Prediction of Fracturing and Dynamic Roof Failure in Platinum Mines

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Introduction

- Roof collapses occur occasionally in platinum mines and are devastating events
- When roof collapse occurs, large slabs of the rock fracture
- These roof collapses compromise miner safety and are very costly to the mine
- By understanding how a roof collapse occurs, we can predict its occurrence as well as attempt to mitigate the risks, thus avoiding unnecessary loss

Introduction



Introduction



Crack Formation

Given slabbing, what factors contribute to/determine the uniform thickness of the slabs in non-sedimentary rock?

Definitions

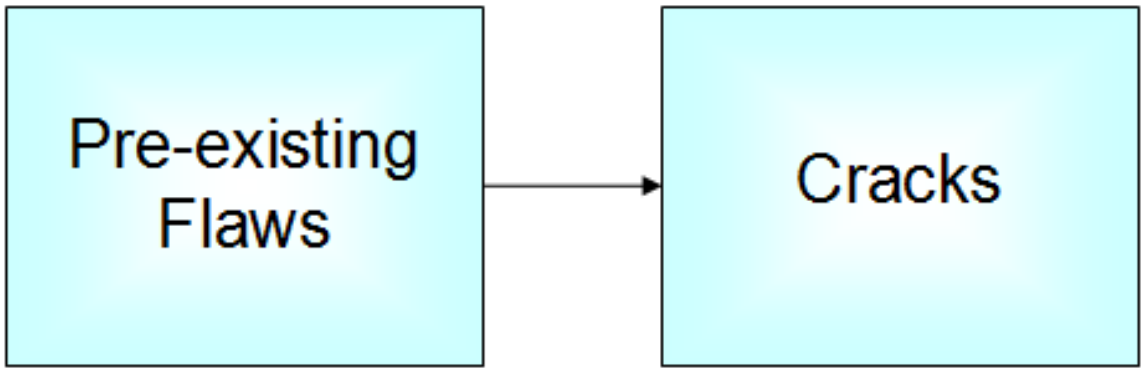
- Slabbing: A phenomenon whereby a rock mass peels off in uniform layers
- Thickness: Distance from free surface to the fracture

Crack Formation

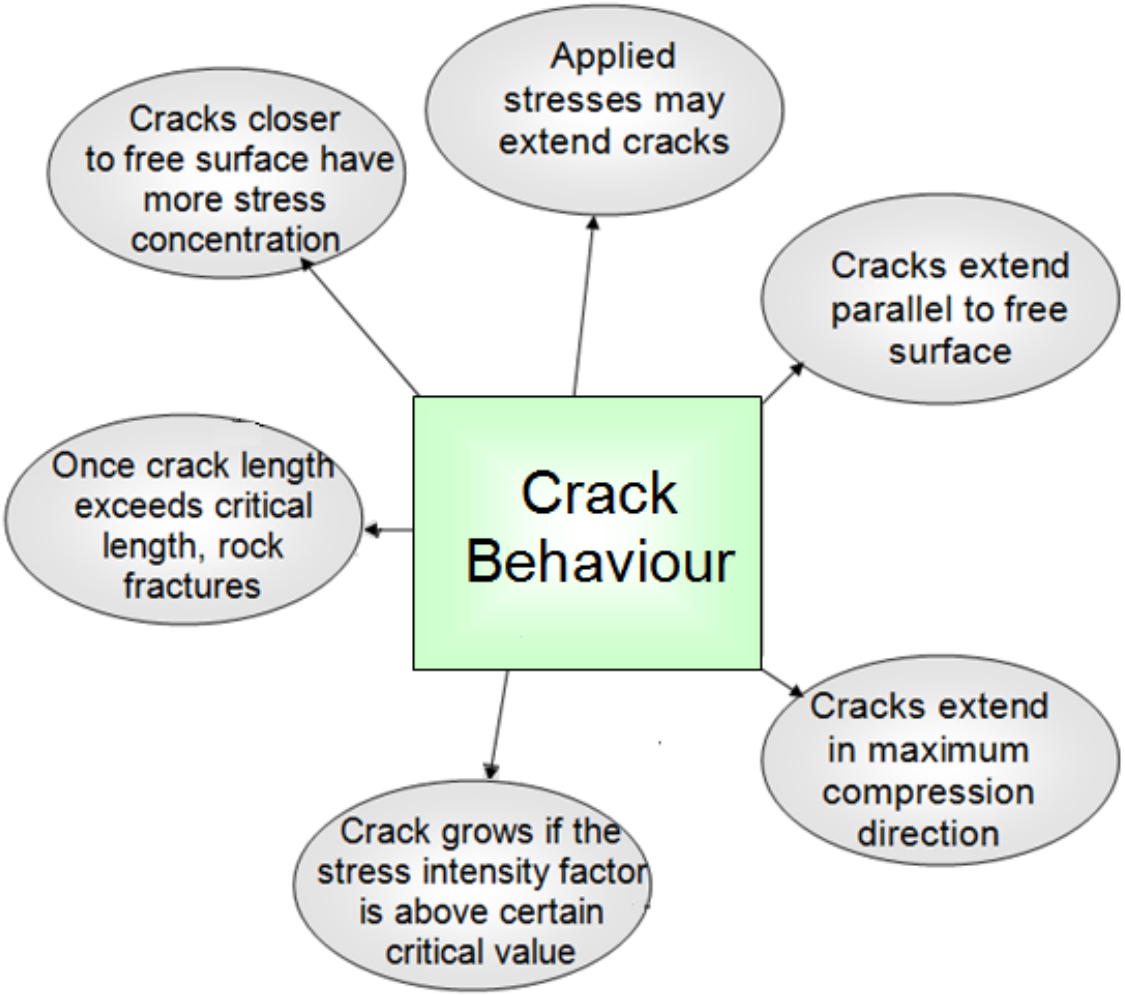


Crack Formation

Considerations



Crack Formation



Crack Formation

Possible Slabbing Explanations

- $P \propto H$, so the pressure closer to the lower free surface is the greatest. This is why we have roof collapse.
- The geometric distribution of pre-existing cracks may influence the manner in which the rock fractures.
- Seismicity induced by drilling, explosions and natural effects causes a re-distribution of stresses which contributes to crack extension.

