b)}{(N_b^b + N_t^b)}. \tag{12}$$

Of particular interest is the total flux of vehicles given by

$$F^{\text{total}} = F^n + F^b. \tag{13}$$

Also of interest is the average travel speed for all passengers which can be obtained by taking into account the average occupancy of cars, buses and taxis (n_c, n_b, n_t):

$$\bar{V}^p = \frac{[n_c N_c^n + n_t N_t^n)]\, V^n + \left[n_b N_b^b + n_t N_t^b)\right] V^b}{\left[n_c N_c^n + n_t N_t^n + n_b N_b^b + n_t N_t^b\right]}. \tag{14}$$

The required results are now in place and various scenarios can be examined.

For the scenario depicted in Figure **??** we have pre-switching conditions given by

$$N_t^b = 0,\ N_t^n = \mathcal{N}_t^n \text{ and with } \quad N_c^n = \mathcal{N}_c^n,\ N_b^b = \mathcal{N}_b^b,$$

with the $\mathcal{N}$'s prescribed. The post-switching conditions (if all taxis switch lanes) are given by

$$N_t^b = \mathcal{N}_t^n,\ N_t^n = 0, \quad \text{and with} \quad N_c^n = \mathcal{N}_c^n,\ N_b^b = \mathcal{N}_b^b.$$

3.2 Results

The status of the system before and after lane switching is best presented on a scaled flux/scaled density plot. We scale the densities:

$$N^n = N_{max}^n \rho^n,\ N_t^n = N_{max}^n \rho_t^n,\ N_c^n = N_{max}^n \rho_c^n, \tag{15}$$

$$N^b = N_{max}^b \rho^n,\ N_t^b = N_{max}^b \rho_t^n,\ N_c^b = N_{max}^b \rho_c^n, \tag{16}$$

where (N_{max}^n, N_{max}^b) are the maximum densities in the two lanes respectively, which will normally be different. With this choice of scale the range of scaled densities in both lanes is 0 to 1, with maximum flux conditions realised when $\rho = 1/2$. We will use N_{max}^n as a scale for specifying the total density

$$N^{\text{total}} = N^n + N^b = N_{max}^n(\rho^{\text{total}}) = N_{max}^n(\rho^n + \mathcal{R}\rho^b) \quad where\ \mathcal{R} = \frac{N_{max}^b}{N_{max}^n}\ .$$

The maximum density ratio, $\mathcal{R}$, is an important dimensionless group for the problem.

The maximum speeds in the two lanes are likely to be the same (V_{max} say), but (because of the different maximum densities) the maximum flux levels (F_{max}^n, F_{max}^b) will normally be different. For the linear model case the maximum flux ratio $F_{max}^b/F_{max}^n = N_{max}^b/N_{max}^n = \mathcal{R}$. For purposes of comparison it is useful to use the same scale F_{max}^n for both lanes. Thus we use the scaling:

$$F^n = F_{max}^n f^n(\rho^n) \quad F^b = F_{max}^n f^b(\rho^b), \tag{17}$$

and the net flux through both lanes is given by

$$F^{\text{total}} = F_{max}^n(f^n + \mathcal{R}f^b) \equiv F_{max}^n f^{\text{total}}. \tag{18}$$

For the prescribed densities ρ^n, ρ^b the effect of shifting lanes is displayed in two cases: a small initial traffic density case and a (peak hour) larger density case, see Figure **??**. The total flux on the road is given by the sum of the

