FLOW OF A YIELD STRESS FLUID THROUGH A NOTCH

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Abstract

The flow of a yield stress liquid through a notch is examined. It is shown that the method employed to analyse Newtonian flows, namely using Bernoulli's streamline theory, is not appropriate for this type of fluid. The correction factors used by experimentalists to determine the flux are explained in terms of eddies and viscous heating.

1 Problem description

Consider a Newtonian fluid flowing over a rectangular cross-section weir. Sufficiently far upstream the fluid may be considered quiescent, so $\mathbf{u} = \mathbf{0}$. We denote the position of the free surface as $z = h_a$ At the upstream edge of the weir the height of the free surface can easily be measured, but the velocity is unknown. Following a free surface streamline, so $p = p_a$ we may apply Bernoulli's streamline theorem to obtain

$$u = \sqrt{2\rho g(h_a - h)} = \sqrt{2\rho g H} \tag{1}$$

where h is the height of the free surface above the weir. The flux over the weir is then determined by integrating the velocity, to find $Q \sim H^{3/2}$. Due

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to it's simplicity, involving a single height measurement, this is a standard method employed by engineers to measure flow rates. The problem posed at the Study Group was to extend this method to deal with yield stress fluids and verify the result against experimental data for flow through rectangular or triangular notches.

The current practice for measuring the flux through a notch relies on the Newtonian formula with the addition of three correction factors giving

$$Q = \frac{2}{3} C_d \sqrt{2g} (B + K_b)(H + K_h)^{3/2}$$

where B is the width of the notch and C_d, K_b and K_h are the introduced correction factors. The correction factors K_b and K_h purportedly represent viscous and surface tension effects respectively. Both K_b and C_d are observed to vary with the ratio of the width of the notch to that of the tank, B/W and the sign of K_b changes as B/W decreases. The correction factor K_h increases with viscosity η.

In the experiments carried out at the Cape Peninsula University of Technology (CPUT), fluid was pumped into the bottom of a tank and allowed to flow over the weir at the top. For a Newtonian fluid the results conformed to the standard result. For the yield stress fluid they did not and, of particular interest was the fact that the free surface near the notch rose above the quiescent free surface.

2 Conclusion?

The first and most important result of the Study Group deliberations was that this experiment could not work. The Bernoulli calculation relies on the presence of a free surface streamline. At a free surface the shear stress is zero (or at least negligible in this case) and so the free surface in the quiescent region is a solid plug. One cannot then follow a streamline from the quiescent region to the weir. None exist.

Experiments in the literature for flow of a yield stress fluid from a pipe into an expansion (and subsequently a contraction) show that the fluid will actually tunnel through the expansion region [1]. At the edges there will be a solid plug and between the plug and central flow region eddies may occur. For the present problem the inlet to the tank may be viewed as an expansion and the notch as a contraction. At the top of the tank the fluid is exposed to the air, so a free surface exists. The standard free surface condition is that the shear stress is zero and consequently a solid plug exists at the top of the tank. From these observations one may gain a simple picture of the flow in the tank. The fluid forms a tunnel joining the inlet pipe to the outlet notch. On either side there is a solid plug (there will also be a small central plug near the symmetry line within the flow). That such a tunnel exists is apparent from the experiments at CPUT where the fluid actually shoots higher than the free surface.

Two of the correction factors may also be explained with reference to the experimental observations:

- the variation of K_b with B/W is a result of two types of edge effect. Firstly, when $B/W \sim 1$ there will be an effect due to the fact that flow at the edge of the tank (where the shear is high) differs from that in the central region (where shear is low and the central plug may exist). As B/W decreases the edge of the tank becomes less important and the dominant effect comes from the contraction at the notch. Eddies will form on either side of the notch which were not present when the notch reached to the edge of the tank. The fact that there are two different mechanisms affecting the flow can explain why the sign of K_b changes as B/W decreases.

- C_d varies with B/W and the ratio of the height of the flow above the notch to that of the tank. Again this may be explained by eddies forming at the sides and below the notch.

In an attempt to extend the Bernoulli equation to the flow in question we viewed it as a form of energy equation, rather than coming from the standard inviscid Euler equations. In this case the obvious effect of viscosity is viscous heating. This requires knowledge of the flow, since the temperature rise is related to the shear rate. Our calculations showed that the effect of viscous heating was to introduce a term $(H + c\Delta T)^{3/2}$ to the flux expression and so we may identify K_h as $c\Delta T$, where ΔT is the temperature rise and c the heat capacity. This is verified by the fact that K_h increases with η and our calculations showed that $\Delta T \propto \eta$. However c turns out to be large (of the order 10^3) so a very small temperature rise can have a significant effect. In practice this means that extremely accurate temperature measurements would be required to calculate K_h. Note, that our deliberations led to a form of the *steady-flow energy equation*, [2], including the often-neglected viscous work term.

3 Conclusion

The correction factors C_d and K_b appear to be a result of eddy formation, which removes energy from the main flow. The factor K_h appears to be a result of viscous heating, not surface tension. In fact, given that the free surface must be a solid it is not even clear what the surface tension could be. The success of the standard method for measuring flux relies on the existence of a near surface streamline. This does not exist in the current experiment. Even so, it is possible to extend Bernoulli's streamline theorem to include viscous heating, however in practice the temperature rise would be so small as to make measurement virtually impossible.

It therefore appears that the current experimental setup is not an appropriate method to measure flow of a yield stress fluid.

References

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