Exfoliation Problem

Exfoliation of surface rocks is a complementary phenomenon that has been observed. We believe it to be a buckling beam problem and that it also corresponds to the eigenvalue problem, as the effect of gravity was seen to be negligible.

Exfoliation Problem



Exfoliation Problem



Exfoliation Problem

The beam equation is given by

$$EI\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} = q,$$

where q is the sum of the body force and the surface traction per unit length.

For the exfoliation problem, q=0 thus resulting in

$$\frac{d^4w}{dx^4} + B^2\frac{d^2w}{dx^2} = 0.$$

Exfoliation Problem

Finding the general solution to our equation yields

$$w(x) = A\cos(Bx) + C\sin(Bx) + \frac{D}{B^2}x + \frac{F}{B^2},$$

subject to the boundary conditions for a beam clamped at both ends:

$$\begin{aligned} w(0) &= 0, \\ w(1) &= 0, \\ w'(0) &= 0, \\ w'(1) &= 0. \end{aligned}$$

Exfoliation Problem

In order to produce non-trivial solutions, we impose our boundary conditions to obtain a homogeneous system of equations in the form $H\underline{x} = \underline{0}$:

$$\begin{pmatrix} 1 & 0 & 0 & B^{-2} \\ \cos(B) & \sin(B) & B^{-2} & B^{-2} \\ 0 & B & B^{-2} & 0 \\ -B\sin(B) & B\cos(B) & B^{-2} & 0 \end{pmatrix} \begin{pmatrix} A \\ C \\ D \\ F \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

We want the determinant of the matrix to be equal to 0.

$$\det(H) = 4B^5 \sin\left(\frac{B}{2}\right)\left(\sin\left(\frac{B}{2}\right) - \frac{B}{2}\cos\left(\frac{B}{2}\right)\right) = 0$$

Exfoliation Problem

Case 1: $\sin\left(\frac{B}{2}\right) = 0$

Solving for B gives $B = 2n\pi$ $n \in \mathbb{Z}$.

We substitute this into the matrix H:

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{(2n\pi)^2} \\ 1 & 0 & \frac{1}{(2n\pi)^2} & \frac{1}{(2n\pi)^2} \\ 0 & 2n\pi & \frac{1}{(2n\pi)^2} & 0 \\ 0 & 2n\pi & \frac{1}{(2n\pi)^2} & 0 \end{pmatrix}.$$

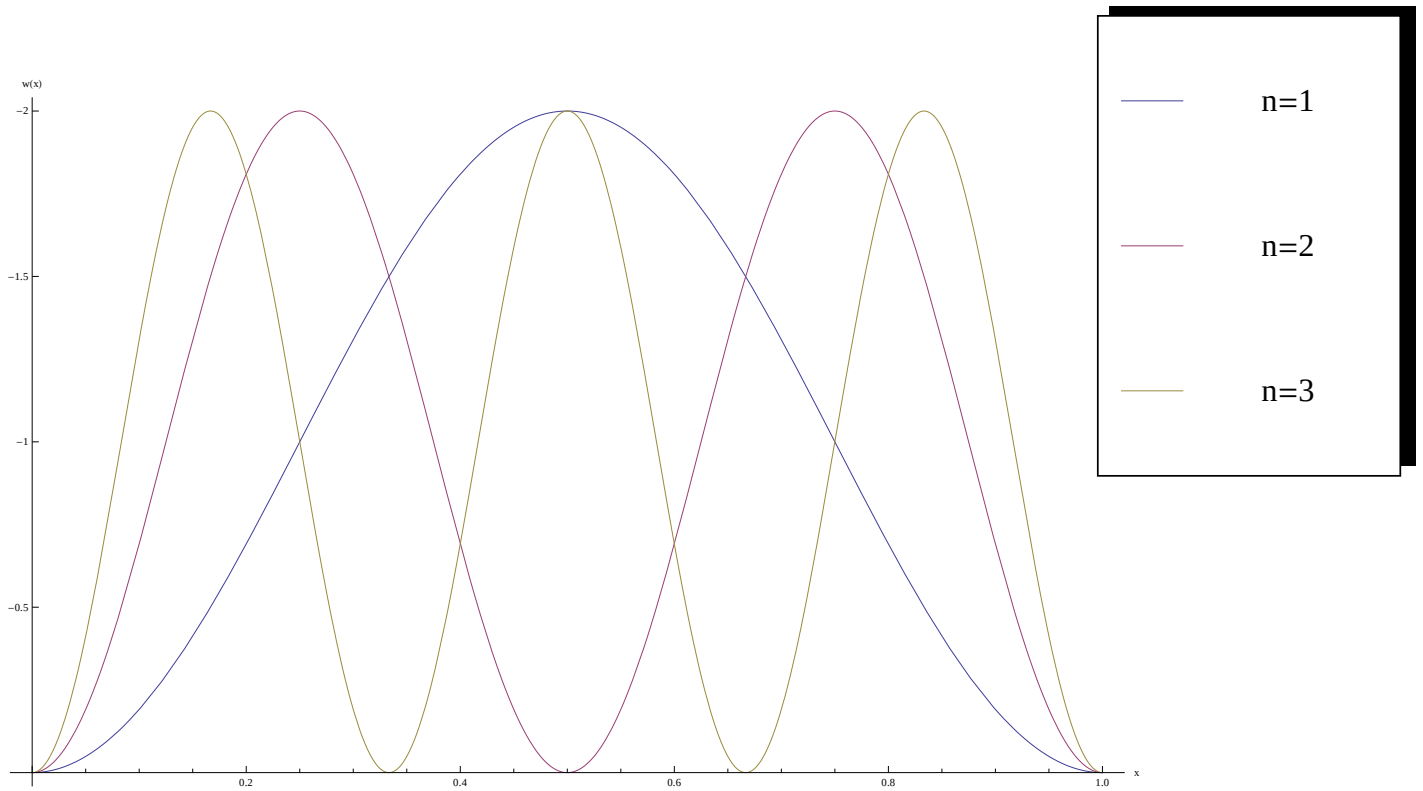
Solving the resulting matrix system with $B = 2n\pi$ gives