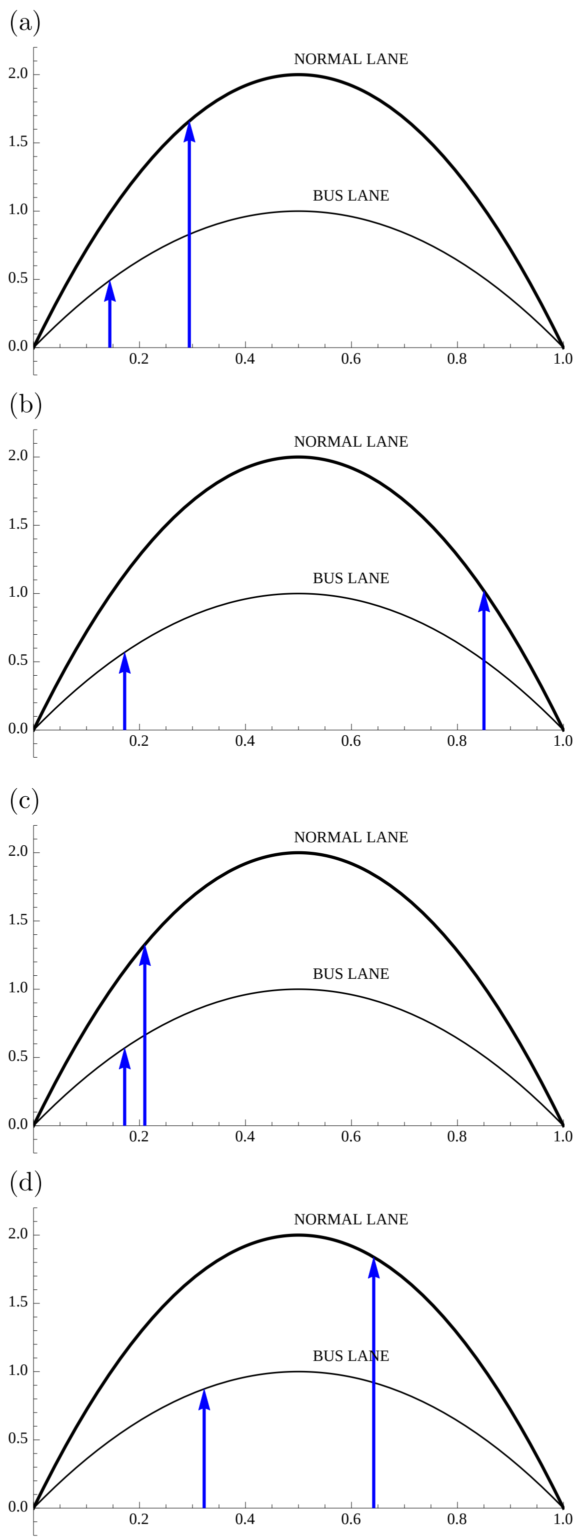

Figure 7: Scaled flux levels $(f^b(\rho^b), f^n(\rho^n))$ as a function of scaled density levels before and after switching: (a) and (b) represent low and high density conditions prior switching while (c) and (d) represent the post switching situation.

two arrows and the result obtained as a function of total initial density is displayed in Figure ??. In the case of a low initial density the effect of lane switching reduces the total vehicle flux, whereas for an initially high density the effect of lane switching is to increase the total vehicle flux. Evidently there is a critical traffic density $\rho_{crit}^{\text{total}}$ above which lane shifting improves the outcome; the intersection of the curves determines $\rho_{crit}^{\text{total}}$; an explicit solution can be obtained for $\rho_{crit}^{\text{total}}$ and the associated flux as a function of the maximum flux and maximum density on the two lanes. A knowledge of this value or the associated initial flux levels can be usefully used to determine the appropriate time to allow or enforce switching as peak hour is approached.

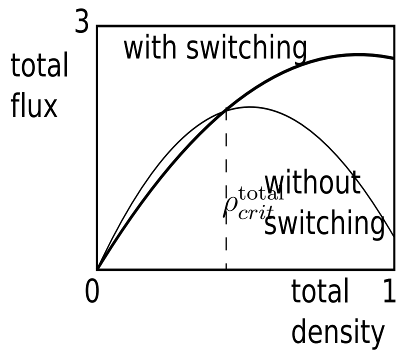

Figure 8: Total (scaled) flux f^{total} through the network as a function of the average initial traffic density ρ^{total}, (a) with (thick curve), or (b) without (thin curve) taxis switching into the bus lane. Note that switching produces a better outcome for initial density levels greater than $\rho_{crit}^{\text{total}}$.

3.3 The dynamics

As indicated earlier steady state conditions will not be immediately established after initiating switching, whether or not the process is initiated using traffic robots. The flow situation can be determined using the method of characteristics as described in Section ??, see Figures ??, ??. If the flow of traffic is not interrupted by a traffic robot, and a fixed proportion α of vehicles (the taxis) travelling in the normal lane transfer into the bus lane, the appropriate conditions to impose across the switching point $x = 0$ are the flux transfer conditions

$$F^n(0^+, t) = (1 - \alpha)F^n(0^-, t) \text{ and, } F^b(0^+, t) = F^b(0^-, t) + \alpha F^n(0^-, t). \quad (19)$$

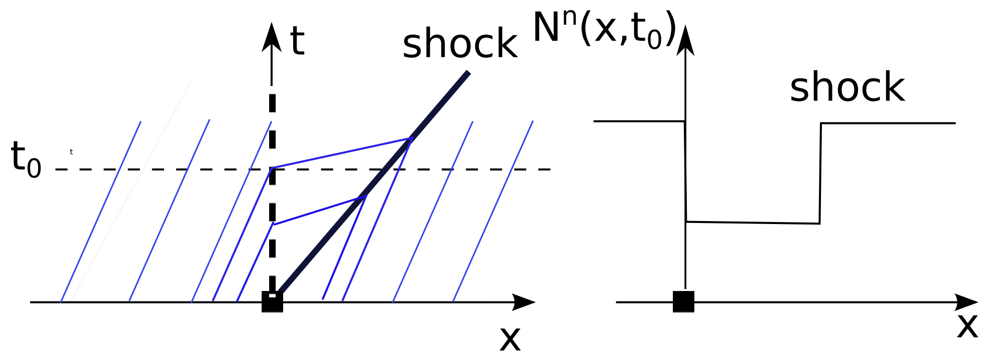

Figure 9: Flow in the normal lane: the solution using characteristics. *Left:* Characteristics are shown as thin lines, contact discontinuities as dashed lines, shocks as continuous thick lines. *Right:* There is an immediate reduction in density at the switching point (and cars immediately increase velocity across a 'contact surface' (dashed)). Past the switching point there is a constant density region followed by a shock transition (and contact surface) back to the pre-switching solution.