$$w(x) = A(\cos(2n\pi x) - 1),$$

where A is an arbitrary constant.

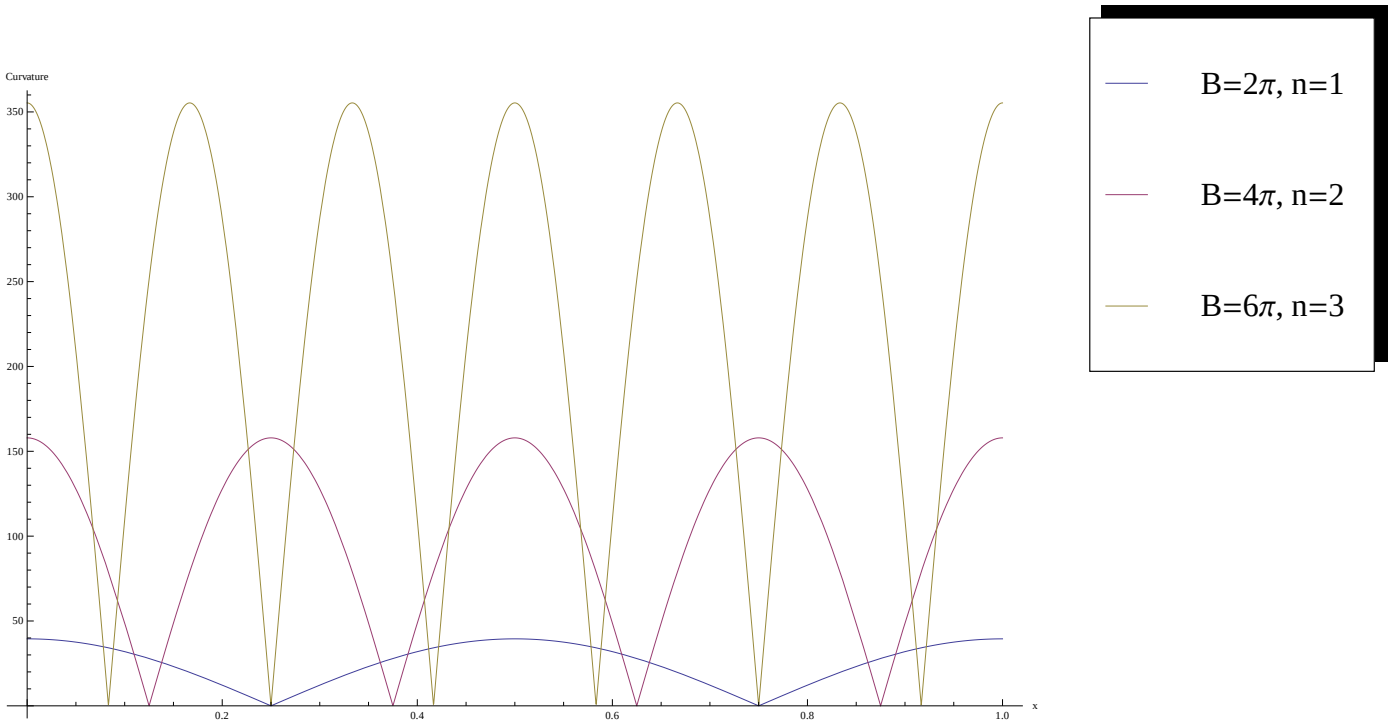
Exfoliation Problem

Plot of the beam deflection



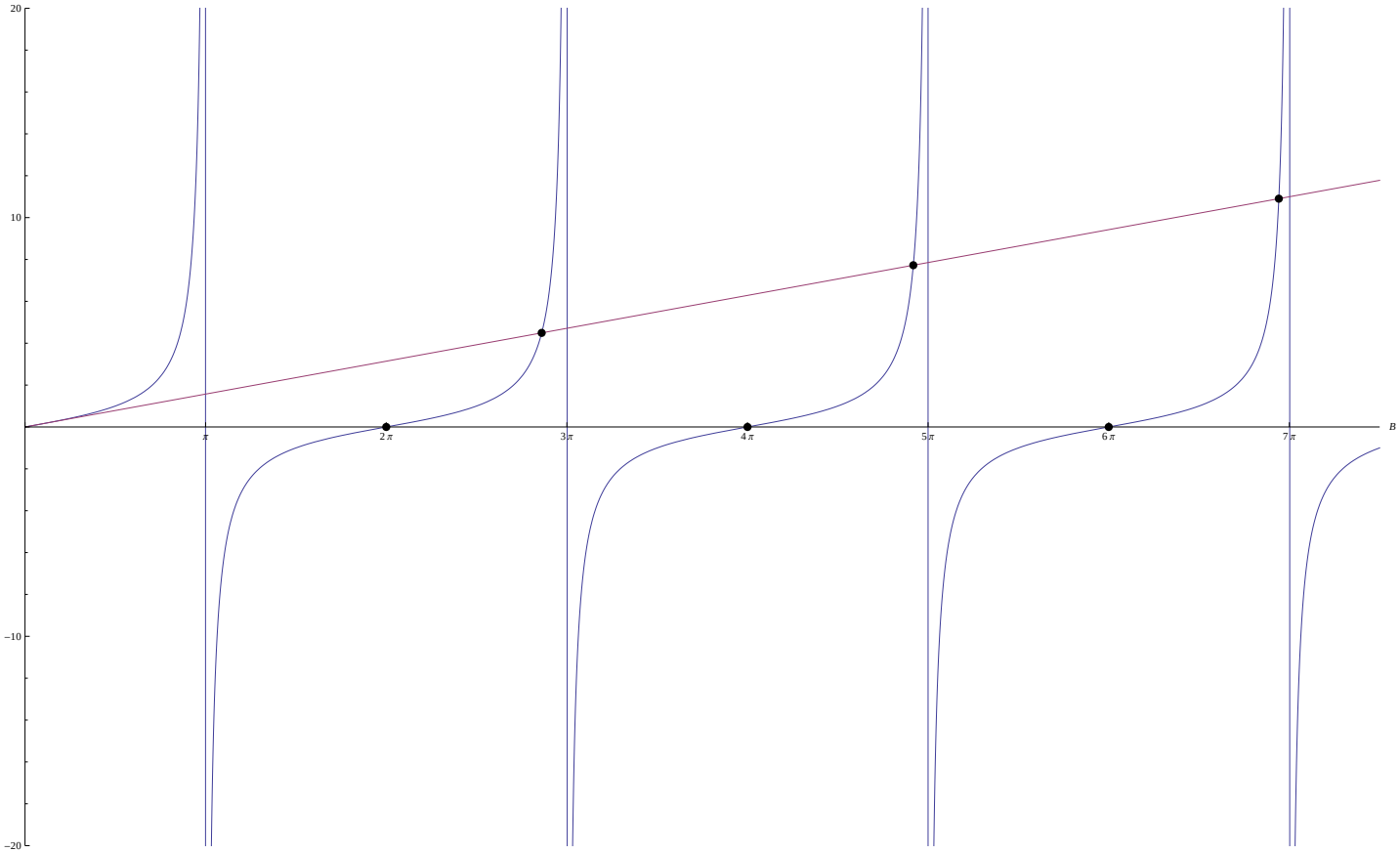
Exfoliation Problem

Plot of the beam curvature



Exfoliation Problem

Case 2: $\left(\sin\left(\frac{B}{2}\right) - \frac{B}{2}\cos\left(\frac{B}{2}\right)\right) = 0 \Rightarrow \tan\left(\frac{B}{2}\right) - \frac{B}{2} = 0$



Exfoliation Problem

From $\tan\left(\frac{B}{2}\right) = \frac{B}{2}$, we can derive

$$\sin B = \frac{B}{\frac{B^2}{4} + 1}$$

$$\cos B = \frac{\frac{B^2}{4} - 1}{\frac{B^2}{4} + 1}$$

Exfoliation Problem

We substitute these expressions into the matrix H, and apply row operations, we obtain

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1-\frac{B^2}{4} & B & 1+\frac{B^2}{4} & 1+\frac{B^2}{4} \\ 0 & B & 1 & 0 \\ -B^2 & B(1-\frac{B^2}{4}) & 1+\frac{B^2}{4} & 0 \end{pmatrix}.$$

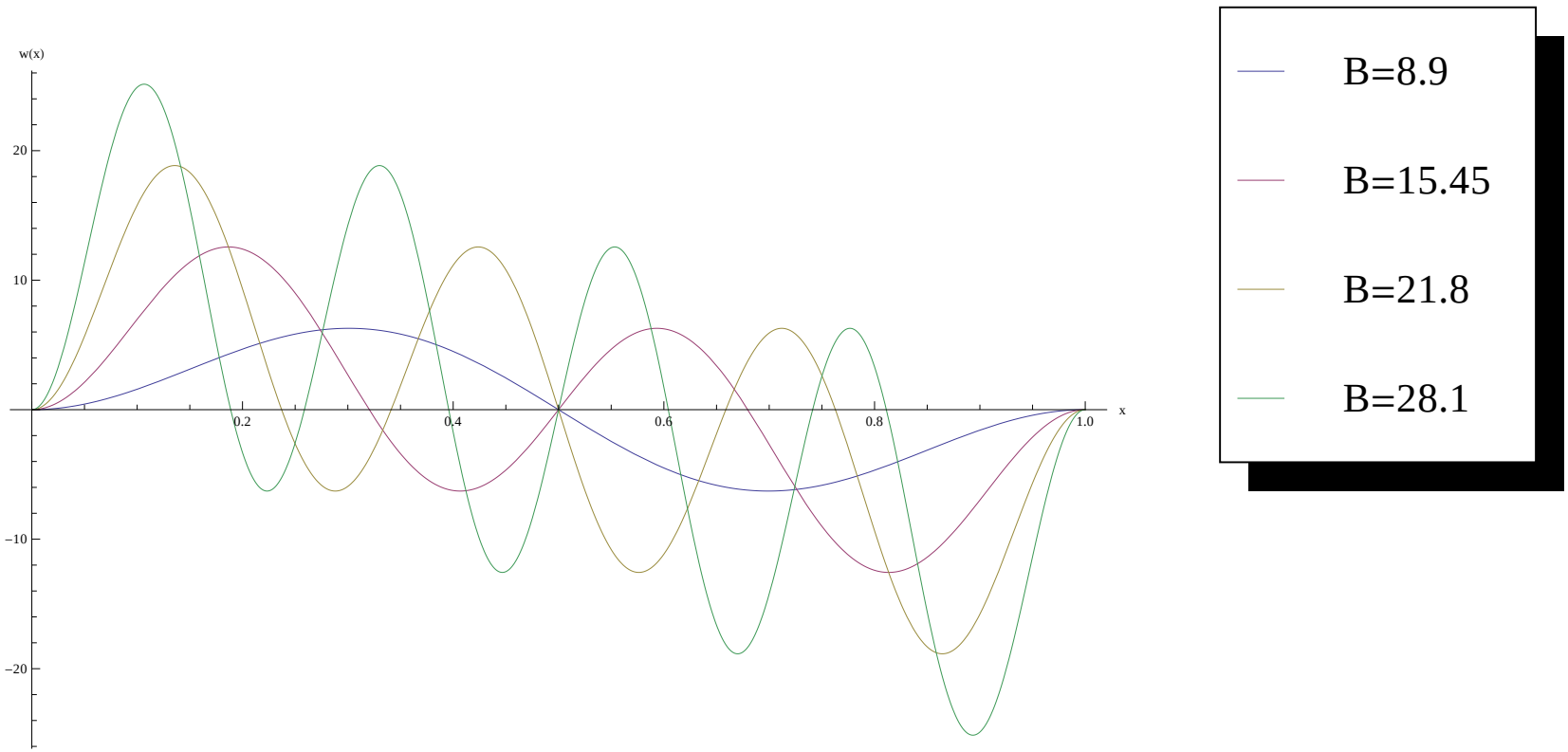
Solving the resulting matrix system gives