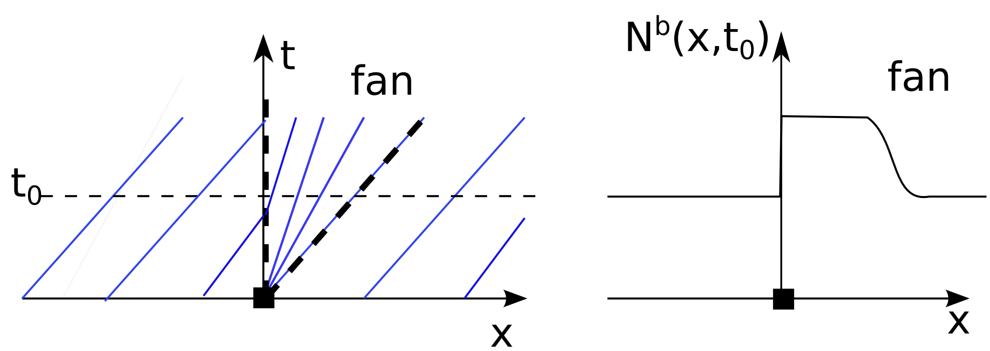

Figure 10: Flow in the bus lane: *Left:* the solution using characteristics. *Right:* There is an immediate density transition across a 'contact surface' at the switching point, followed by a constant density region, and then a centred fan transition back to the pre-switching state, again across a 'contact surface'.

As indicated earlier the simple single lane model assumes that after the switch the drivers *immediately* adjust to the changed flux circumstances by adjusting the spacing between vehicles and thus the lane density. Thus equations (**??**, **??**) determine the post-switch traffic densities $(N^b(0^+, t), N^n(0^+, t))$, and the switch location $x = 0$ corresponds to a contact discontinuity in the flow. (These (local) results correspond to the steady state solutions already obtained.) The implication of this result is that the switching process occurs smoothly. (One might expect the switching process to generate a shock wave in the bus lane travelling back into the oncoming stream, but this does not happen.)

In the normal lane situation the transition to the constant pre-switch state solution occurs across a shock with speed given by (**??**), whereas in the bus lane this transition occurs via a centred rarefaction fan. Thus drivers in the normal lane speed up after passing by the switching point and catch up (across a shock) with slower moving drivers ahead on the lane. In the bus lane drivers slow down after the switch, but after travelling at constant speed for a while (until they hit the fan edge), then find they can gradually increase speed until they travel again at the pre-switch speed.

Aside The Lighthill and Whitham model assumes that drivers reaction times are negligible and in context there is the assumption of a 'perfect' interchange. A more sophisticated model may be necessary to determine the effect of interchange details on the local flow.

As can be seen in Figures **??**, **??** *Right*, all the above dynamics effects travel out of the system, leaving behind the steady state situation with a discontinuous transition across the switching point. Subsequently dynamic changes will occur as a result of changes in inflow (for example, as a result of increasing traffic as peak hour is approached) or locally generated disruptions within the traffic streams. All these disruptions will produce dynamic changes similar to those described above, with such changes travelling through the system with approximately the signal speed at the background traffic density. These dynamic effects become more pronounced for high traffic density situations, because of the decreasing signal speed. Under conditions close to maximum flux conditions the signal speed goes to zero so any disturbances are likely to remain within the system.

4 Conclusions/Suggestions

Using the Lighthill and Whitham single lane traffic flow model a simple scenario was examined to investigate the effect of lane switching on traffic flow. For traffic density levels greater than a critical level, the switching process will result in increased flows. Explicit solutions were obtained for this 'critical density' in terms of the flow characteristics of the two lanes and this result may be used to optimise the system's performance. Dynamic fluctuations caused by disturbances can be of concern for safety and these effects are likely to be more acute under switching circumstances. The effect of allowing taxis to switch lanes in terms of the reliability of bus timetables can be assessed using models of the above type but real data is necessary.

In order to apply the above work for the practical situation one needs:

- Information (data) about the flux verses density functions (bus, normal routes)
- Information about the proportion of taxis, buses, cars travelling on routes and passenger occupancy
- Information about the suitable objectives
- Specific information about one experimental route. This route should be monitored (fluxes, densities) before and after lane switching.

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