$$w(x) = D(-\frac{B}{2}\cos(Bx) + \sin(Bx) - Bx + \frac{B}{2}),$$

where D is an arbitrary constant.

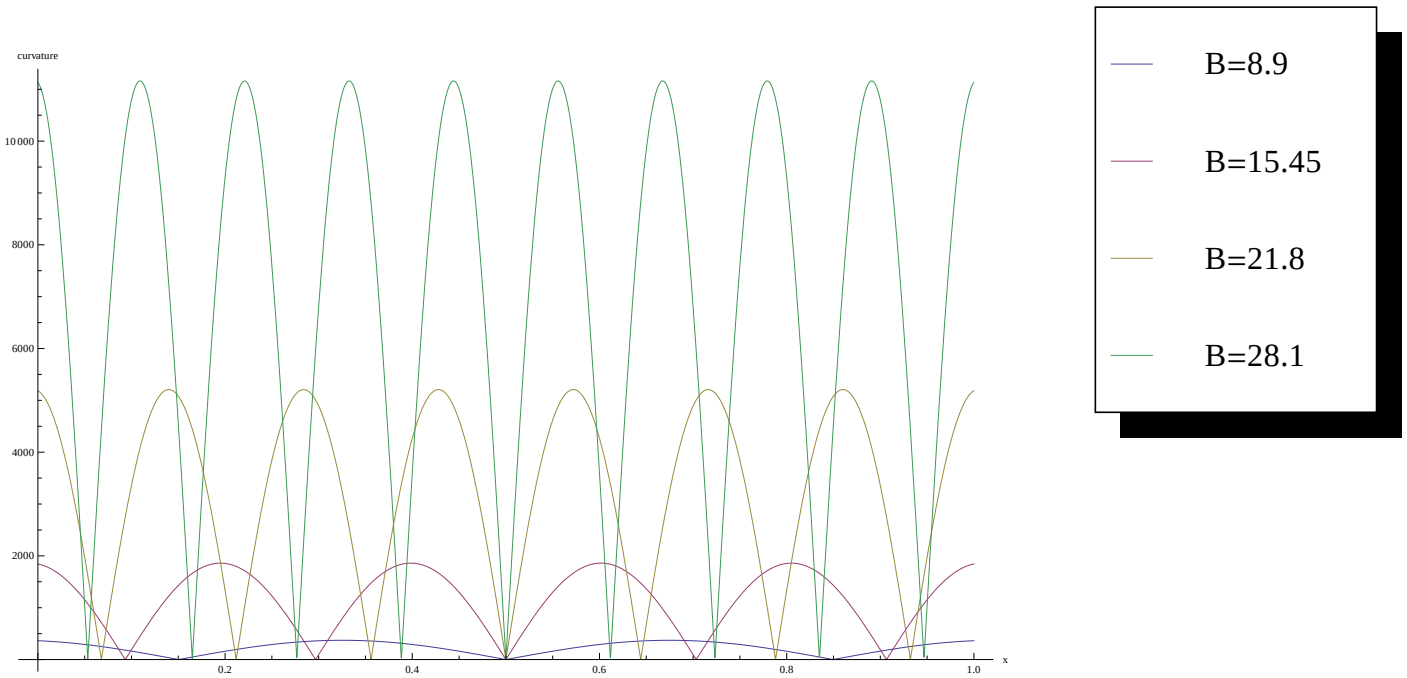
Exfoliation Problem

Plot of the beam deflection, for varying B values



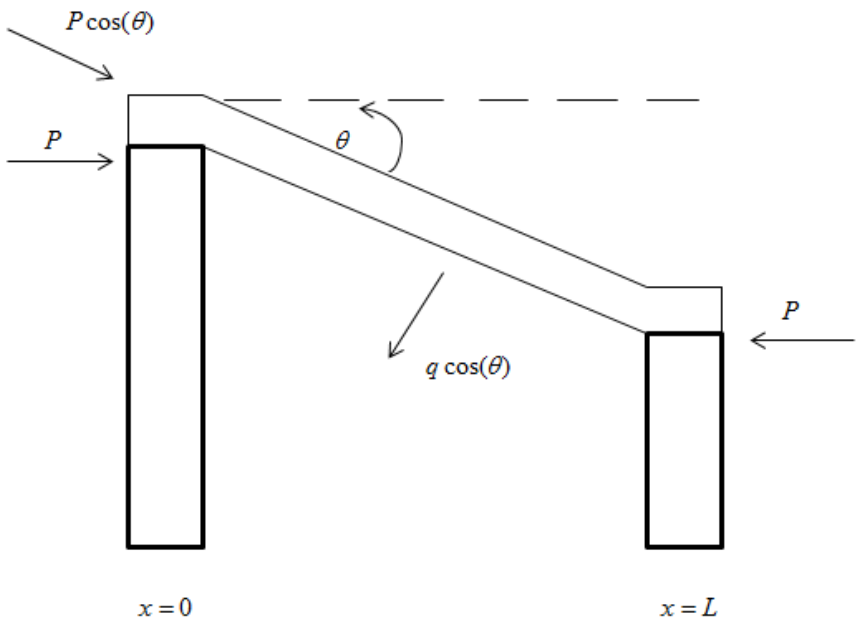
Exfoliation Problem

Plot of the beam curvature, for varying B values



Slanting Beam Problem

Blasted tunnels have roofs that seem to behave as beams; however, they are at an angle, say θ. We thus felt that examining how the roof in a platinum mine fractured would be equivalent to examining the slanted beam equation.



Slanting Beam Problem



Slanting Beam Problem

Once resolving the components of force, we deal with the following equation

$$EI\frac{d^4w}{dx^4} + P\cos\theta\frac{d^2w}{dx^2} = q\cos\theta.$$

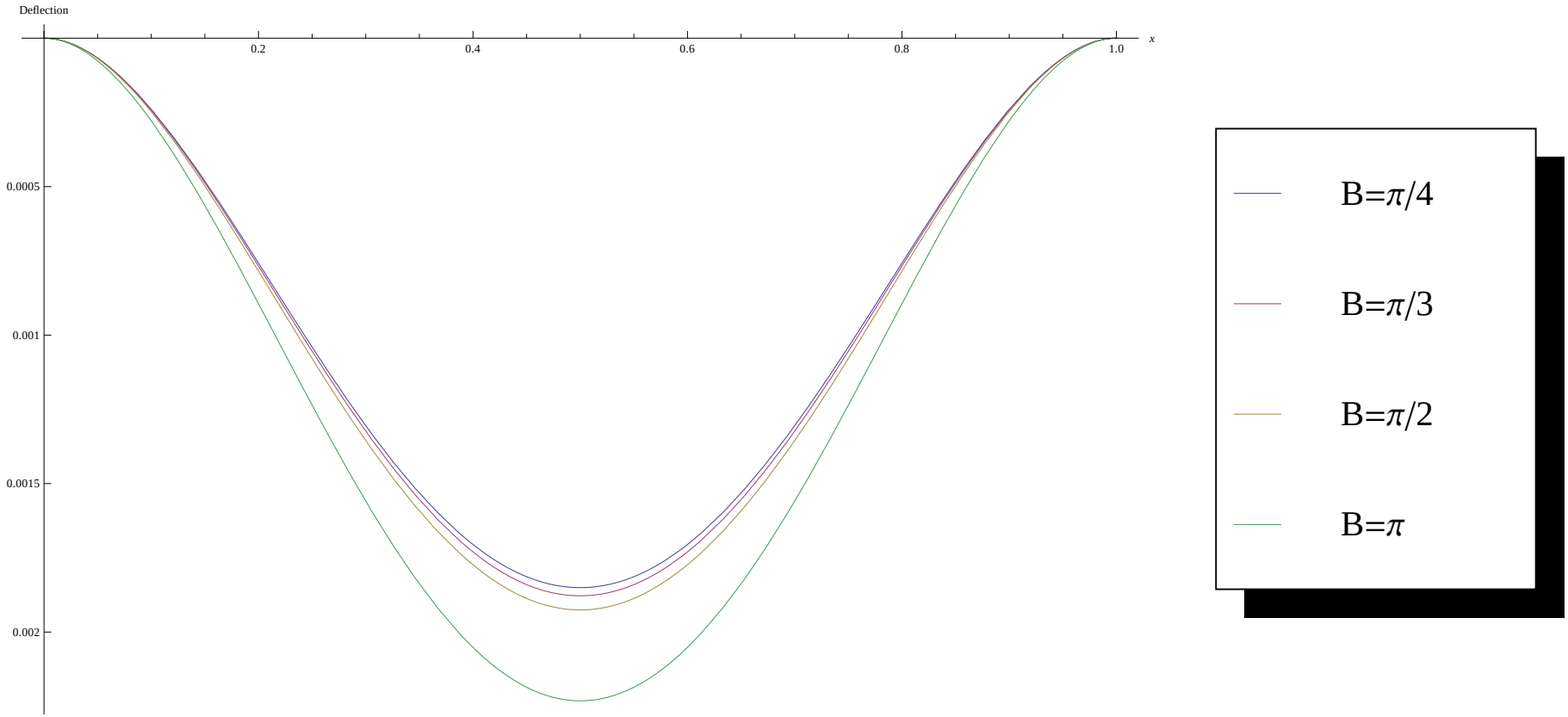
Upon non-dimensionalising this, we obtain

$$\frac{d^4w}{dx^4} + B^2\cos\theta\frac{d^2w}{dx^2} = \cos\theta.$$

This equation governs the slanting roof beams of interest.

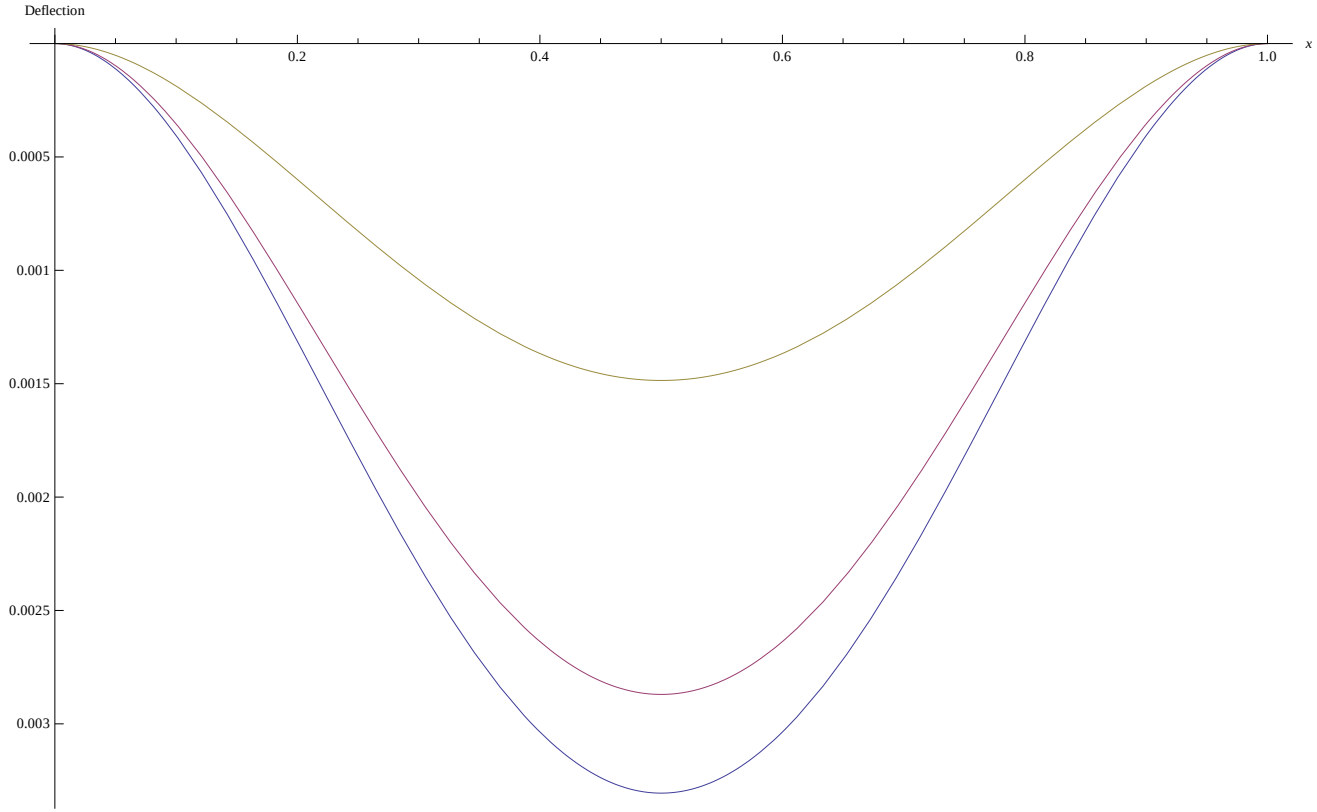
Slanting Beam Problem

The graphical representation of our deflection, for different Beam number values with constant θ, is given by the following plot:



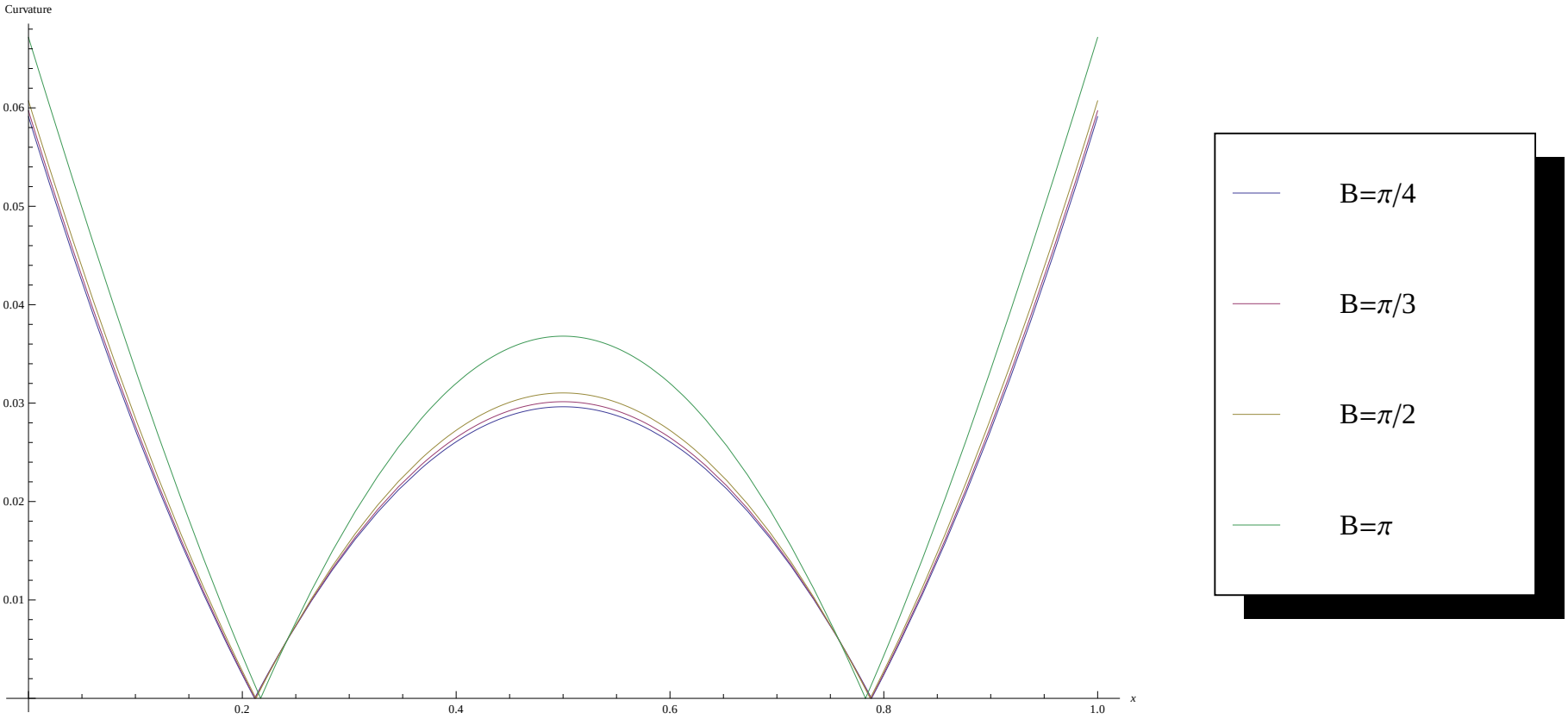
Slanting Beam Problem

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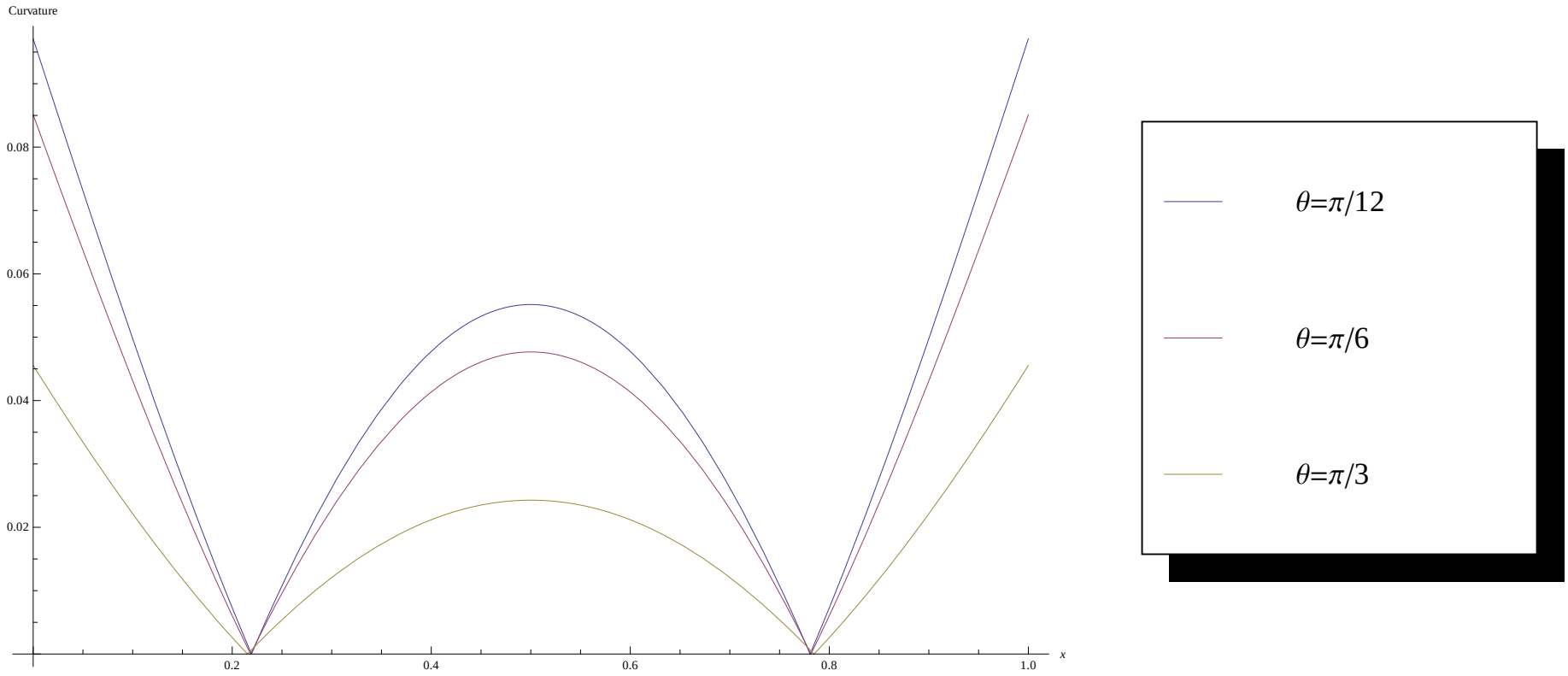
Slanting Beam Problem

The graphical representation of the curvature, for different Beam number values with constant θ, is given by the following plot:



Slanting Beam Problem

The graphical representation of the curvature, for different θ values with constant B, is given by the following plot:



Conclusions and Further Research

- We need to find a mathematical model to explain the formation and propagation of fractures
- For the exfoliation problem, we need to improve our model in order to find the arbitrary constant